

THE INDUCED FLOW FIELD BY INTERNAL SOLITARY WAVE AND ITS ACTION ON CYLINDRICAL PILES IN THE STRATIFIED OCEAN

Youliang Cheng^{1,2}, Jiachun Li¹, Liansuo An², Yufeng Liu³

¹DES, Institute of Mechanics, The Chinese Academy of Sciences, Beijing 100080, China

²Dept. of Power Engineering, North China Electric Power University, Baoding 071003, China

³Research Center, Chinese Ocean Oil Co., Beijing 100080, China

Abstract: The induced flow fields by internal solitary waves and its actions on cylindrical piles in density stratified ocean with a basic density profile and a basic velocity profile are investigated. Some results, such as the time evolution of flow fields and hydrodynamic forces on the piles are yielded both by theoretical analysis and numerical calculation for general and specific cases. Several kinds of ambient sea conditions of the South China Sea are specified for numerical simulation. Moreover, the effects of relative density difference, depth ratio and wave steepness on maximal total force and total torque are analyzed.

Keywords: induced flow field, internal solitary wave, force, torque, piles, stratified ocean

1. INTRODUCTION

Oceanic internal wave is a kind of subsurface wave existing in the pycnocline of seawater. It can transmit energy and momentum throughout the ocean, not only laterally but also vertically. So the investigation of internal waves is very important in the oceanic dynamics.

The internal waves, especially internal solitary waves can bring about huge energy of sea water and sudden strong current, which can cause severe threat to the ocean engineering structures, such as oil drilling platform and pipeline. It seems that some of designs can withstand the strong current of internal solitary waves. However, very few researches on the details of the flow fields of internal solitary wave are available up to now^[1,2].

In this paper, we consider a density stratified fluid flow with a basic density profile and a basic velocity profile as the density stratified ocean. The two-layer fluids bounded by two horizontal rigid walls (use WW as the symbolic mark) or one horizontal rigid wall and one free surface (use SW as the symbolic mark) can be treated as its special cases. Due to limited space, only some of the results of SW model are given. The mathematical description of the model is in Section 2. Section 3 presents the analytic solutions of the flow fields of solitary wave and its action on piles. The numerical simulations of the marine states for SW model are shown in Section 4. Finally, the conclusions are made in Section 5.

2. MODEL AND EQUATIONS

Suppose that the density stratified fluid is incompressible, inviscid fluid bounded by one horizontal rigid wall $z=0$ and one horizontal rigid wall or one free surface being in equilibrium at $z=h$ with a basic density profile $\rho_0(z)$ and a basic velocity profile $u_0(z)$, where z is a Lagrangian vertical coordinate. Let z^* be an Euler vertical coordinate and x be a horizontal coordinate moving with wave phase speed c so that the

flow becomes steady. Supposing η is a vertical displacement of streamline as shown in Fig.1, we have the following nondimensional equation and boundary conditions

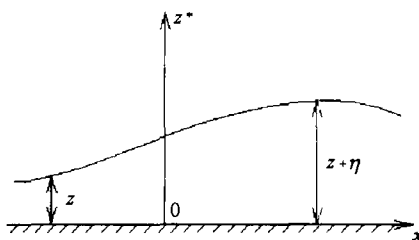


Fig.1 Nondimensional wave coordinate frame

$$\left\{ \rho_0(c-u_0)^2 \left[1 - \frac{1}{(1+\eta_z)^2} \right] \right\}_z + 2\mu^2 \rho_0(c-u_0)^2 \left(\frac{\eta_z}{1+\eta_z} \right)_x - \mu^2 \left[\rho_0(c-u_0)^2 \frac{\eta_z^2}{(1+\eta_z)^2} \right]_z$$

$$+ 2\rho_0 N^2 \eta = 0, \quad 0 < z < h; \quad (1a)$$

$$\eta = 0, \quad z = 0; \quad (1b)$$

$$2\eta = b\sigma(c-u_0)^2 \left[1 - \frac{1}{(1+\eta_z)^2} - \frac{\mu^2 \eta_z^2}{(1+\eta_z)^2} \right], \quad z = h. \quad (1c)$$

where N is the nondimensional Brunt-Väisälä frequency, σ is the Boussinesq parameter, which is a small parameter under the Boussinesq supposition. $b=0$ or $b=1$ if the upper boundary is a horizontal rigid wall or a free surface. For the latter, h is the nondimensional height of upstream. $\mu=H/L$ (H is vertical characteristic length, L is characteristic horizontal length) is a small parameter denoting nonlinearity. Using some transforms, Eqs.(1a)~(1c) and the nondimensional velocitys are obtained from Long equation^[1,3].

3. SOLUTIONS

We expand η into power series of μ^2 according to the shallow water assumption, and then derive the KdV equation, which amplitude $A(x)$ must obey, where $\eta = Ur\mu^2 A(x)\phi(z)$, $\phi(z)$ is the solution of eigenvalue problem, Ur is the Ursell Number. The nondimensional velocity induced by soliton looks like

$$u = \frac{u_0 + c\eta_z}{1 + \eta_z} = \frac{u_0 + cUr\mu^2 A(x)\phi_z(z)}{1 + Ur\mu^2 A(x)\phi_z(z)} \quad (2)$$

$$w = -\frac{(c-u_0)\eta_x}{1 + \eta_z} = -\frac{(c-u_0)Ur\mu^2 A'(x)\phi(z)}{1 + Ur\mu^2 A(x)\phi_z(z)} \quad (3)$$

which is agreement with Ref.[3] as $Ur=1$.

Supposing a cylindrical pile is at the place where $x=0$, and its top and base are overlapped with the upper surface and the horizontal wall respectively, and its diameter is 5 m, and z axes is used as its symmetrical axes. Using the Morison formula^[4], The force profile per length is obtained. Integrating over the whole vertical cylinder, the total forces and total torques on this pile are calculated.

4. CASE STUDY

As an example, the two-layer fluid bounded by one horizontal rigid wall $z^*=0$ and one free surface (SW) with the equilibrium position $z^*=h_2+h_1$ is considered. The basic density and velocity profiles and the

Boussinesq parameter are supposed respectively to be $\rho_0(z) = \rho_1 H(z-h_2) + \rho_2 H(h_2-z)$, $u_0(z) = 0$ and $\sigma = 2(\rho_2 - \rho_1) / (\rho_2 + \rho_1)$, where H is the Heaviside function. Therefore, we have c_0 and $\phi(z)$ by solving the boundary value problem about them. Then, the first-order interfacial elevation, the corresponding velocities, the force profile per length, the total forces and total torques on this pile are all calculated.

Several ambient oceanic conditions corresponding to the South China Sea^[5] can be selected for numerical simulation. Due to limited space, only one condition, i.e., $h_1 = 60\text{m}$, $h_2 = 440\text{m}$, $r = 0.136364$, $\sigma = 0.003$, $A = 92.8\text{m}$, $L = 10000$ for $b = 1$ are given. Where $r = h_1 / h_2$, the ratio of depths, σ is the relative density difference being approximately the Boussinesq parameter, L is the characteristic wawewidth, A is the wave height of the internal solitary wave.

From Fig.2 to Fig.4, it is founded that the directions of the velocity vectors and the forces on the pile of the upper and lower layers are opposite and the pile is subjected to a strong impact during the passage of a soliton.

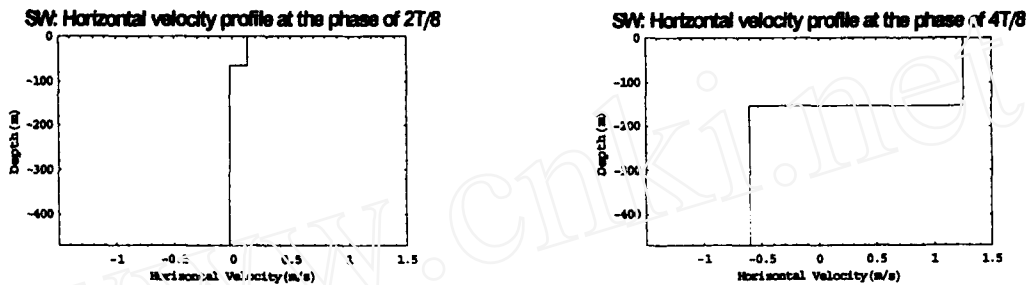


Fig.2 The horizontal velocity profiles at different moment within the time as the internal solitary move through about half a wawewidth

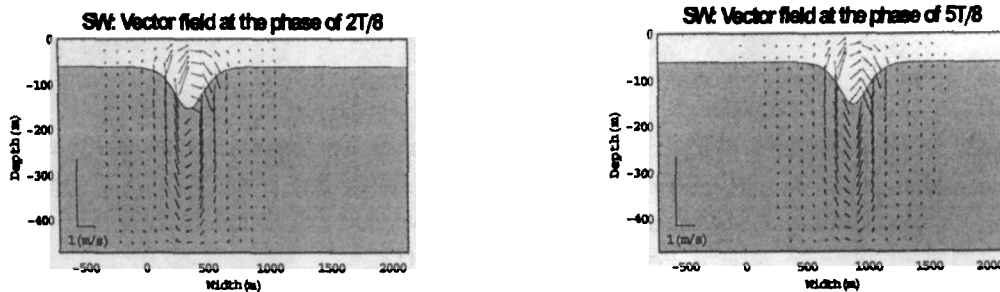


Fig.3 The velocity vector field at different moment

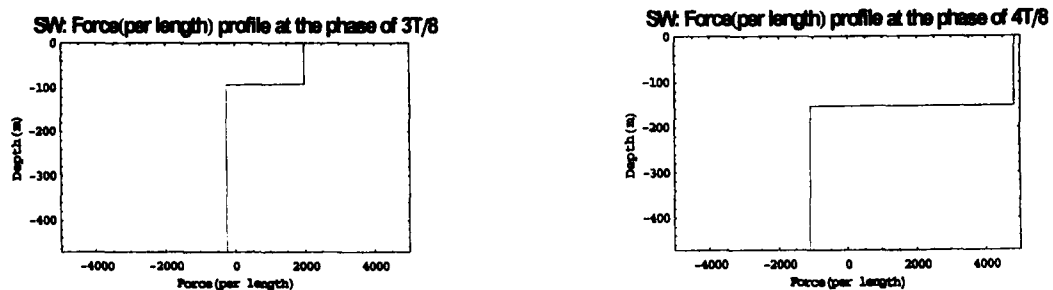


Fig.4 The force profiles per length on a 5m-diameter pile at different moment within the time as the internal solitary move through about half a wawewidth

The effects of the relative density difference, the depth ratio and the wave steepness on the maximal force and torque also are analyzed. It is founded that the maximal force and torque increase as the relative density difference and the wave steepness increase. On the contrary, the maximal total force and torque decrease as the depth ratio is less than the critical one.

5. CONCLUSIONS

The induced flow fields by internal solitary waves and its actions on cylindrical piles in density stratified ocean are investigated. The boundary problem about the displacement of streamline is obtained, and then, some results, such as the temporal evolution of flow fields and hydrodynamic forces on the piles are yielded for general and specific cases. Several kinds of ambient sea conditions are specified for numerical simulation. The effects of relative density difference, depth ratio and wave steepness on maximal total force and total torque are analyzed. It is founded that the directions of the velocity vectors and the forces on the pile of the upper and lower layers are opposite and the action on the pile is a strong impact. So, the piles should withstand strong impact shearing force. It is shown that the maximal force and torque increase as the relative density difference and the wave steepness increase. On the contrary, the maximal total force and torque decrease as the depth ratio is less than the critical depth ratio.

Thus, as the stratification becomes stronger, the wave steepness larger, and the depth ratio tends to the critical depth ratio, the actions of internal solitary wave on the pile should be paid more attentions in the oceanic structure design.

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