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Monte-Carlo Simulation of Surface Crack Growth Rate for Offshore Structural Steel E36-Z35

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Abstract — The Monte-Carlo method is used to simulate the surface fatigue crack growth rate for offshore structural steel E36-Z35, and to determine the distributions and relevance of the parameters in the Paris equation. By this method, the time and cost of fatigue crack propagation testing can be reduced. The application of the method is demonstrated by use of four sets of fatigue crack propagation data for offshore structural steel E36-Z35. A comparison of the test data with the theoretical prediction for surface crack growth rate shows the application of the simulation method to the fatigue crack propagation tests is successful.

Key words: *Monte-Carlo method*; *simulation*; *surface crack*; *crack growth rate*; *offshore structural steel*; *fatigue crack propagation test*

1. Introduction

Offshore steel structures are widely used for explotation and exploration of offshore oil. The environmental loads acting on offshore structures excite cyclic variation of stresses and result in fatigue and damage in structural components. In general, the design life of an offshore platform is about 25 years experiencing about 100 million cycles of wave loads (Rao *et al.*, 1994). As well known, most fatigue failure in welded structures and components often develops from surface flaws. Hence, surface fatigue crack growth analysis is one of the major tasks in the fatigue life prediction of welded structures. However, owing to undeterminable factors from random enviornmental loads, welding process which results in stress concentration and residual stresses, micro-structural and mechanical properties of different components and materials, etc., the growth rates of surface cracks exhibit considerable statistical variability even in well-controlled laboratory conditions. Therefore, statistical methods are usually applied to the analysis of fatigue crack propagation (Virkler *et al.*, 1979; Lin and Yang, 1983; Yang *et al.*, 1983; Itagak and Shinozuka, 1972; Madson *et al.*, 1987). However, fatigue crack propagation (FCP) testing is very expensive and time-consuming as well, making it more difficult to implement a large group of tests for statistical analysis. In such cases, the Monte-Carlo technique is considered to be an effective method, which consists of repeated numerical random samplings of a given model with an objective of estimating the unknown statistical properties of the model. A large number of samples of random variables can be produced by the Monte-Carlo technique to replace complicated and expensive experiments. Moreover, the mathematical concept of this method is explicit, no matter how difficult the problems are. We can solve these problems once their probabilistic models are constructed according to this method. Recently, different simulation methods based on the Monte-Carlo technique have been widely applied to reliability analysis

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and life estimation of structures (Rubinstein, 1981; Harbitz, 1983; Ayyub and Lai, 1989; Melchers, 1989 and 1990; Engelund and Rackwitz, 1992; Ding and Liu, 1996a, 1996b and 1997).

In this paper, the Monte-Carlo method is used to simulate the surface fatigue crack growth rate for offshore structural steel E36-Z35, and to determine distributions and relevance of the parameters in the Paris equation. The application of the method is demonstrated by use of four sets of FCP data for offshore structural steel E36-Z35.

2. FCP Tests on Offshore Structural Steel E36-Z35

In order to investigate the surface fatigue crack growth of steel E36-Z35, experiments on E36-Z35 steel (EZS) and welded plates (EZWP) are conducted and the results are compared with Monte-Carlo simulation.

2.1 Material and Specimen

The tested material is Z-direction steel E36-Z35 provided by Shanghai Jiangnan Shipbuilding Factory. The chemical composition of this material is given in Table 1, while the mechanical properties shown in Table 2. The specimens are three-point bending ones of 28 mm in thickness, 85 mm in width and 370 mm in length. By use of a spark discharge machine, all specimens are pre-cracked to 1 mm depth and 2 mm width under constant amplitude loading at room temperature.

Table 1 Chemical composition of steel E36-Z35

C	Si	Mn	P	S	Cu	Al	Nb
0.16	0.33	0.34	0.10	0.01	0.02	0.49	0.35

Table 2 Mechanical properties of steel E36-Z35

σ_S (MPa)	σ_b (MPa)	δ (%)
411.6	558.6	34

2.2 Testing Conditions

All FCP tests were conducted at room temperature in a servo-hydraulic material testing system MTS810.12. The tests were load controlled at a frequency of 10 Hz in a three-point bending load mode as shown in Table 3.

2.3 Crack Growth Monitoring

Taking into account the difficulty in direct measurement of the depth of surface fatigue crack, the beach mark method was used in the tests. Beach marks were made on the fracture surface by reducing the load amplitude by half but keeping the average load unchanged. It was found that the beach mark was visible once there was a small propagation during the reduced-load program.

Table 3 Loading conditions

Specimen No.	Specimen type	P_{max} (kg)	P_{min} (kg)
101	EZS	4700	336
103	EZS	5455	1091
104	EZS	7273	2909
105	EZS	8728	4364
107	EZS	4909.5	981.9
108	EZS	6546	1309.2
109	EZS	4364	872.8
301	EZWP	4700	336
302	EZWP	5455	1091
304	EZWP	7273	2909
305	EZWP	6546	1309.2
307	EZWP	4909.5	981.9
308	EZWP	4364	872.8
309	EZWP	5455	1091

3. Analysis of the Test Results

3.1 Stress Intensity Factors for Three Point Bending Specimens

As well known, the Paris equation is proved to be a universal law for fatigue crack propagation in both materials and structures. It is recently mathematically derived on the basis of dislocation dynamics, thermal activation theory and rate process theory (Duan, 1995; Duan *et al.*, 1999). For a semi-elliptical surface crack in a three-point bending specimen, the propagation along the depth and width can be described as

$$\frac{dX_j}{dN} = \varepsilon_j (\Delta K_j)^{m_j}, \qquad (j = 1,\ 2), \tag{1}$$

where X_j stands for a half crack length c or depth a, ΔK_j the corresponding stress intensity factor range, and ε_j, m_j test constants. The stress intensity factor ranges are given as follows (Newman and Raju, 1981):

$$\Delta K_1 = Z_n S H_1 M \sqrt{\frac{\pi a}{Q} \cdot \frac{a}{c}} \tag{2a}$$

$$\Delta K_2 = Z_n H_2 M \sqrt{\frac{\pi a}{Q}} \tag{2b}$$

where Z_n is the bending stress amplitude in the load history, H_1, H_2 are boundary correction factors, a crack depth, c half surface crack length, Q the shape factor for elliptical crack, and

$$M = \left\{ 1.13 - 0.09\left(\frac{a}{c}\right)\right\} + \left\{ -0.54 + 0.89\left[0.2 + \left(\frac{a}{c}\right)\right]^{-1} \right\}\left(\frac{a}{B}\right)^2 + \left\{0.5 - \left[0.65 + \left(\frac{a}{c}\right)\right]^{-1} + 14\left(1 - \frac{a}{c}\right)^{24}\right\}\left(\frac{a}{B}\right)^4 ;$$

$$S = 1.1 + 0.35\left(\frac{a}{B}\right)^2 ;$$

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} ;$$

$$H_1 = 1 - \left[0.34 + 0.11\left(\frac{a}{c}\right)\right]\left(\frac{a}{B}\right);$$

$$H_2 = 1 - \left[1.22 + 0.12\left(\frac{a}{c}\right)\right]\left(\frac{a}{B}\right) + \left[0.55 - 1.05\left(\frac{a}{c}\right)^{0.75} + 0.47\left(\frac{a}{c}\right)^{1.5}\right]\left(\frac{a}{B}\right)^2 .$$

And B is the thickness of the specimen.

Substituting Eq. (2) into Eq. (1) gives

$$\frac{dc}{dN} = \varepsilon_1 \left[Z_n S H_1 M \sqrt{\frac{\pi a}{Q} \cdot \frac{a}{c}}\right]^{m_1} , \tag{3a}$$

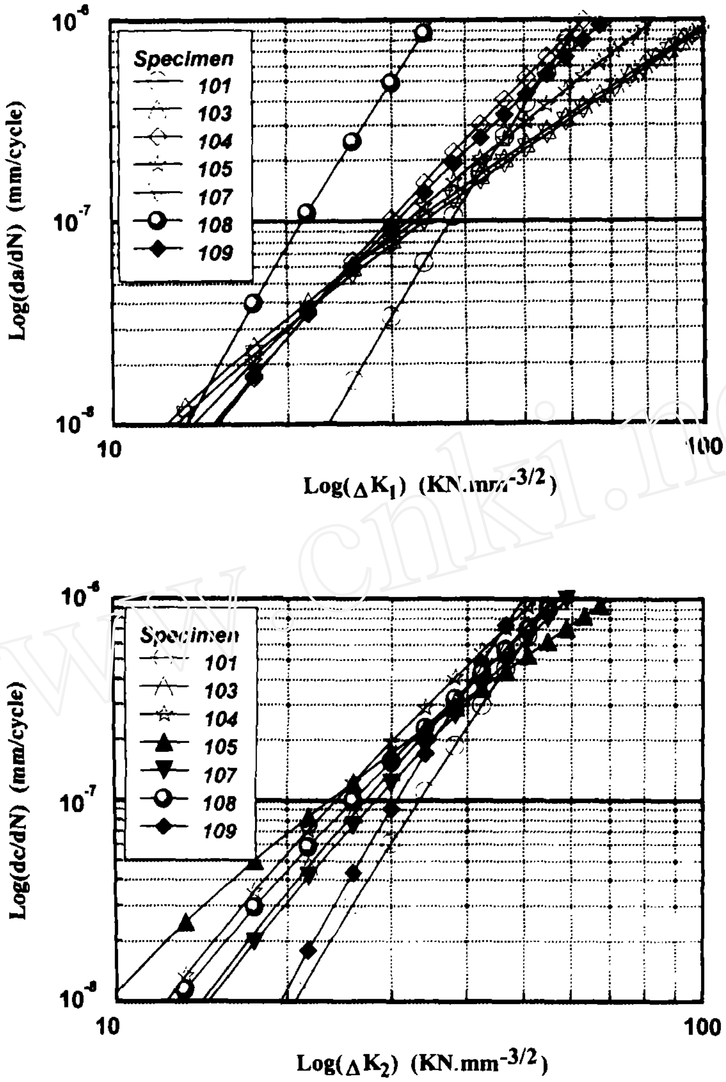
$$\frac{da}{dN} = \varepsilon_2 \left[Z_n H_2 M \sqrt{\frac{\pi a}{Q}}\right]^{m_2} . \tag{3b}$$

3.2 Regression of the Test Data

The fatigue crack propagation data for all the specimens tested and presented in log-log plots of ΔK_j and dX_j/dN as shown in Figs. 1 and 2. The crack growth rates are calculated by use of seven-point polynomial fit and the stress intensity factors are given by Eqs. (2a) and (2b). The values of ε_j and m_j in Eq. (3) are obtained by performing a least fit of the data. The regression results are illustrated in Table 4 where the relative coefficient r and standard deviation s are presented respectively.

Table 4 Regression results of the experimental data

Material	E36-Z35 steel		E36-Z35 welded plate	
	Group 1	Group 2	Group 3	Group 4
m_j	2.6291	2.6851	2.4791	2.5133
log ε_j	−10.6814	−10.6437	−10.5777	−10.2846
r	0.9226	0.9385	0.9286	0.9278
s	0.5202	0.5517	0.5144	0.5546



A

B

Fig. 1 Surface crack growth rate for E36-Z35 steel.

4. Monte-Carlo Simulation

4.1 Distribution Function of Stress Intensity Factor Range

In order to use Monte-Carlo simulation, we must first determine the distribution functions of random variables before carrying out random sampling of the distribution functions. As shown in Table 4, $\log da/dN$ and $\log dc/dN$ present a linear random function with constant standard deviation, but its coefficient of variation cannot be used as a simulation parameter. The coefficients of variation of $\log da/dN$ and $\log dc/dN$ are large and larger than zero,

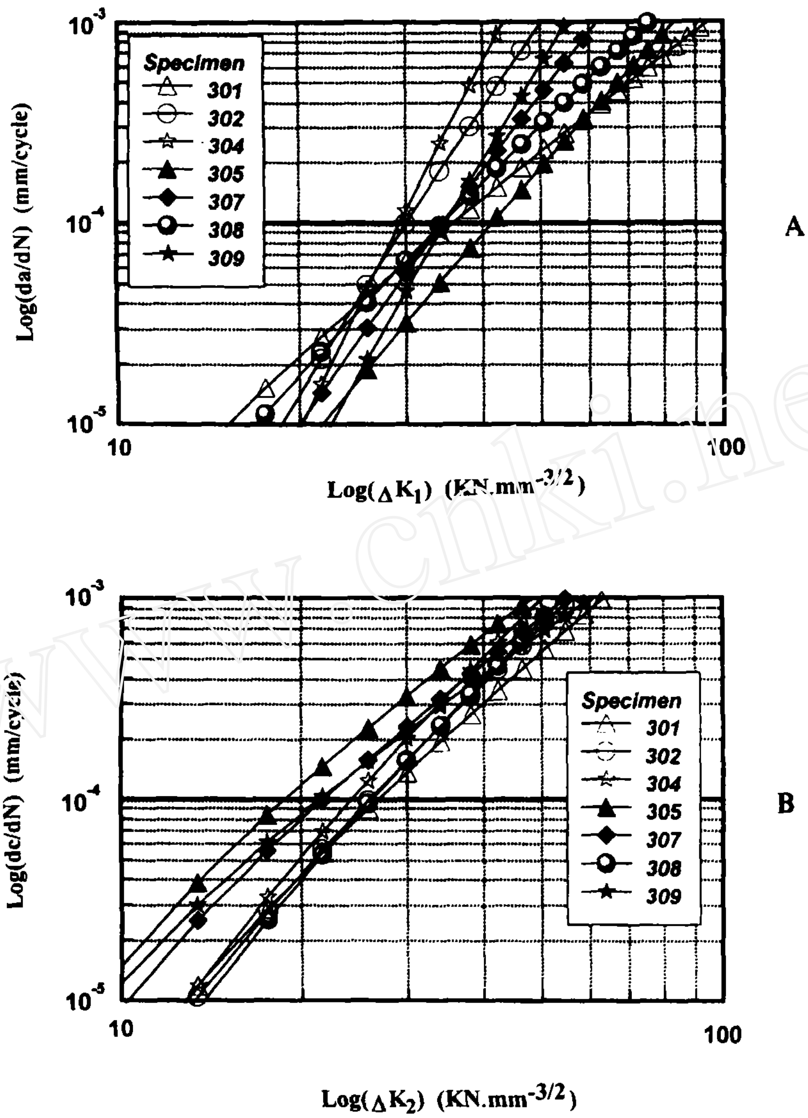


Fig. 2 Surface crack growth rate for E36-Z35 steel plate.

therefore the error induced by the use of normal distribution is comparatively large. Usually, the lognormal distribution and the Weibull distribution are used, and from the point of physics. the Weibull distribution is more reasonable. However, it is convenient to use normal distribution to cope with practical problems. The values of these two kinds of distributions are nearly the same in some of the distribution zones, therefore, the log-normal distribution is chosen to process the data of $\log da/dN$ and $\log dc/dN$ in this paper. Newman and Raju (1981) estimated K_1 around semi-elliptical surface cracks in rectangular beams of width $2B_1$ and depth B_2 under remote bending moment M for $2c \geqslant a$;

$$K_1(\varphi) = H \cdot \frac{6M}{B_1 B_2^2} \frac{\sqrt{\pi a}}{E(k)} \cdot F\left(\frac{a}{B_2}, \frac{a}{c}, \frac{c}{B_1}, \varphi\right) \tag{4}$$

where H is a boundary correction factor; E the complete elliptic integral of the second kind; $k^2 = 1 - a^2/c^2$, F a polynomial of a/B_2, a/c, c/B_1, and a simple trigonometric function of φ, the parametric angle of the ellipse. Apparently, Eqs. (2a) and (2b) are the extremities of Eq. (4) for $\varphi = 0°$, or $\varphi = 90°$. Because the variables in Eq. (4) are in normal distribution, ΔK also takes normal distribution.

4.2 Distribution Function and Relevance of ε and m

As shown by the test results, the surface fatigue crack growth rate dX_j/dN exhibits considerable statistical variability. Such variability should be taken into account in reliability analysis. In particular, the distribution functions of ε_j and m_j must be defined, because they are the basis of life reliability analysis. Usually, statistical analysis of many groups of experiments must be conducted to gain the functions, although their sample sizes are small and the confidences are low. Comparatively, the Monte-Carlo method is an economical and efficient numerical method, reducing the amount of experiments by means of calculation and simulation. First, ε_j and m_j can be obtained by least square fit of one set of test data. The coefficient of variation of every variable in $m_j \log(\Delta K_j) + \log \varepsilon_j$ is smaller than 0.2, therefore $m_j \log(\Delta K_j) + \log \varepsilon_j$ can be taken as the mean value of $\log dx_j/dN$. Furthermore, s in Table 4 is considered as the standard deviation of $\log dX_j/dN$, the distribution function of $\log dX_j/dN$ can then be defined.

The coefficient of variation of ΔK can be calculated by (Zhang, 1989)

$$C_{V_k} = \sqrt{C_{V_P}^2 + C_{V_{B_1}}^2 + 2C_{V_{B_2}}^2 + \frac{1}{4} C_{V_a}^2}\ . \tag{5}$$

According to observation of some experiments and their statistical analysis, the coefficients of variation of load P, width B_1 and length B_2 of the specimen can all be taken as the same value (Zhang, 1989) marked by σ. In general, the coefficients of variation are calculated according to $3\sigma = 5\%$, yielding

$$C_{V_P} = C_{V_{B_1}} = C_{V_{B_2}} = 0.017\ .$$

For a crack length in an experiment, the crack was measured by a travelling microscope thirty times, obtaining the coefficient of variation C_{V_a} as 0.013 which yields $C_{V_k} = 0.0346$. Hence the distribution functions of ΔK_j and dX_j/dN are as follows

$$\Delta K_j \sim N\left[\overline{(\Delta K_j)_i},\ C_{V_k}(\Delta K_j)_i\right], \tag{6}$$

$$\log\frac{dX_j}{dN} \sim N\left[\overline{\log \varepsilon_j + m_j \log(\Delta K_j)_i},\ s\right]. \tag{7}$$

Sampling can be carried out according to this distribution. At every point of $[(\Delta K_j)_i, (\frac{dX_j}{dN})_i]$, we can sample five random two dimension numbers, and a least square fit is done to obtain the values of ε_j and m_j. The sampling is repeated a hundred times and the statistical analysis is conducted. The simulation results are listed in Table 5.

Table 5 Monte-Carlo simulation results for $C_{V_k} = 0.0346$.

		Group 1	Group 2	Group 3	Group 4
m_j	Mean	2.5746	2.5573	2.3959	2.3996
	Standard deviation	1.6593	1.7417	1.4406	1.5074
$\log \varepsilon_j$	Mean	−10.7170	−10.5634	−10.6767	−10.2493
	Standard deviation	0.4274	0.7621	0.0705	0.1586
r	Mean	0.9187	0.9220	0.9216	0.9218
	Standard deviation	0.06277	0.05936	0.05983	0.05990
s	Mean	0.5196	0.5404	0.5054	0.5464
	Standard deviation	0.3459	0.5895	0.3069	0.2799

It can be seen from the comparison of Table 4 with Table 5 that the results of Monte-Carlo simulation and those of experiments show only a little difference. That is due to the randomness of simulating experiments and the limited number of simulations. However, the difference has little effect on the results.

According to the sample obtained, the distribution pattern of ε_j and m_j can be determined. Because ε_j and m_j are statistically related, only one of them is taken for consideration. Here ε_j is taken, and χ^2 inspection is used to inspect the normal and log-normal distributions of ε_j. The calculation results are shown in Table 6.

Table 6 Inspection results of χ^2 for ε_j

	Group 1	Group 2	Group 3	Group 4
Lognormal	8.7453	10.2148	9.1742	11.2132
Normal	10.7234	15.4728	13.2867	19.6275

The calculation method for χ^2 is as follows:

Firstly, the statistic value x_i ($x_i = (\varepsilon_j)_i$ in normal inspection, and $x_i = \log(\varepsilon_j)_i$ in lognormal inspection) is normalized as

$$u_i = \frac{x_i - \bar{x}}{s} . \tag{8}$$

Then the interval of [−4, 4] is divided into eight sub-intervals and the frequency of u_i belonging to the ith sub-interval is calculated, leading to

$$\chi^2 = \sum_{i=1}^{8} \frac{(n_i - 100p_{i0})^2}{100p_{i0}} \tag{9}$$

where p_{i0} is the theoretical probability belonging to the ith sub-interval.

Taking the significance level $\alpha = 0.05$, freedom degree $\gamma = 8 - 1 = 7$ gives

$$\chi^2_{0.95}(7) = 14.067 .$$

Compared with Table 5, the four groups of test data are in log-normal distribution, moreover, χ^2 value of log-normal distribution is smaller than that of normal distribution, therefore, ε_j takes a log-normal distribution.

The relation of ε_j and m_j can be expressed as follows:

$$\log \varepsilon_j = \frac{1}{8} \sum_{i=1}^{8} \log \left(\frac{dX_j}{dN}\right) - \frac{m_j}{8} \sum_{i=1}^{8} \log (\Delta K_j)_i \tag{10}$$

which illustrates that ε_j and m_j are interrelated. The experiments also attest that ε_j and m_j are negatively dependent. In general, ε_j and m_j satisfy the following formula (Duan *et al.*, 1997)

$$\log \varepsilon_j = Am_j + B \tag{11}$$

where A and B are constants. In our cases, A and B are obtained by least square process of the test data and the results are listed in Table 7. It can be seen that r is approximately equal to -1, indicating a very high correlation of the two parameters ε_j and m_j. Moreover, m_j follows a normal distribution as seen from Eq. (11).

Table 7 Correlation of ε_j and m_j

	Group 1	Group 2	Group 3	Group 4
A	−6.9662	−6.5266	−7.2509	−6.8312
B	−1.4558	−1.5786	−1.4287	−1.4244
r	−0.9951	−0.9952	−0.9940	−0.9947
s	0.2382	0.2701	0.2249	0.2204

5. Conclusion

The Monte-Carlo technique as a practical engineering method is presented for simulating the surface fatigue crack growth rate and determining distribution functions and relevance of the parameters in the Paris equation. The technique is demonstrated by use of four sets of FCP data for offshore structural steel E36-Z35. The comparison of the experimental data with theoretical predictions shows good agreement. The difference between the Monte-Carlo simulation and experimental results could be minimized by increasing the number of simulations.

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