Numerical Simulation of R-M Instability

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Summary. In order to capture shock waves and contact discontinuities in the field and easy to program with parallel computation a new algorithm is developed to solve the N-S equations for simulation of R-M instability problems. The method with group velocity control is used to suppress numerical oscillations, and an adaptive non-uniform mesh is used to get fine resolution. Numerical results for cylindrical shock-cylindrical interface interaction with a shock Mach number Ms=1.2 and Atwood number A=0.818, 0.961, 0.980 (the interior density of the interface/ outer density $\rho_1/\rho_2 = 10, 50, 100$, respectively), and for the planar shock-spherical interface interaction with Ms=1.2 and $\rho_1/\rho_2 = 14.28$are presented. The effect of Atwood number and multi-mode initial perturbation on the R-M instability are studied. Multi-collisions of the reflected shock with the interface is a main reason of nonlinear development of the interface instability and formation of the spike-bubble structures In simulation with double mode perturbation vortex merging and second instability are found. After second instability the small vortex structures near the interface produced. It is important factor for turbulent mixing.

1 Introduction

Consider a material interface between two different media. As an incident shock interacts with the material interface the interface becomes unstable due to shock acceleration. The small disturbances at the interface start to grow. This kind of instability is called as a Richtmyer-Meshkov instability(R-M instability). The R-M instability problems can be met in many important practical applications, for example, the propagation of sound boom in turbulent atmosphere, the Inertial Confinement Fusion(ICF), and the explosion of the supernova. The R-M instability is also a model problem for studying the physical mechanism of fluid motion from instability to turbulence. Many works have been done for studying R-M instability, but mainly for the case of interaction between plane shock and plane interface with perturbation, interaction between plane shock and cylindrical interface with single mode perturbation. In order to get numerical results of the R-M instability with high density ratio a new algorithm is developed in this paper, and it is used to simulate cylindrical shock-cylindrical material interface interaction and the planar shock-spherical interface interaction. The shock is going from the heavy gas through the interface to the light gas. The flow structures of shock refraction, reflection, interaction of the reflected shock with the material interface, and effect of Atwood number and the initial perturbation modes on R-M instability are investigated.

2 Numerical Method

A sixth order traditional difference approximation with group velocity control(GVC) is used to simulate the physical problem. Consider the convection term for x-direction in the N-S equations $\partial f/\partial x$ discretized with $F_j/\triangle$ for positive characteristics:

$$F_j^{(6,+)} = H_{j+1/2}^{(6,+)} - H_{j-1/2}^{(6,+)} \quad (1)$$

$$H_{j+1/2}^{(6,+)} = \frac{1}{2}[1 + SS(u_{j+1/2})]\overline{h}_{j+1/2}^{(6,+)} + \frac{1}{2}[1 - SS(u_{j+1/2})]h_{j+1/2}^{(6,0)} \quad (2)$$

$$\overline{h}_{j+1/2}^{(6,+)} = (1+\sigma)h_{j+1/2}^{(7,+)} - \sigma h_{j+1/2}^{(6,0)} \quad (3)$$

$$h_{j+1/2}^{(6,0)} = [f_{j+3} - f_{j-3} - 9(f_{j+2} - f_{j-2}) + 45(f_{j+1} - f_{j-1})]/60$$

$$h_{j+1/2}^{(7,+)} = h_{j+1/2}^{(6,0)} - \delta_x^6 f_j/140$$

$$\delta_x^6 f_j = f_{j+3} + f_{j-3} - 6(f_{j+2} + f_{j-2}) + 15(f_{j+1} - f_{j-1} - 20f_j$$

The upper index (k,i) shows that the scheme is kth order accurate, and it is symmetrical for case of i=0 and upwind biased for case i=+. The scheme with $h_{j+1/2}^{(7,+)}$is SLW, and a parameter $\sigma > 0$ is introduced to make the scheme MXD. The function $SS(u)$ is used to control the group velocity[1] .

3 Numerical results of R-M instability

3.1 Interaction of Cylindrical Shock with Cylindrical Interface

Consider R-M instability produced by interaction of a cylindrical shock with cylindrical material interface. The shock is going inward from the heavy gas with density ρ_1 into the light gas with density ρ_2 . This problem is solved by using the two dimensional compressible N-S equations in the cylindrical coordinate discretized with above presented method. The interface is tracked with time according to the maximal gradient of scale function g: $[gradg]_{max}$.

General characteristics of R-M instability

When the interface is perturbed with small amplitude the interface becomes unstable after collision of the incident shock with the interface. Now we are going to discuss the general characteristics of R-M instability with interaction between a cylindrical shock and a cylindrical material interface based on initial single mode perturbation. Consider a case with Ms=1.2 ,Re=50000,$\rho_1/\rho_1 = 10$. The initial cylindrical surface with perturbation is defined as $r_0 = 1 + \alpha\cos(n\varphi)$,where $\alpha = 0.033$, $n = 12$, and the length scale is normalized with the radius of initial interface without perturbation. Fig.1 shows the density contours at different times. We see that at t=0.10 the

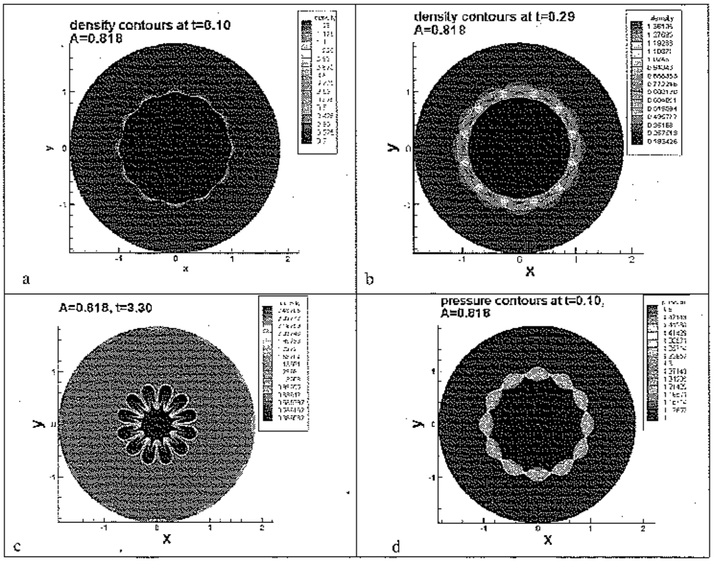


Fig. 1. Density and Pressure contours: a. Density contours at t=0.10; b. Density contours at t=.0.29 c. Density contours at t=3.30; d. Pressure contours at t=0.10.

discontinuity bifurcates into reflected rarefaction wave and refracted shock after collision with interface. The discontinuity bifurcation can be seen clearly from Fig.1d for pressure contours at t=0.10. The perturbation starts to grow linearly, and the interface starts to deform. Existence of perturbation on the interface leads to non- parallelism of the pressure gradient $[gradp]$ with the density gradient $[grad\rho]$ which produces the vorticity near the interface. Further development nonlinear growth of perturbation leads to change the phase of the perturbed surface(see Fig.1b). The transmitted shock reaches the center and reflects. Collision of this reflected shock with the interface increases nonlinear development of the perturbation. After multiple collision of the reflected shock with interface the spike-bubble structures are formed(Fig.1c). The spike is a portion of heavy gas penetrating into the light gas, and the bubble is a light gas penetrating into the heavy gas. fig1

Effect of Atwood number on R-M instability

The cases Ms=1.2Re=50000 with different Atwood number ($A = [\rho_1 - \rho_2]/[\rho_1 + \rho_2]$) are computed in order to study the effect of density ratio on R-M instability. The initial cylindrical surface with perturbation is defined as above. The variation of pressure with the time at the cylinder center for different Atwood number is given in Fig.2a-c. From it we can see that when the transmitted shock approaches the center the center pressure increases, and then reflected shock is formed. This reflected shock is going outward,

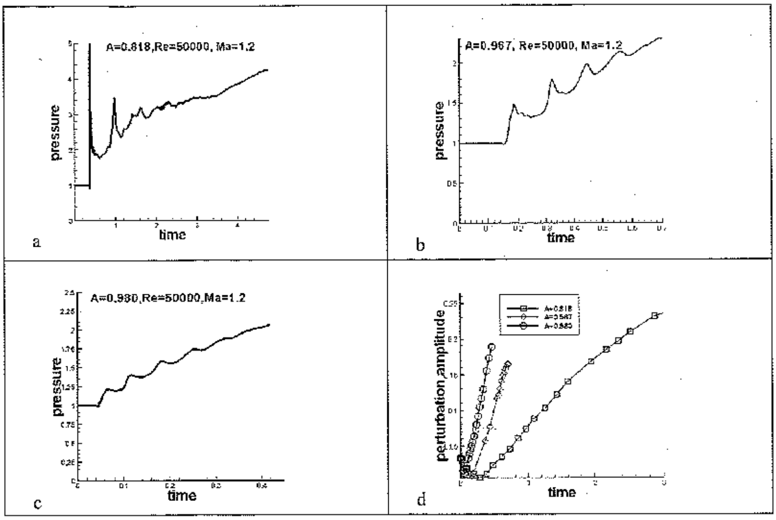
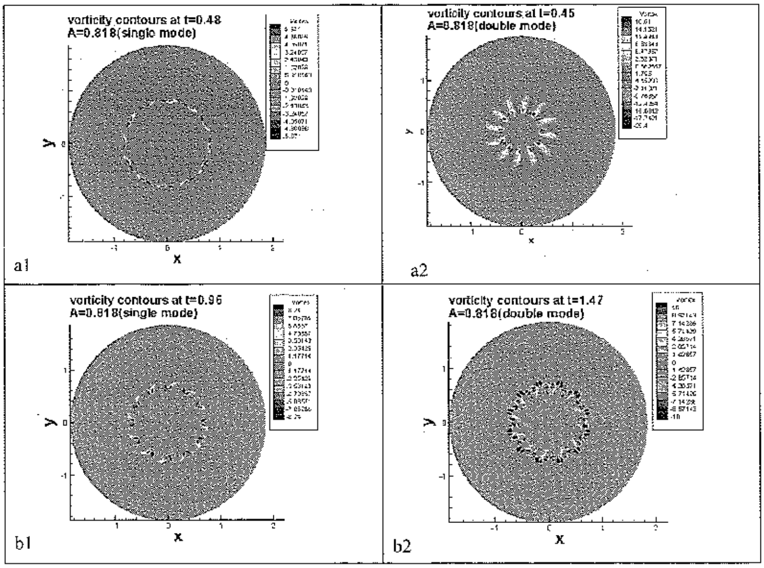


Fig. 2. Variation of pressure at the cylinder center and perturbation amplitude with different Atwood numbers (A: a. A=0.818;b. A=0.967; c. A=0.980; d: perturbation amplitude).

and the gas in central region starts to rarefy with pressure decreasing. This outward going shock in light media will collide the interface behind which the gas is heavy. After interaction an inward going reflected shock is formed. The inward going shock will reach the center, then it reflects, and it collides with the interface again.From Fig.2a we can see that there are four times of this kind of collisions for the case A=0.818. After the collisions the pressure at the center increases fast. For the case of A=0.967 and 0.980 there are five six times of this kind of collisions respectively, but the pressure increases much less than the case of A=0.818 (see Fig. 2a-c). It means that pressure increase decays with Atwood number increasing. In Fig.2d the variation of perturbation amplitude with the time for different Atwood number is given. It can be seen that at first stage before the phase change of the interface the perturbation amplitude decreases, after phase changing it is increases. The perturbation amplitude with large Atwood number grows much faster because of faster sound speed in the light gas.

Effect of initial perturbation on R-M instability

The cases Ms=1.2, Re=50000, ρ_1/ρ_2=10 with both single and double initial perturbation modes at interface are computed. The initial perturbed interface for single mode is as above. The double mode perturbed interface is given as $r_0 = 1 + a[cos(n_1\varphi) + cos(n_2\varphi)]$ where $n_1 = 12$,$n_2 = 60$, $a = 0.033$. The vorticity contours with two different initial perturbation mode are given in Fig.3. Comparing the results with different initial perturbation modes we



a. Single mode b. Double mode

Fig. 3. Vorticity contours with two initial mode perturbations at different time.

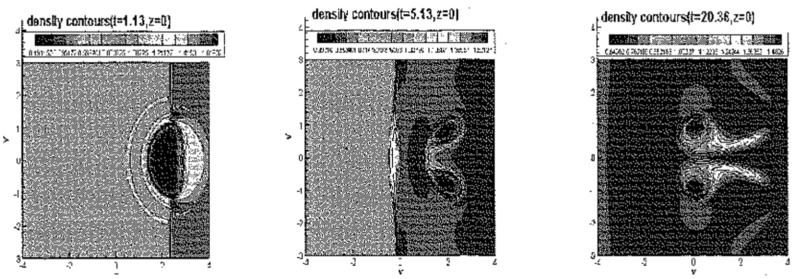


Fig. 4. Density contours on section z=0 for different time (t=1.33, 5.13, 20.36).

see that at first stage the initial subharmonic is appended on the basic mode perturbation, and vorticity is much increased . With nonlinear growth of perturbation secondary instability occurs, and vortex merging can be seen in Fig. 3a2. With further development of perturbation in Fig.3b2 we can see that small vortex structures near the interface are formed. This is important factor for turbulence development.

3.2 Interaction of Planar Shock with Spherical Interface

The numerical method presented above is used to solve the 3-D compressible N-S equations in the Cartesian coordinate for simulating interaction of a planar shock with spherical interface. The Mach number is and the density

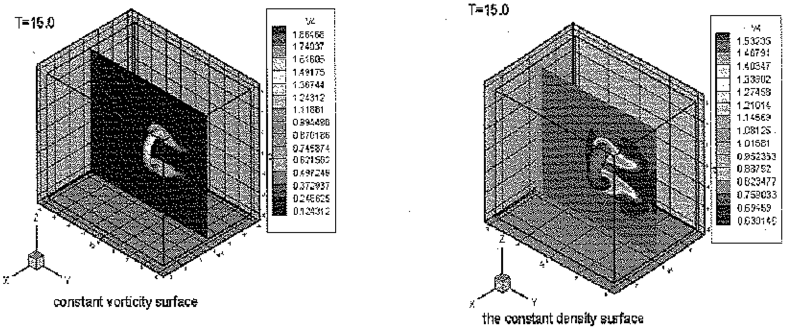


Fig. 5. Three Dimension numerical results.

ration is . Figure 4 shows density contours on section z=0 for different times (t=1.33, 5.13, 20.36). From Fig.4a it can be seen that as the shock sweeps over the interface the shape of the interface at the upstream face changes due to compression by the shock and the interfacial instability. From here also we can see that first and second transmitted waves at the left are followed by the interface and the reflect waves are going to the right. At the same time due to the shock going from the heavy gas to the light gas, the gas inside the interface has a higher wave speed, and the effect of the interfacial instability, the incident shock is deformed. This phenomenon can be seen from the pressure contours (there is not presented). From Fig.4b it can be seen that after interaction due to interface instability the heavy air jet with conical shear layer is formed, then this jet impinges on the downstream interface and pierces it (see Fig. 4c). During the time of interaction the vorticity is produced due to non-parallelism of the gradients for pressure and density. Because the shock is incident from the right to the left and the light gas is relatively easier to accelerate, clockwise vorticity is produced at the top and anticlockwise vorticity is at the bottom of the interface. With further development of flow structure a vortex ring is formed. Fig.5 shows the constant vorticity surface and vorticity contours on section z=0 at t=15.0. This vortex ring for the sphere-interface is more distinct than the case for the cylinder interface. From numerical results we can also see that most of the vorticity in the flow is concentrated in the vortex ring and along the conical shear layer at the boundary of the heavy air jet. Obtained numerical results agree well with experiment results [2].

Acknowledgements

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References

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