Numerical investigation of toroidal shock waves focusing

H.H. Teng[1], Z. Jiang[1], Z.Y. Han[2], S.H.R. Hosseini[3], and K. Takayama[3]

[1] *LHD, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080, China*
[2] *Dept. of Modern Mechanics, University of Science and Technology of China, Hefei, Anhui 230026, China*
[3] *Shock Wave Research Center, Institute of Fluid Science, Tohoku University, Sendai 980-8577, Japan*

Abstract. In this paper, focusing of a toroidal shock wave propagating from a shock tube of annular cross-section into a cylindrical chamber was investigated numerically with the dispersion-controlled scheme. For CFD validation, the numerical code was first applied to calculate both viscous and inviscid flows at a low Mach number of 1.5, which was compared with the experiment results and got better consistency. Then the validated code was used to calculate several cases for high Mach numbers. From the result, several major factors that influent the flow, such as the Mach number and the viscosity, were analyzed detailedly and along with the high Mach number some unusual flow structure was observed and explained in detail.

1 Introduction

Shock wave focusing has been a fundamental topic in shock wave research for many years. Especially, in axisymmetric cases, the cylindrical shock wave focuses at a point, instead of a planar shock focusing in a plane. The phenomenon results in a small region of high temperature and pressure around a certain point, which leads to vortex generation, turbulence mixing and new pattern of shock wave reflection. So, the relevant research is not only meaningful for fundamental study, but also has wide applications in industry and medicine. Sod [1] studied the focusing of cylindrical shock waves by numerical simulation as early as in 1977. Takayama et al. [2] investigated the stability of cylindrical shock waves in air experimentally in 1987. In 1989, Groenig [3] had reviewed shock waves focusing and pointed out the focusing of cylindrical shock waves to be one of the research topics worth trying in future. Jiang et al. [4] discussed the focusing of the toroidal shock wave discharged from a coaxial annual shock tube. Hamid et al. [5] investigated the focusing of the toroidal shock wave in the shock tube experimentally with Mach number 1.5. Because of the difficulty of generating high quality toroidal shock waves, there are no experimental results of higher Mach number focusing published, which is more complex and practical.

This paper is devoted to numerical simulation of the toroidal shock wave focusing in the cylindrical shock tube, including both viscous and inviscid flow. For numerical validation, the dispersion-controlled scheme [6] is used to solve both Euler equation and Navier-Stokes equation for inviscid and viscous axisymmetric flow respectively at a low Mach number 1.5, which the experiment has been accomplished. Then some cases for high Mach numbers are simulated and through the analysis of numerical results, the focusing process is investigated and also the influence of the Mach number and the viscosity are discussed.

2 Problem description

The model investigated here is a toroidal shock wave propagating from a shock tube of annular cross-section into a cylindrical chamber, as shown in Fig 1. The shock wave will diffract over the bent corner first and then implode toward the axis of symmetry, and finally focus at a certain point on it. The focusing point is singular point and without regard to the viscosity pressure and temperature on it will become infinite. In practice the effect of the viscosity and real gas decreases the value of pressure and temperature to being definite. When carrying on numerical simulation, because the grid can not be infinitesimal, the results is also definite, which is usually lower than actual results. But the results of numerical simulation, which show the process of the focusing and interaction of complex wave system, still play an important role in research and application of shock waves focusing.

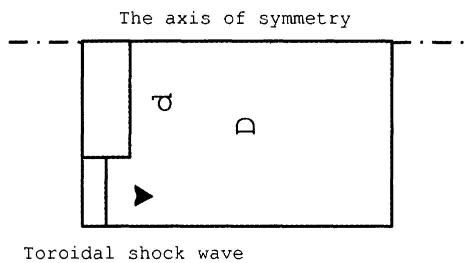

Fig. 1. Computation domain with D:d=5:4

3 Numerical methods

In this numerical investigation, Euler equation and Navier-Stokes equation are solved separately in the axisymmetric flow field. In the inviscid flow, the dispersion-controlled scheme is used to solve the Euler equation, which has been demonstrated to be capable of predicting complex flow fields, including shock wave interactions, without non-physical oscillations and any need for artificial viscosity. For the viscous flow, the dissipation terms, which introduce the effect of the viscosity, are discretized with the simple central difference. As the initial condition, the toroidal shock wave is moving and up to enter the cylindrical shock tube from the annular shock tube. As the boundary condition the reflection and slip boundary conditions are applied to the solid wall when the Euler equation is solved, while no-slip and adiabatic conditions are used on the solid wall when the Navier-Stokes equation is solved.

4 Solution verification

For the numerical validation, the density contour is integrated into the 'numerical holographic interferogram' [7] and compared with the experimental holographic interferogram directly. In Fig. 2, the experimental result(a) [5], viscous numerical result(b) and inviscid numerical result(c) are shown when focusing. It can be found out that not only the number of fringes but also the distribution of individual fringe are very close, so the

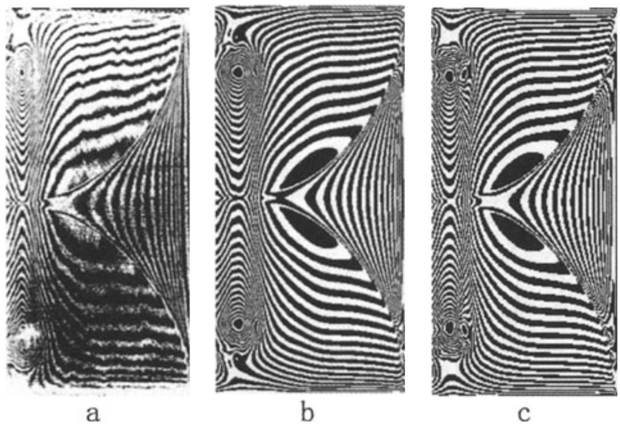

Fig. 2. Holographic interferograms when focusing for $M_s = 1.5$

calculation results are credible and the code can be used to investigate the shock wave focusing. Through comparing the viscous and inviscid numerical result, it's found that near the wall the inviscid result is not so accurate. If taking into account of the viscosity, the departure will be improved.

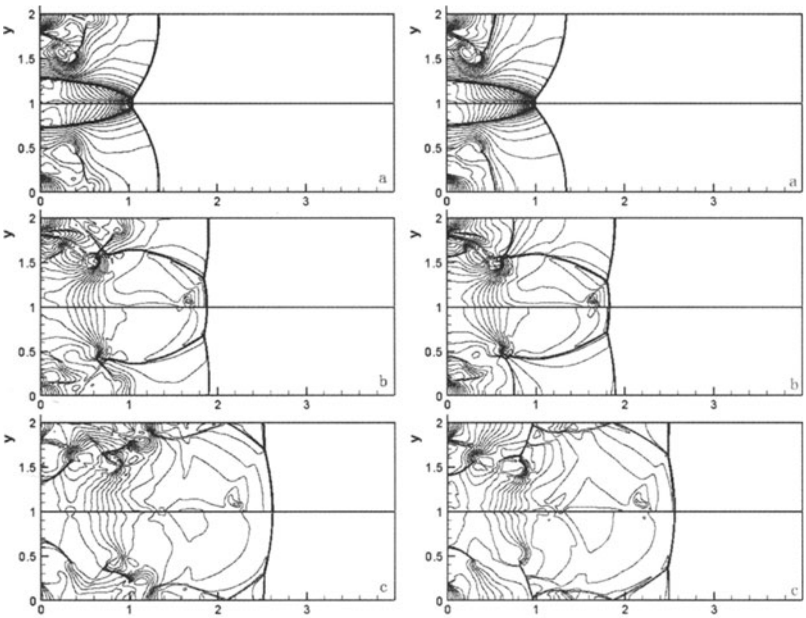

Fig. 3. Isobars(lower half) and isopycnics (upper half) for viscous and inviscid flow ($M_s = 2.0$)

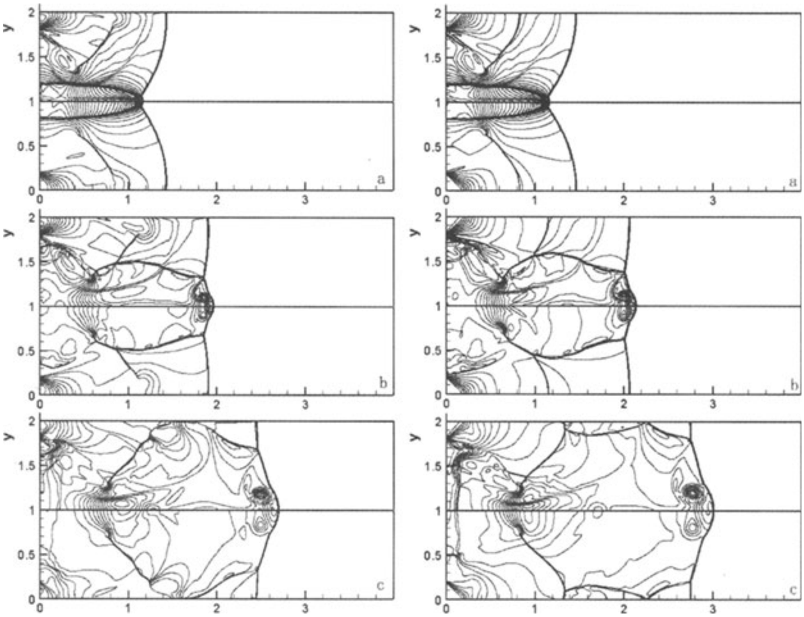

Fig. 4. Isobars(lower half) and isopycnics (upper half) for viscous and inviscid flow ($M_s = 3.0$)

5 Results and discussion

5.1 Case 1: $M_s = 2.0$

The case for a shock Mach number of $M_s = 2.0$ is simulated and shown in Fig. 3, which viscous results are on the left column from Fig.3a to c and inviscid ones on the right. In every frame of the figure, isobars are shown on the lower half and isopycnics are shown on the upper half . The same display is also used in other cases. The toroidal shock wave, which is discharged from a annular shock tube to a co-axial cylindrical shock tube, diffracts and implodes toward the axis of symmetry. At first the reflecting pattern on the axis is regular reflection, with the lapse of time it quickly transits to Mach reflection as show in Fig.3a and the pressure behind the shock wave decreases rapidly. From Fig.3a to b, the Mach stem propagates forward and the triple point moves away from the axis of symmetry. In the axisymmetric case, the triple point is actually a ring and becomes bigger and bigger. So in Fig.3c, the shock wave surface gradually becomes a plane which is mainly the Mach stem. From the figure, the slip line derived from the Mach stem can be observed by comparing the isobars and isopycnics. Through comparing the left column and the right, differences between viscous and inviscid flow can be found out easily. As shown in Fig. 3, the flow structure taking into account the effect of viscosity on the left is similar to that inviscid one on the right approximately. But near the wall of the shock tube and the entry plane of the toroidal shock wave, the shock wave and contact surface become illegible.

5.2 Case 2: $M_s = 3.0$

In this case, the flow changes induced by better focusing effect on the axis of symmetry. As shown in Fig. 4, instead of decaying back to plane like the case $M_s = 2.0$, it can be found that the Mach stem is further accelerated near the axis so that the curved Mach stem develops. Behind the curved Mach stem, vortex and the second contact surface can be observed, which is peculiar for higher Mach number. To investigate these phenomenons deeply, higher Mach number cases are carried out in the next section.

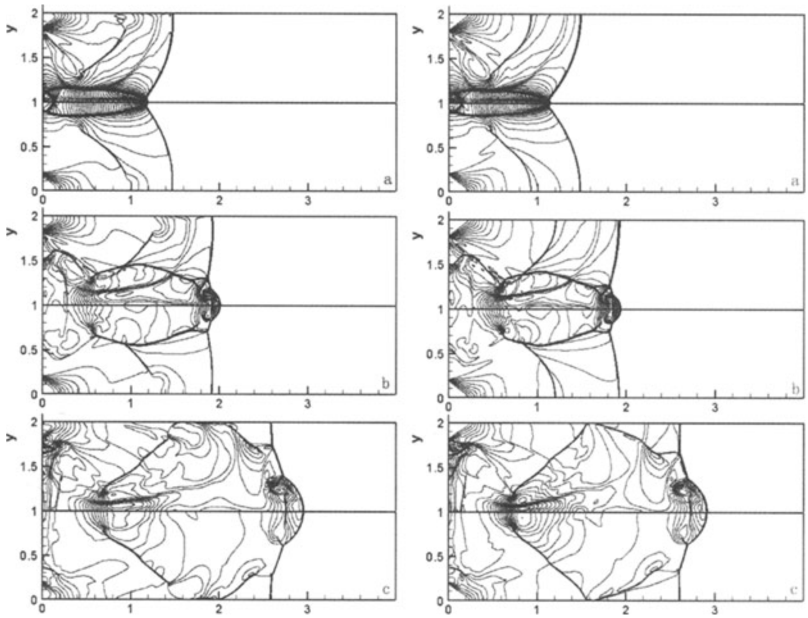

Fig. 5. Isobars(lower half) and isopycnics (upper half) for viscous and inviscid flow ($M_s = 4.0$)

5.3 Case 3: $M_s = 4.0$

In this case, numerical results for $M_s = 4.0$ are shown in Fig. 5. The peculiar phenomenons along with the higher Mach number become more clear. The Mach stem is curved forward nearly to be spherical around the axis of symmetry, and the second triple point and contact surface are generating at a certain point of the first one. So the shock wave front surface is divided into three sectors: the outside original diffracting shock wave, the middle first Mach stem and the inside second Mach stem. This kind of reflection, with the second Mach stem spherical, is called spherical double Mach reflection. Behind the inside Mach stem, there are very complex interactions between shock wave, vortex, contact surface and perhaps jet. From the Fig. 5 vortex and another short shock wave generation can be observed .

The pressure value of every point on the axis of symmetry is checked for each time step and the maximum ones are recorded, as shown in Fig . 6. It can be found out that the maximum pressure of the focusing point, which is also the peak pressure of the axis of symmetry, mainly depends on the incident Mach number and the focusing effect can be improved rapidly with the incident shock wave Mach number increasing. The viscosity influences the result slightly, which decrease the focusing effect. Interestingly, we find

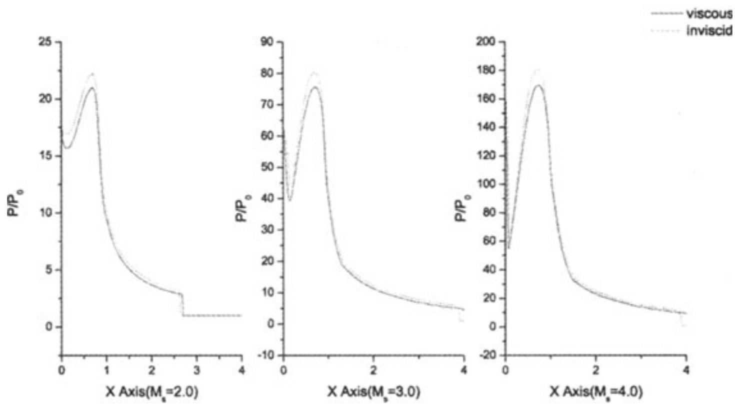

Fig. 6. Historic maximum pressure of every point on the centerline

there are two peak values. One is on the back wall and the other is not the first arriving point of diffracting shock waves but after it.

6 Conclusions

From the numerical investigation, some results are summarized as follows: Being different from low Mach number focusing, in which shock wave front decays back to planar after focusing, the shock wave front surface around the axis of symmetry is curved forward and generate spherical double Mach reflection. High incident Mach number also induces some complicated phenomenons and generate two peak focusing pressure. When taking the flow viscosity into account, the flow structure appears similar to the inviscid one but the dissipation induced by viscous terms weakens the effect of shock wave focusing slightly.

References

1. G.A. Sod: A numerical study of a converging cylindrical shock. J.Fliud Mech. **83**, 795 (1977)
2. K. Takayama, H. Kieine and G. Groenig: An experimental investigation of the stability cylindrical shock wave in air. Exp. in Fluid. **5**, 315 (1987)
3. H. Groenig: 'Past, present and future of shock wave focusing research'. In: *Proceeding of the Int. Workshop on Shock Wave Focusing,Sendai,Japan. 1*, (1989)
4. Z. Jiang and K. Takayama: Reflection and focusing of toroidal shock waves from coaxial annular shock tubes. Computers Fluids. **27**, 553 (1998)
5. S. Hamid, R. Hosseini and K. Takayama: 'Study of shock wave focusing and reflection over symmetrical axis of a compact vertical co-axial diaphragmless shock tube'. In: *ISSW23*, (2001)
6. Z. Jiang, K. Takayama and Y.S. Chen: Dispersion conditions for on-oscillatory shock capturing schemes and its applications. CFD Journal. **4**, 137 (1995)
7. Z. Jiang and K. Takayama: An Investigation into the Validation of Numerical Solution of Complex Flowfields. J.Comput.Phys. **151**, 479 (1999)