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Acta Astronautica 59 (2006) 40 – 55

ACTA ASTRONAUTICA

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Oscillatory instabilities of two-layer Rayleigh–Marangoni–Benard convection

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Available online 24 May 2006

Abstract

The Rayleigh–Marangoni–Bénard convective instability (R–M–B instability) in the two-layer systems such as Silicone oil (10cSt)/Fluorinert (FC70) and Silicone oil (2cSt)/water liquids are studied. Both linear instability analysis and nonlinear instability analysis (2D numerical simulation) were performed to study the influence of thermocapillary force on the convective instability of the two-layer system. The results show the strong effects of thermocapillary force at the interface on the time-dependent oscillations at the onset of instability convection. The secondary instability phenomenon found in the real two-layer system of Silicone oil over water could explain the difference in the comparison of the Degen's experimental observation with the previous linear stability analysis results of Renardy et al.

1. Introduction

Convection structure and instability in two-layer liquid system heated from below have been extensively studied in the past two decades [1–4]. It has been well known that the convection in two-layer system has many new features, which have no counterpart in a single-layer system [5,6]. Exchange of stabilities has been proved valid for the classical Rayleigh–Bénard problem [1] and for Marangoni convection (surface-tension-driven convection) in one-layer system [7]. However, the exchange of stabilities was assumed without any proof in the first linear stability analysis for two-layer liquids heated from below performed by Zeren and Reynolds in 1972 [8]. They did not get any real eigenvalue for the depth ratio $H_r = 1.22$ (see Table 3 in Ref. [8]). From the view of our recent theoretical studies, at this point the overstability of system takes place physically.

Most studies focus on the oscillatory onset of convection from conductive state in a two-layer system heated from below. There are three different cases where the appearance of oscillatory instability has been predicted. The first one is recognized as interfacial mode. When the deviations of density caused by heating are comparable with density differences between the two liquids, oscillatory convection sustains with interfacial deformation (interfacial mode) [9–11]. The other mechanism of oscillatory instability (Rayleigh–Bénard instability), which neglects interfacial deformations and Marangoni effect, exists in a very narrow region [10,12]. When liquid depth is small (i.e. ∼1 mm on earth) or in microgravity environment, the Marangoni effect plays an

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0094-5765/$ - see front matter © 2006 Published by Elsevier Ltd.
doi:10.1016/j.actaastro.2006.02.042

important role on the convective instability. Both linear stability analysis [13–15] and numerical simulation study [16] have shown that the oscillatory region is enlarged through the coupling of Rayleigh instability and Marangoni effect.

For a long time, some scientists tried but failed to find out oscillatory onset in two-layer liquid system experimentally [10]. Recently, many studies [14–18] focused on the two combinations of two-layer fluid system: Silicone oil (10cSt) over Fluorinert (FC70) and Silicone oil (2cSt) over water, since Degen et al. [17] have conducted the experimental investigations on these two different pairs of fluids. In Degen's experimental studies, for the two-layer system of Silicone oil (10cSt) over Fluorinert (FC70), the oscillation convection observed at the onset are in good agreement with the linear stability analysis results when considering or not considering the Marangoni effect [14,15,18]. However, the oscillatory instability behavior in the two-layer system of Silicone oil (2cSt) over water could not be predicted by the previous linear stability analysis. For this system, Degen et al. have observed the oscillatory convection in a large region $H_{\mathrm{r}} = 0.41$–0.67 while Renardy et al. [18] had not found oscillatory onset in their linear stability analysis disregarded Marangoni effect.

In the present study, we investigated theoretically and numerically the interaction between the Rayleigh–Bénard instability and Marangoni instability in a two-layer system, and more attention was paid to the oscillatory convection in the nonlinear regime, where the oscillation is due to a secondary instability of the initial static state.

2. Physical model and basic equations

A two-layer fluid system considered here is shown schematically in Fig. 1. The two-layer liquids with the total depth, $H = H_1 + H_2$, are aligned horizontally in a rectangular cavity and its dimension is $L \times H(H_1 + H_2)$. The thickness ratio of two-layers is defined as $H_{\mathrm{r}} = H_1/H_2$. The system is heated from below and the upper wall is held at a fixed temperature T_{c}. By using ν_2/H, H^2/ν_2, H and ΔT as the scales for velocity, time, length and temperature, respectively, the dimensionless governing equations for two-dimensional convection in such a two-layer system are [16]

$$\nabla \cdot \mathbf{V}_i = 0, \tag{1}$$

$$\frac{\partial \mathbf{V}_i}{\partial t} + \mathbf{V}_i \cdot \nabla \mathbf{V}_i = -C_i^{\rho} \nabla p_i + C_i^{\nu} \nabla^2 \mathbf{V}_i + C_i^{\alpha} \mathbf{g}\theta, \tag{2}$$

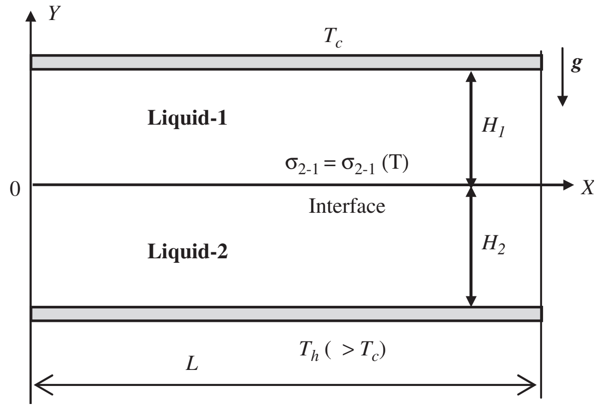

Fig. 1. Schematic diagram of two-layer liquids.

$$\frac{\partial \theta_i}{\partial t} + \mathbf{V}_i \cdot \nabla \theta_i = C_i^{\kappa} \nabla^2 \theta_i, \tag{3}$$

where the subscripts 1 and 2 refer to the upper and the lower layers, respectively. $\mathbf{V}_i = (u_i, 0, w_i)$ is the dimensionless velocity, $\mathbf{g}$ is gravitational acceleration, $\theta_i = (T_i - T_{\mathrm{c}})/\Delta T$, dimensionless temperature, and p_i, dimensionless pressure. The constants in the right-hand side of above dimensionless Eqs. (1)–(3) are, respectively:

$$C_1^{\rho} = 1/\rho^*, \quad C_1^{\nu} = \nu^*,$$

$$C_1^{\alpha} = Ra\beta^*/Pr, \quad C_1^{\kappa} = \kappa^*/Pr,$$

$$C_2^{\rho} = 1, \quad C_2^{\nu} = 1, \quad C_2^{\alpha} = Ra/Pr, \quad C_2^{\kappa} = 1/Pr,$$

where $Pr = \nu_2/\kappa_2$ is the Prandtl number corresponding to the physical properties of liquid-2. $Ra = g\beta_2 \Delta T H^3/(\nu_2\kappa_2)$ is the Rayleigh number. In a system of two-layer fluids, there are other two Rayleigh numbers and two Marangoni numbers corresponding to the upper liquid-layer and the lower liquid-layer which are defined as, respectively:

$$Ra_1 = g\beta_1 \Delta T_1 H_1^3/(\nu_1\kappa_1), \quad Ra_2 = g\beta_2 \Delta T_2 H_2^3/(\nu_2\kappa_2).$$

The corresponding boundary conditions are:

(a) At the two vertical side-walls ($x = 0$ and $x = L/H$):

$$u_i = v_i = 0, \quad \partial\theta_i/\partial x = 0. \tag{4}$$

(b) At the horizontal walls ($y = -H_2/H$ and $y = H_1/H$):

$$u_i = v_i = 0, \tag{5}$$

$$\theta_2 = ct \ (y = -H_2/H), \quad \theta_1 = 0 \ (y = H_1/H_{\mathrm{r}}), \tag{6}$$

where c is a constant which represents the heating rate. In order to find out the bifurcation process, $c = 360\kappa_2/(H_2^2 \Delta T)$ corresponding to a heating rate of 0.1 °C/h was taken in this study.

(c) At the interfaces ($y = 0$):

$$u_1 = u_2, \quad v_1 = v_2 = 0, \tag{7}$$

$$\frac{\partial u_2}{\partial y} - \rho^* v^* \frac{\partial u_1}{\partial y} = -Ma \frac{\partial \theta_2}{\partial x}, \tag{8}$$

$$\theta_1 = \theta_2, \quad \chi^* \frac{\partial \theta_1}{\partial y} = \frac{\partial \theta_2}{\partial y}. \tag{9}$$

The dimensionless ratio of the fluid properties are $\rho^* = \rho_1/\rho_2$ (density), $v^* = v_1/v_2$ (kinematical viscosity), $\beta^* = \beta_1/\beta_2$ (coefficient of thermal expansion), $\kappa^* = \kappa_1/\kappa_2$ (thermal diffusivity), $\mu^* = \mu_1/\mu_2$ (dynamic viscosity), and $\chi^* = \chi_1/\chi_2$ (thermal conductivity), respectively. The Marangoni number is defined as

$$Ma = (-\partial\sigma/\partial T)\Delta T H/(\mu_2 \kappa_2).$$

The Bond number

$$Bo_{\mathrm{d}} = Ra/Ma = g\beta_2\rho_2 H^2/(-\partial\sigma/\partial T)$$

is introduced in this paper.

3. Linear instability analysis

For the linear instability analysis of the problem in a two-layer system, we considered the base state of the system with a flat interface at $y=0$, a zero velocity field, and a temperature field that varies linearly with z in each fluid. At first, we analyzed the oscillatory instability regime of R–M–B convection in the system of Silicone oil (10cSt) and Fluorinert (FC70) with a total depth $H = 6$ mm and in the ground gravity condition $g = 9.8\,\mathrm{ms}^{-2}$ for a larger various range of two-layer thickness ratios H_{r} from 0.2 to 5.0. In the case of the two-layer depth $H = 6$ mm, the Bond number Bo_{d} $(=Ra/Ma) = 15.35$ is about four times less than the ratio value $Bo_{\mathrm{d}} = 61.38$ for the same liquids system when $H = 12$ mm used in the Degen's experiment [17] and analyzed theoretically by Renardy et al. [18].

When $H = 6$ mm, the oscillatory instabilities at the onset are found in the region of $1.5 \leqslant H_{\mathrm{r}} \leqslant 2.1$, in which the maximum of the image part of λ, $\lambda_i = 0.085$ corresponds to the $H_{\mathrm{r}} = 1.8$ shown in Fig. 2 (see the lower part). The critical Rayleigh numbers vary from 25 010 to 21 520 when H_{r} increases from 1.5 to 2.1, and at $H_{\mathrm{r}} = 1.8$ for $Ra_{\mathrm{c}} = 22\,983$, $k_{\mathrm{c}} = 5.025$ the convective oscillation is the most intensity. The oscillatory instability at the onset emerges from $H_{\mathrm{r}} = 1.5$ which is over, but not far away the intersection ($H_{\mathrm{r}} = 1.361$) of the two neutral curves of Ra_1 and Ra_2. The layer depth ratio $H_{\mathrm{r}} = 1.361$ is the balance point of the Rayleigh numbers of the liquid layers, Ra_1 and Ra_2 for this two-layer system.

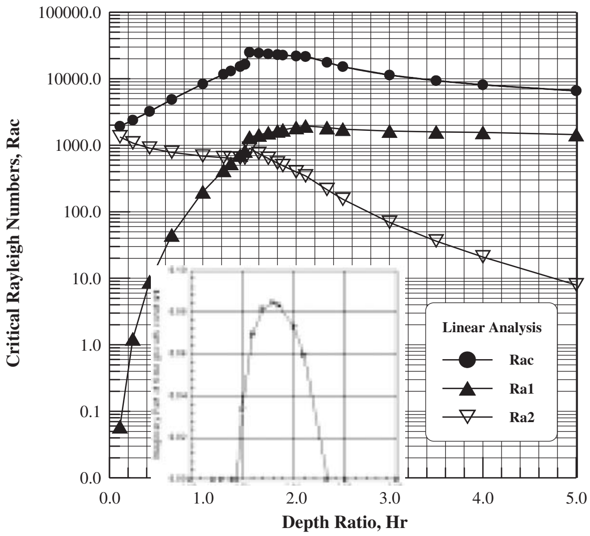

Fig. 2. Linear instability analysis: variation of the critical parameters Ra_{c}, Ra_i, at the onset of R–M–B convection in the system $H = 6$ mm and $g = 9.8\,\mathrm{ms}^{-2}$ for different depth ratios H_{r}.

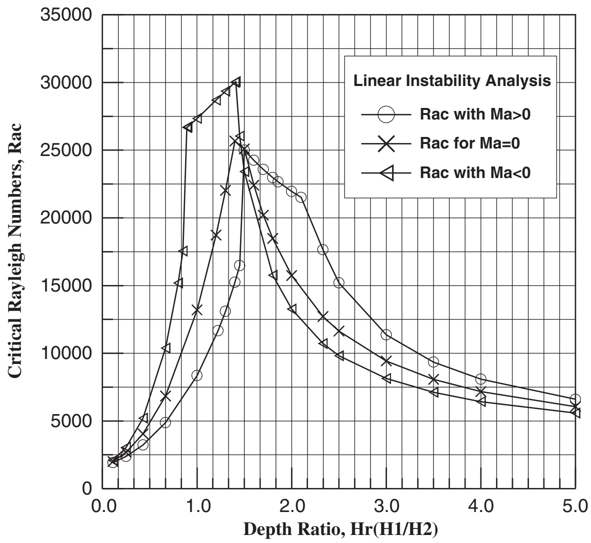

Fig. 3. Comparison of the neutral stability curves for the system with different Marangoni effects $Ma > 0$, $Ma < 0$ and without Marangoni effect ($Ma = 0$).

Fig. 3 shows the influence of the thermocapillary effect on the instability of the two-layer system. When we consider the existent Marangoni effect at the interface, the neutral stability curve of the system displaces to right in comparison with the Rayleigh–Bénard instability states of the system without the Marangoni effect ($Ma = 0$), corresponding to the case considered in Colinet and Legros' work [12]. For this system, a very narrow oscillatory instability region of $1.425 \leqslant H_r \leqslant 1.433$ for the Rayleigh–Bénard convection when $Ma = 0$ was found in the neutral stability curve of Ra_c–H_r plane. It is notable that a more larger oscillatory regime for $1.5 \leqslant H_r \leqslant 2.1$ found in the R–M–B convective instability of the system replaces the very narrow oscillatory onset gap in the Rayleigh–Bénard instability of the system when neglecting the thermocapillary effect ($Ma = 0$). On the contrary of positive Marangoni effect, the oscillatory regime at the onset will displace to the left in the region of $0.892 \leqslant H_r \leqslant 1.41$ when we assume $\partial\sigma/\partial T = 4.46 \times 10^{-5}$ N/mK for $Ma < 0$.

4. Nonlinear instability and a Hopf bifurcation

Direct numerical simulation was carried out by a finite difference method in two dimensions. The validation of our numerical code was established by comparing numerical results with both experiments and linear stability analysis (see Ref. [16]). The control parameters of the two-layer system such as Silicone oil (2cSt) (liquid-1) and water (liquid-2) are $Pr = 7.1$, $\rho^* = 0.867$, $\nu^* = 2.0$, $\beta^* = 5.66$, $\kappa^* = 0.78$, $\chi^* = 0.184$, and $H = 12$ mm, $L = 78$ mm, respectively. The surface tension is assumed to vary linearly with temperature at the interface: $\sigma = \sigma_0 - \partial\sigma/\partial T(T - T_0)$ with $\partial\sigma/\partial T = -1.0 \times 10^{-4}$ N/mK. Then we have $Bo_d = 2.9$.

For the Rayleigh–Marangoni–Bénard instability in the Silicone oil (2cSt)–water system, we found numerically that it will take two bifurcation processes from static state to time-dependent convection, and then the oscillatory regime is enlarged compared to linear stability analysis. A typical time-dependent convection behavior of the system is shown in Fig. 4 for the uniform heating rate of 0.1 °C/h, and two bifurcations appeared. For an initially static of the system, it loses its stability and onsets to steady convection at the critical Rayleigh number, $Ra_c = 1.09 \times 10^4$, corresponding to the point I in Fig. 4. Then it undertakes a secondary bifurcation to time-dependent pattern at the point II where $Ra = 2.21 \times 10^4$.

We summarized different results obtained by several authors in Fig. 5. Linear stability analysis predict that the Silicone oil (2cSt)–water system will lose its stability and onset to the steady convection, except only one point $H_r = 0.59$ where the oscillatory onset takes place. Our 2D simulation found oscillatory convection exists

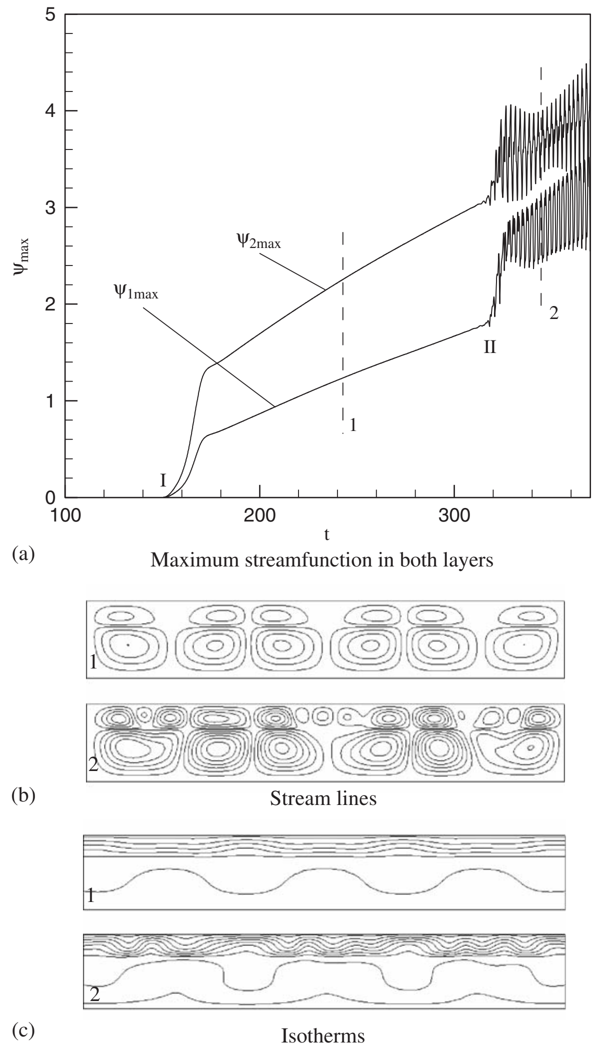

Fig. 4. Time variation of maximum stream functions in upper and lower layer, and corresponding streamlines and isotherms at two instants in silicone oil–water system for depth ratio $H_r = 0.5$, $Bo_d = 2.9$, with $H = 12$ mm and $A = L/H = 6.5$. The two pictures from top to bottom in (b) and (c) are corresponding to the points 1–2 in (a).

in a larger gap region $0.49 \leqslant H_r \leqslant 0.67$ (symbols △ in Fig. 5). Degen et al. observed the oscillation in the region $0.41 \leqslant H_r \leqslant 0.67$ in which they did not find steady convection in their experiments. So the observed oscillatory convection in their experiment was taken as the primary instability of static state. Our present theoretical study show that the oscillatory instability observed in the region $0.41 \leqslant H_r \leqslant 0.67$ should be the secondary Rayleigh–Marangoni–Bénard instability in this system.

The main reasons of our above explanation about the difference with Degen's experimental observation are as follows: (1) The shadowgraph method used by

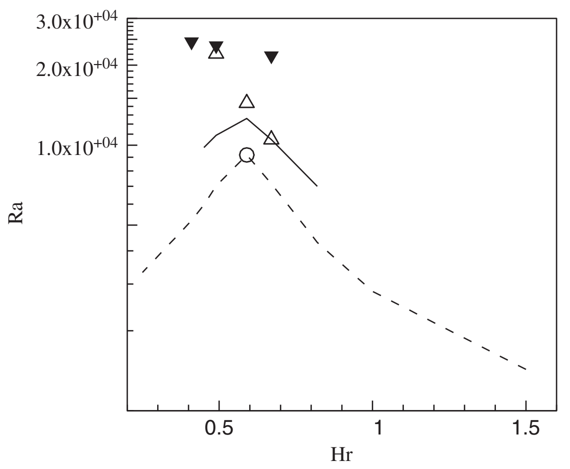

Fig. 5. Comparison of critical *Ra* for the onset of steady and oscillatory convection in the Silicone oil–water two-layer system obtained by different methods: - - - - - - onset of steady convection (linear stability), ◦ onset of oscillatory convection (linear stability), —— onset of steady convection (2D simulation), △ onset of oscillatory convection (2D simulation), and ▼ onset of oscillatory convection (Degen's exps.).

Degen et al. would not detect the steady convection at the onset due to "unfortunately, using water presents another difficulty in that its variation of index of refraction with temperature is small" said in Ref. [17]. This kind of difficulty in experiment is confirmed by our numerical simulation from the evolution of temperature distributions in the two-layer system shown in Fig. 4. (2) The onset of oscillatory convection happens at a much higher *Ra* than the critical number Ra_c corresponding to the premier steady convective onset (see the comparison in Fig. 5). In this paper, *Ra* number corresponding to the oscillation convection lies in the range $Ra_c < Ra < 2.1Ra_c$ $(0 < \varepsilon < 1.1)$. In fact, the secondary oscillatory instability $(\varepsilon = 0.1)$ has been found also in the Degen's experiments for the two-layer system of Silicone oil (10cSt) over Fluorinert (FC70) [17].

It should be noted that the secondary oscillatory instability in two-layer system is a result of the competition between the instability in both layers. Before the oscillatory convection takes place, the convection initiates in water layer and then drags the upper Silicone oil layer through viscous forces. The convection in upper layer is weaker than that in the water layer and could be considered as passive. In Fig. 4, one can see that after the onset of oscillatory convection, maximum stream function in the upper layer (Silicone oil) increases suddenly (see point II). It means the intrinsic instability in the Silicone oil liquid layer takes place and flow is intensified evidently.

5. Conclusion

In summary, both linear instability analysis and 2D numerical simulation results presented in the present paper show that the onset oscillatory instability region is enlarged through the coupling of Rayleigh instability and Marangoni effect for different depth ratios of two-layer liquids (with reducing the Bond number Bo_d $(=Ra/Ma)$). The results are different from the previous study on the Rayleigh–Bénard instability and show the strong effects of the interfacial thermocapillary force on the time-dependent oscillations arising at or after the onset of instability convection.

For the Silicone oil–water system, the main finding from our numerical simulation is that convective instability of the system will take two bifurcation processes from the static state to time-dependent convection. In this case, the two-layer system will lose stability and onset at first to the steady convection, and then the steady convection of the system bifurcates to oscillatory convection with the increasing of *Ra* number for some depth ratio. The oscillatory convection coming from the interaction between the upper and lower layer is the secondary instability. This secondary oscillatory instability mechanism explains the difference between the experimental observation of Degen et al. [17] and the linear stability analysis of Renardy and Stoltz [18].

Acknowledgments

This research was supported financially by the National Nature Science Foundation of China (10372105, 10432060) and in part by the Knowledge Innovation Program of Chinese Academy of Sciences (KJCX2-SW-L05, KGCX-SW-409).

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