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Multiscale Method for Turbulent-Flow Computations

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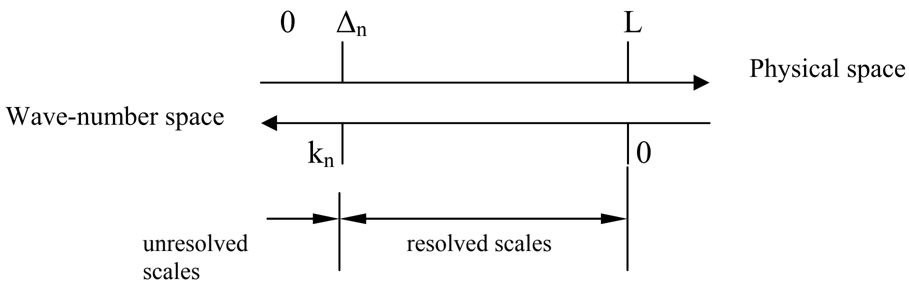
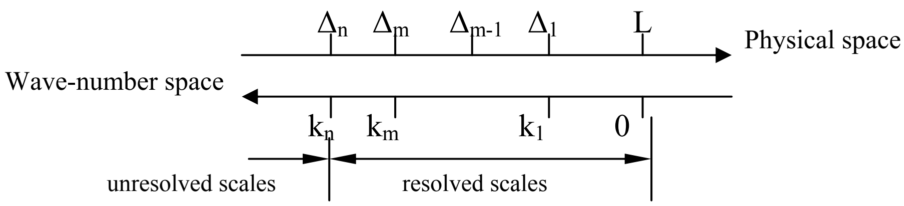
A main method of predicting turbulent flows is to solve LES equations, which was called traditional LES method. The traditional LES method solves the motions of large eddies of size larger than filtering scale Δ_n while modeling unresolved scales less than Δ_n (see Fig.1). Hughes et.al. argued that many shortcomings of the traditional LES approaches were associated with their inabilities to successfully differentiate between large and small scales. One may guess that a priori scale-separation would be better, because it can predict scale-interaction well compared with posteriori scale-separation. To this end, a multi-scale method was suggested to perform scale-separation ab initio.



a)



b)

Fig.1 A sketch of traditional LES (*a*) *and multiscale Method* (*b*)

The primary contents of our multiscale method are: (i) A space average is used to differentiate scale. (ii) The basic equations include the large scale equations and fluctuation equations. The basic unknown quantities for an incompressible turbulent flow are U_{1i}, P_1, ($U_{mi} - U_{m-1,i}$) and ($P_m - P_{m-1}$), i=1,2,3 and m=2,…,n. where

$$U_{mi} = (\prod_{l} \Delta x_{ml})^{-1} \int u_i dv, \quad V_m = \prod_{l}^{3} \Delta x_{ml} \quad p_m = (\prod_{l} \Delta x_{ml})^{-1} \int p dv \ \text{(m=1, 2,…, n)} \tag{1}$$

(iii) The large-scale equations and fluctuation equations are coupled through turbulent stress terms. The large-scale equations are

$$\frac{\partial U_{1i}}{\partial x_i}=0 \tag{2.a}$$

$$\frac{\partial U_{1i}}{\partial t}+U_{1j}\frac{\partial U_{1i}}{\partial x_j}=-\frac{\partial p_1}{\partial x_i}+\frac{1}{R_e}\frac{\partial^2 U_{1i}}{\partial x_i \partial x_i}-\frac{1}{\mathrm{V}_1}\int (U_{nj}-U_{1j})\frac{\partial}{\partial x_j}(U_{nj}-U_{1i})dv \qquad \text{(i=1,2,3)} \tag{2.b}$$

The middle m-levels fluctuation equations (m≥2, m≤n-1) are

$$\frac{\partial}{\partial x_i}(U_{mi}-U_{m-1,i})=0 \tag{3.a}$$

$$\begin{aligned}&\frac{\partial}{\partial t}(U_{mi}-U_{m-1,i})+(U_{mj}-U_{m-1,j})\frac{\partial}{\partial x_j}(U_{mi}-U_{m-1,i})=-\frac{\partial p_1}{\partial x_i}+\\&\frac{1}{R_e}\frac{\partial^2}{\partial x_i \partial x_i}(U_{mi}-U_{m-1,i})-U_{m-1,j}\frac{\partial}{\partial x_j}(U_{mi}-U_{m-1,i})-(U_{mj}-U_{m-1,j})\frac{\partial U_{m-1,i}}{\partial x_j} \qquad \text{(i=1,2,3)}\\&-\frac{1}{V_m}\int (U_{nj}-U_{mj})\frac{\partial}{\partial x_j}(U_{ni}-U_{mi})dv+\frac{1}{V_{m-1}}\int (U_{nj}-U_{m-1,j})\frac{\partial}{\partial x_j}(U_{ni}-U_{m-1,i})dv\end{aligned} \tag{3.b}$$

and the last n-level fluctuation equations are

$$\frac{\partial}{\partial x_i}(U_{ni}-U_{n-1,i})=0 \tag{4.a}$$

$$\begin{aligned}&\frac{\partial}{\partial t}(U_{ni}-U_{n-1,i})+(U_{nj}-U_{n-1,j})\frac{\partial}{\partial x_j}(U_{ni}-U_{n-1,i})=-\frac{\partial}{\partial x_i}(p_n-p_{n-1})+\\&\frac{1}{R_e}\frac{\partial^2}{\partial x_i \partial x_i}(U_{ni}-U_{n-1,i})-U_{n-1,j}\frac{\partial}{\partial x_j}(U_{ni}-U_{n-1,i})-(U_{nj}-U_{n-1,j})\frac{\partial U_{n-1,i}}{\partial x_j}\\&+\frac{1}{V_{m-1}}\int (U_{nj}-U_{n-1,j})\frac{\partial}{\partial x_j}(U_{ni}-U_{n-1,i})dv-\frac{1}{12}\sum_j\sum_k\frac{\partial U_{nk}}{\partial x_j}\frac{\partial^2 U_{ni}}{\partial x_k \partial x_j}\Delta x_{nj}^2\end{aligned} \tag{4.b}$$

the conditions of the velocities at the boundary walls are

$$U_{mi}=0 \quad \text{(i=1,2,3; m=1,2,…,n)} \tag{5}$$

iv) Some discussions: It should be noted that these turbulent stress formulae in above equations do not contain any empirical relations or constants. The action of unresolved scales less than Δ_n on the n-level fluctuation motions is modeled mainly by the last term of the right hand side of the equations (4b). This term also may be substituted by an empirical subgrid scale model. It should be noted that there is no action of the unsolved scales on the resolved scale ranging from Δ_{n-1} to L. For the case of 2-D shear turbulent flow, this turbulent stress reduces to well-known one of Prandtl mixing length theory. If the y-direction is the normal of the shear flow, the Prandtl mixing length l_p equals to $\frac{1}{2\sqrt{3}}\Delta y^2$. The mixing length l_p, as we know, is an empirical length, but the grid spacing Δy is a definite one. The molecular viscous terms in the equations (2.b) and (3.b) are much smaller than the turbulent stress terms and then they should be neglected. Retaining them is to satisfy $U_{mi}=0$ (i=1,2,3; m=1,2,…,n), i.e. no slid conditions at the walls.

From here we know that there exist obvious difference between the multiscale model and traditional calculation.
The nonlinear dynamics of the m-level fluctuation motions are governed mainly by their interactions with slightly larger scales of (m-1)-level and slightly smaller scales of (m+1)-level have a little effect.

v) Approximate closed differential equations for the large scales larger than Δm are as follows:

$$\frac{\partial U_{mi}}{\partial x_i} = 0 \tag{5.1}$$

$$\frac{\partial U_{mi}}{\partial t} + U_{mj}\frac{\partial U_{mi}}{\partial x_j} = -\frac{\partial p_m}{\partial x_j} + \frac{1}{R_e}\frac{\partial^2 U_{mi}}{\partial x_i \partial x_j} - \frac{1}{12}\sum_j \sum_k \frac{\partial U_{mk}}{\partial x_k}\frac{\partial^2 U_{mi}}{\partial x_k \partial x_j}\Delta x_{mj}^2 \tag{5.2}$$

vi) We use the multiscale equations of n=2, i.e., the large and small scale (LSS) equations, to simulate 3-D evolutions of a channel flow and a planar mixing layer flow. Some interesting results are given. Such as, for both cases of the channel and planar mixing layer flows, the average values of the velocity, fluctuation velocity and turbulent stress given by the multiscale simulations are consistent with those by the NS computations. The reverse-transfer process of the energy from the small scales to the large ones were brought to light in computations of LSS equations. For the case of planar mixing layer flow, the maximum fluctuation velocity, maximum turbulent stress and their twice abrupt-increase processes explored by multiscale computations are not captured in the NS calculations, see Fig.2. The twice abrupt-increase of both the maximum fluctuation velocity and maximum turbulent stress are corresponding to the “burst” phenomena in the transition flow. In addition, a wavelike increase of the momentum thickness of the mixing layer flow was shown by NS computations, see Fig.3, its cause is not clear in the past. The calculations for LSS equations show that a direct-reverse transfer of the energy between the large and small scales results in the wavelike increase of the momentum thickness. These new results should be gains of the multiscale, i.e. a priori scale-separation method. The results of multiscale calculation include both normal data of the flow field and interactions between different resolved scales that play a key role in flow-evolution.

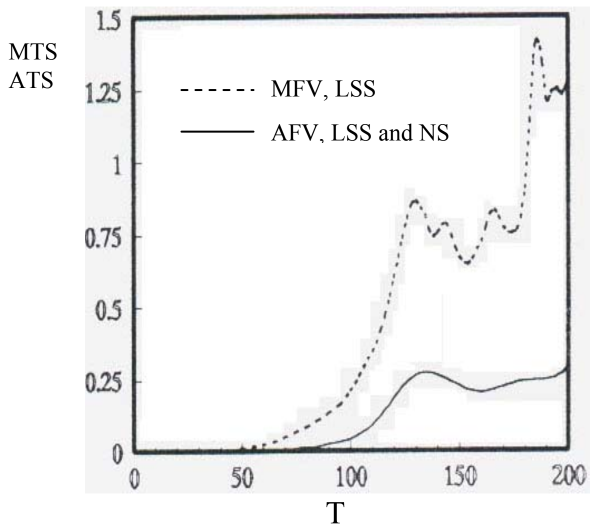


Fig.2 Variations of average fluctuation velocity (AFV) and maximum fluctuation velocity (MFV) of planar mixing layer flow with time T

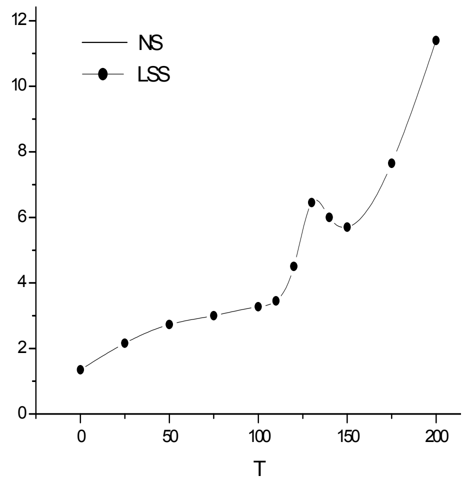


Fig.3 Variations of dimensionless average momentum thickness of planar mixing layer with time T.