Measurement and calculation of vibrational temperature behind strong shock waves

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Abstract. In the present paperthe measured values of vibrational temperature behind strong shock waves are compared with theoretical ones. The histories of vibrational temperature behind strong shock waves in a shock tube were measured using two monochromators. The test gas was pure nitrogen at $100\text{-}300Pa$, and the speeds of shock waves were $5.0\text{-}6.0km/s$. The electronic temperature of N_2^+ was also approximately determined from experiment and compared with the experimental vibrational temperature. The results show that the presented calculational method is effective, and the electronic energy of N_2^+ is excited much faster than its vibrational energy. One Langmuir probe was used to determine the effective time of region 2. The influence of viscosity in the shock tube is also analyzed.

1   Introduction

To correctly predict the flow field around hypersonic vehicles, the calculations should adopt both qualitatively and quantitatively reliable physico-chemical models. Chul Park thoroughly discussed the theoretical models of hypersonic flow[1]. The average temperature model of dissociation has been widely used to predict the hypersonic flow field.

In reference[1], many experimental results were given and compared. The developed two-temperature model failed to explain the experimental data given by Allen[2] concerning vibrational temperature. Allen used the radiation of N_2^+ first negative system emanated from the upper electronic state $B^2\Sigma_u^+$. The experimental results did not show a peak as that of calculation, and the experimental values relax much more slowly. This is rather confusing, because nearly all other radiation behaviors observed experimentally in a flow of nitrogen can be reproduced by the theoretical calculation[1].

To explain the discrepancy, Park surmised that the vibrational levels of the molecule N_2^+ relax differently from those of N_2, and there is little exchange of energies between the vibrational mode of N_2^+ and N_2[1]. Sharma and Gillespie[3] measured the vibrational temperature of the N_2 using its second positive system, and the results were much closer to the calculated T_v. This should be an experimental evidence of the slow exchange of energies between the vibrational mode of N_2^+ and N_2.

Park gave a model to understand the vibrational behavior of $N_2^+(B^2\Sigma_u^+)$ observed experimentally[4]. The reaction of reverse pre-dissociation of N_2^+ was used. The vibrational levels of $N_2^+(B^2\Sigma_u^+)$ are influenced by the free state of $N + N^+$ in the model.

In the present paper, the measured values of vibrational temperature behind strong shock waves are compared with theoretical ones, while the latter were obtained using a new and effective method. The histories of vibrational temperature behind strong shock waves in a shock tube were measured using two monochromators. The test gas was pure nitrogen in $100\text{-}300Pa$, and the speeds of shock waves were $5.0\text{-}6.0km/s$. The calculation is based on three-temperature model and Euler's equations, and the species of N, N_2, N^+, N_2^+, e are included. The electronic temperature of N_2^+ was also approximately determined from experiment and compared with the experimental vibrational temperature. The results show that the presented numerical method is effective, and the electronic energy of N_2^+ is excited much faster than its vibrational energy. To

determine the effective time of region 2, one Langmuir probe was used. The influence of viscosity in the shock tube is also analyzed.

2   Experiment

The shock tube in the experiment has a square test section and the length of the lateral is $132mm$. Two monochromators and two photomultiplier tubes (R456), symmetrically placed at the test point, respectively measured the (0,0) band (band head:3914.4$\mathring{A}$) and (1,2) band (band head:4236.5$\mathring{A}$) of N_2^+ first negative system. The entrance slit had a width of $50\mu m$, and the exit slit had a width of $1mm$. The resolution of the monochromators is $16.6\mathring{A}/mm$. The CN violet could be avoided in the two measured bands.

The optical windows (K9 glasses) had a diameter of $5mm$. This reduced the spatial resolution of the experiment, because the test section is not big enough. The test gas was pure N_2 at 100-$300Pa$. The driving gas was a mixture of H_2, O_2, and N_2, which drove the test gas after ignition. The speeds of shock waves were 5.0-6.0km/s. The environmental temperature was $283.5K$.

To determine the effective time of the experiments, one Langmuir probe was used. The probe was $1mm$ in length, $0.5mm$ in diameter, and was placed at the end of the shock tube. The probe was negatively added a voltage of $10.6V$, and the sampling resistant had a resistance of 1.4Ω. The driven gas was much more ionized than the driving gas, so the signal collected by the probe could be used to clearly determine the effective time.

For the transitions $B^2\Sigma_u^+ \rightarrow X^2\Sigma_g^+$ of N_2^+ first negative system, the ratio of the intensity of the (0,0) band to that of (1,2) band is[5]

$$\frac{I_{(0,0)}}{I_{(1,2)}} = \exp\{-[G_v(v_0) - G_v(v_1)]/kT_v\} \cdot \frac{\nu_{(0,0)}^4}{\nu_{(1,2)}^4} \cdot \frac{q_{(0,0)}}{q_{(1,2)}} \tag{1}$$

In which, k is the Boltzmann constant, q is the Frank-Condon factor, G_v is the spectroscopic term of vibration, v is the quantum number of vibration, and ν is the wave number. In the right-hand-side of Eq.(1), only T_v is the unknown variable. Others are constants, or can be calculated using constants, which can be found in reference[6]. The ratio $r = I_{(0,0)}/I_{(1,2)}$ can be derived from the experiment, so that the vibrational temperature T_v can be determined according to Eq.(1).

3   Calculation

In the calculation, the three-temperature model, preferential dissociation and recombination model, and Park's reaction rates[7] were adopted. The flow was assumed to be inviscid. Five species (N, N_2, N^+, N_2^+, e) are included in the calculation. A spatial marching method, as presented below, was used to solve the problem of the flow after the steady normal shock wave.

From conservation equations, one can get

$$\rho u = c_1 \tag{2}$$

$$\rho uu + p = c_2 \tag{3}$$

$$\rho hu = c_3 \tag{4}$$

In which, c_1, c_2, c_3 are constants determined by the flow before the normal shock wave, and h is the total enthalpy of the gas. From them, three unknown variables can be solved, which are density ρ, velocity u, and translation-rotational temperature T.

From conservation equations, one can also deduce

$$(f_s)_{x+dx} = (f_s)_x + \dot{\omega}_s \cdot dx/c_1 \tag{5}$$

$$(e_v)_{x+dx} = (e_v)_x + \dot{\omega}_v \cdot dx/c_1 \tag{6}$$

$$(e_e)_{x+dx} = (e_e)_x + \dot{\omega}_e \cdot dx/c_1 \qquad (7)$$

In which, $\dot{\omega}_s(s = 1, 2, \ldots, ns)$ are source terms of the chemical reactions, $\dot{\omega}_v$ and $\dot{\omega}_e$ are respectively the source terms of the vibrational and electron-electronic energies, for which the expressions can be found in references[1,2].

From Eqs.(5-7), the species mass fractions, the vibrational energy (so that the vibrational temperature T_v), and the electron-electronic energy (so that the electron-electronic temperature T_e), can be solved. Iterations are needed to solve Eqs.(2-7), so $\dot{\omega}_s, \dot{\omega}_v,$ and $\dot{\omega}_e$ can be set to the corresponding average at station x and $x + dx$ without difficulty. One can see that the solution of this method is nearly exact, and does not need the iterations involving the full flow field.

From Eqs.(2-4) one can see that,in one-dimensional steady adiabatic flow, the same relations are used to describe the conservation of the flow. So the Eqs.(2-4) are valid at the following two arbitrary points: one is before the wave, one is behind the wave, or the two points are both behind the wave. The equilibrium shock wave relations are also the same as Eqs.(2-4).

4   Results and discussion

The typical numerical results are shown in Fig.1a-d. As one can see, all parameters asymptotically approach those of equilibrium. The equilibrium values were derived from Eqs.(2-4), while the nonequilibrium values were derived from Eqs.(2-7). Because the calculated state of gas are $p_1 = 100Pa$ and $V_S = 6.0km/s$, T_e is nearly identical to T_v.
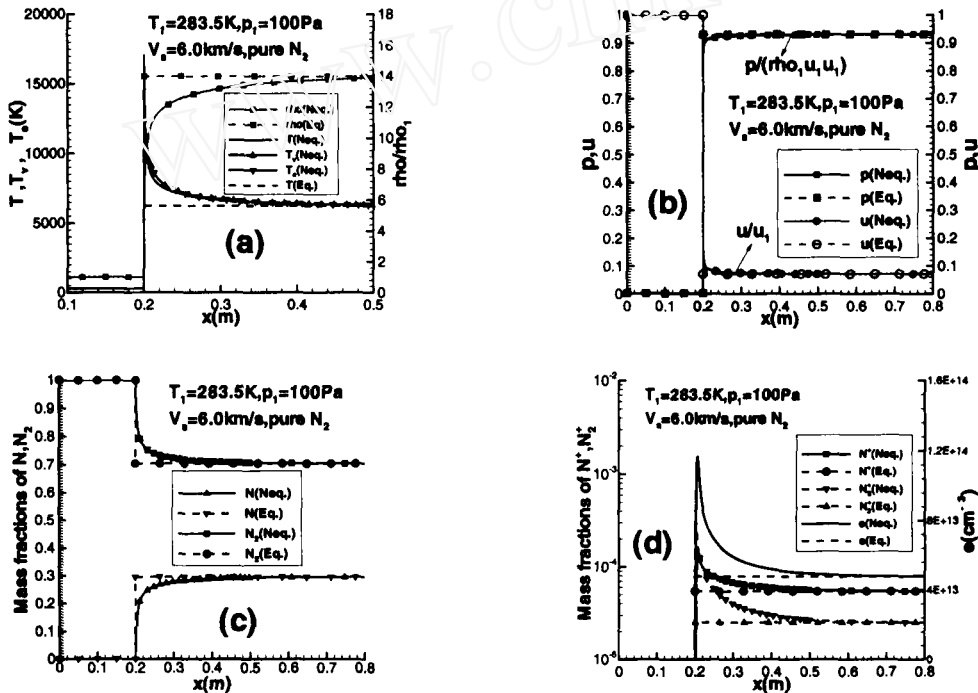
Fig. 1. The distribution of (a) temperatures and density; (b) pressure and velocity; (c) mass fractions of neutral species; (d) mass fractions of ions and number density of electrons
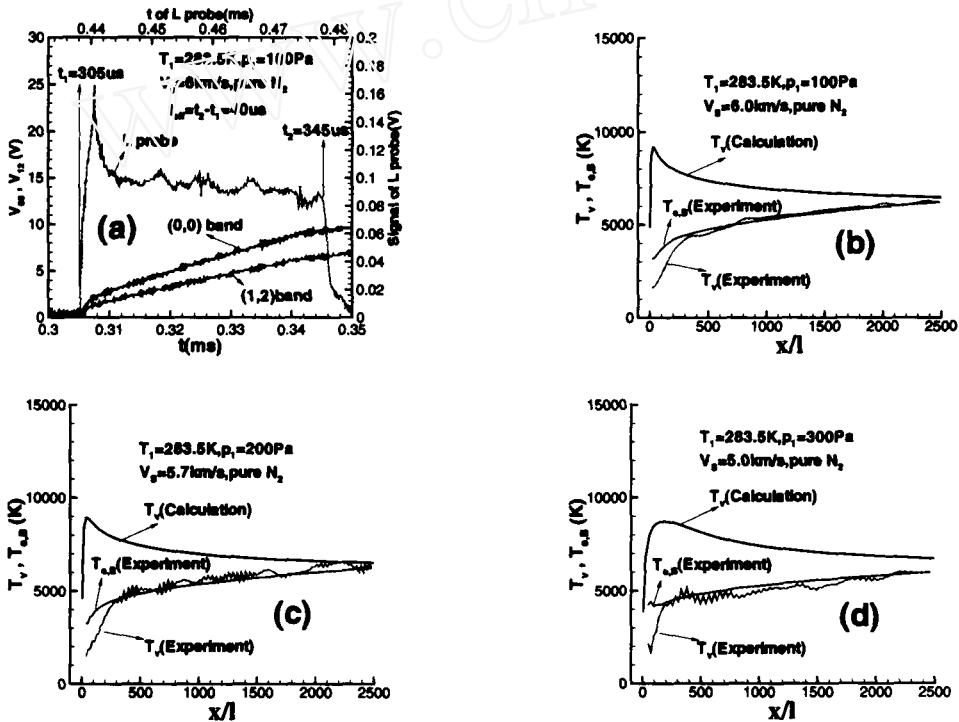
The typical original signal is shown in Fig.2a. As one can see, the signal of Langmuir probe could clearly determine the effective time. Due to lack of spatial resolution, the overshoot of radiations could not be observed. During the run of the shock tube, the speed of shock wave decreased greatly, about $0.4Mach/m$. This resulted in the increase of radiations in the equilibrium region. The shock tube is too thin, so that the wave speed could decrease because of the viscosity during the run of the shock tube. The optical windows were made of K9 glasses, so that the signal ranges of (0,0) and (1,2) bands are not much different. In Table 1, the influence of the shock wave speed decrease on the radiation is analyzed(the intensity of radiation at the wave speed of $6.0km/s$ is set to 1).

Table 1. The equilibrium radiation at different shock wave speeds in $p_1 = 100Pa$

Shock Wave Speed	$5.9km/s$	$6.0km/s$	$6.2km/s$	$6.4km/s$	$6.6km/s$
Equilibrium Radiation	0.8688	1.0	1.314	1.699	2.168

The calculation of Table 1 used the following approximate equation(the influence of vibrational part was neglected).

$$I \propto \rho_{N_2^+} \exp(-36800/T_{eq}) \qquad (8)$$

Fig. 2. The typical original signals (a) and comparisons between experiments and calculations (b-d)

The experimental and numerical T_v are shown in Figs.2b-d. The electronic temperature $T_{e,B}$ of N_2^+ is also shown in these figures. The methods of deriving the experimental T_v and $T_{e,B}$ are followings:

$$\frac{r}{r_{eq}} = \frac{\exp\{-[G_v(v_0) - G_v(v_1)]/kT_v\}}{\exp\{-[G_v(v_0) - G_v(v_1)]/kT_{eq}\}} \qquad (9)$$

$$\frac{I_{(0,0)}}{I_{(0,0)eq}} = \frac{\rho_{N_2^+}}{\rho_{N_2^+,eq}} \cdot \frac{\exp[-36800/T_{e,B}]}{\exp[-36800/T_{eq}]} \qquad (10)$$

From Figs.2b-d in which l are the free stream mean-free-path, one can see that similar trend of vibrational temperature can be obtained as that of Allen[4], although the present spatial resolution is very coarse. The rise of experimental temperatures in equilibrium is the result of the decrease of shock wave speed. This is in agreement with the discussions of the shock wave attenuation by By Bradlay in Ref.[8]. In Eqs.(9-10), equilibrium values at recorded speeds were chosen from numerical results at 2500 times of the free stream mean-free-path behind the wave, so the experimental temperatures are lower than numerical results. As same as the experimental results given by Allen, the present trend of vibrational temperature does not show a peak.

The experimental electronic temperatures $T_{e,B}$ of N_2^+ in Figs.2b-d are somewhat interesting. They show that the electronic energy of N_2^+ is excited much faster than its vibrational energy. From the experimental results given by Allen, Sharma, and the present paper, the exchange of vibrational energy between N_2 and N_2^+ is very slow. There exists no exact theory to explain this to date.

In Fig.2d ($p_1 = 300Pa$), the experimental temperatures are a little flatter than those in Fig.2b ($p_1 = 100Pa$) and Fig.2c ($p_1 = 200Pa$). This is in agreement with the analysis of the viscous influence during the run of the shock tube.

5   Concluding remarks

Some conclusions can be made from the experiment and calculation:

- The presented numerical method is effective.
- The electronic energy of N_2^+ is excited much faster than its vibrational energy in the flow behind strong shock waves.
- The viscosity can cause the decrease of the shock wave speed during the run of the shock tube and influence the measured parameters.

More careful researches need to be made on the vibrational and electronic temperature behind strong shock waves.

Acknowledgement

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