

含有限裂纹弹性体中二次反射波的影响¹⁾

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摘要 处理了无界体中一无限长、有限宽的平面应变裂纹, 对任意入射膨胀波的散射问题. 这里采用了 Wiener-Hopf 技术及标准迭代方法, 得到了二次反射波到达后的应力强度因子的解析表达式, 并给出了数值结果.

关键词 动态断裂, 反射波, 迭代, 动态应力强度因子

引 言

由于数学上的困难, 动态断裂问题中应力波的散射问题很少得到解决. Loeber 和 Sih^[1,2] 处理了含有限裂纹和有限钢条的无限弹性体对简谐变化的水平剪切波 (SH) 的散射问题, 得到了裂纹尖端场和钢条尖端场的近场结果; Sih^[3] 对同样的裂纹体研究了瞬态波的散射问题. 在上述工作中, 主要通过级数展开求解积分方程, 得到适合于远场和近场的解. Thau 和 Lu^[4,5] 采用积分变换和 Wiener-Hopf 技术将问题化成一对对偶奇异积分方程, 用标准迭代求解的方法, 分别研究了二维情形下 SH 波和 P 波在含有限裂纹弹性体中的响应, 每一次迭代得到的解在一定时间范围内是准确的. Freund^[6] 采用运动位错方法分析了同样问题受瞬态 I 型载荷作用的特殊情形, 得到相同的结果. 他们进行了一次迭代, 得到了一次入射波与裂纹尖端作用产生的一次反射波到达所考察裂尖时的应力强度因子的解析解. 后面各次反射波的影响尚未予以考虑. 在此本文借鉴了 Stephen Thau 和 Tsin-Hwi Lu^[4,5] 的方法分析了二维含有限裂纹的无限弹性体对任意入射膨胀波

的散射问题, 考察了二次反射波对应力强度因子的影响.

1 基本控制方程和边界条件

如图 1 所示: 无限弹性体中存在一无限长、有限宽的平面裂纹, x, y 为直角坐标系, 裂纹位于 $y = 0, 0 < x < l$. 一平面瞬态膨胀波作用在裂纹面上, 其传播方向与 z 轴垂直, 与 x 轴正方向成 α 角.

此问题可作为平面应变问题处理. 位移与 z 坐标无关, 并可用膨胀波势 $\varphi(x, y, t)$ 和剪切波势 $\psi(x, y, t)$ 表示

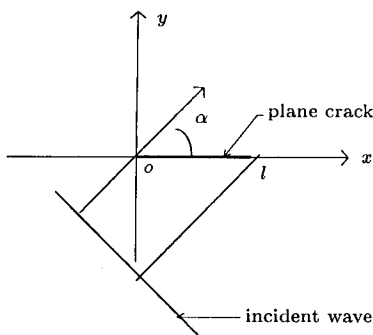


图 1 弹性体的几何形状
Fig.1 Geometrical configuration
of the elastic solid

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$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x}, \quad w = 0 \quad (1.1)$$

运动方程可表示为

$$\nabla^2 \varphi = \frac{1}{c_d^2} \frac{\partial^2 \varphi}{\partial t^2}, \quad \nabla^2 \psi = \frac{1}{c_s^2} \frac{\partial^2 \psi}{\partial t^2} \quad (1.2)$$

其中 c_d, c_s 分别为膨胀波速和剪切波速. $y = 0$ 处的边界条件为

$$v(x, 0, t) = 0, \quad x > l \text{ 或 } x < 0 \quad (1.3)$$

$$\sigma_{xy}(x, 0, t) = 0, \quad -\infty < x < \infty \quad (1.4)$$

$$\sigma_{yy}(x, 0, t) = \begin{cases} \tau_1(x, 0, t), & x > l \\ -p_0 H(t - x \cos \alpha / c_d), & 0 < x < l \\ \tau_0(x, 0, t), & x < 0 \end{cases} \quad (1.5)$$

另外, 无穷远处位移条件

$$\lim_{x^2+y^2 \rightarrow \infty} [u, v] = 0 \quad (1.6)$$

2 求解方法

下面用积分变换方法求解, 下方程通过膨胀波势 φ 表示, 剪切波势 ψ 可作类似处理.

首先引入关于时间 t 的单边 Laplace 变换, 然后关于 x 作双边 Laplace 变换, 其变换参量为 s, η

$$\bar{\varphi}(\eta, y, s) = \int_{-\infty}^{\infty} \int_0^{\infty} \hat{\varphi}(x, y, s) e^{-st} e^{-s\eta x} dt dx \quad (2.1)$$

关于势函数 φ 和 ψ 的控制方程经过二重变换后, 其变换域中的适当解为

$$\bar{\varphi}(\eta, y, s) = A \exp(-s\alpha x), \quad \bar{\psi}(\eta, y, s) = B \exp(-s\beta x) \quad (2.2)$$

其中

$$\alpha^2 = a^2 - \eta^2, \quad \beta^2 = b^2 - \eta^2, \quad a = 1/c_d, \quad b = 1/c_s \quad (2.3)$$

复平面 η 沿割线 $a < |\operatorname{Re}(\eta)| < \infty, \operatorname{Im}(\eta) = 0$ 切开, 使得在割开的复平面中, 对每一值 η , 均有 $\operatorname{Re}(\alpha) \geq 0$, 类似地有 $\operatorname{Re}(\beta) \geq 0$. 对边界条件 (1.5) 和 (1.4) (1.3) 作二重 Laplace 变换, 如果 A, B 从这些方程中消去, 结果为

$$-\frac{\mu}{b^2} \frac{R(\eta)}{\alpha(\eta)} V(\eta, s) = p_0 \frac{e^{-s(\eta+a \cos \alpha)} - 1}{\eta + a \cos \alpha} + \gamma_- + e^{-s\eta} \gamma_+ \quad (2.4)$$

其中

$$\left. \begin{aligned} R(\eta) &= 4\eta^2 \alpha \beta + (b^2 - 2\eta^2)^2 \\ V(\eta, s) &= s^3 \int_{-\infty}^{+\infty} \int_0^{\infty} v(x, t) e^{-st} e^{-s\eta x} dt dx \\ \gamma_+(\eta, s) &= s^2 \int_{-\infty}^{+\infty} \int_0^{\infty} \tau_1(x, s) e^{-st} e^{-s\eta x} dt dx \\ \gamma_-(\eta, s) &= s^2 \int_{-\infty}^{+\infty} \int_0^{\infty} \tau_0(x, s) e^{-st} e^{-s\eta x} dt dx \end{aligned} \right\} \quad (2.5)$$

下标 +, - 分别指 η 平面上在区域 $\text{Re}(\eta) > -a$ 和 $\text{Re}(\eta) < a$ 上解析的函数. 如下确定: τ_0 和 τ_1 分别代表从裂尖 $x = 0, l$ 处产生的波, 因而包含指数因子 $\exp(sx), \exp[-s(x-l)]$, 研究可得出: τ_0 积分变换在 - 区域上解析, 即 $\text{Re}(\eta) < a, \tau_1$ 的积分变换在 + 区域上解析, 即 $\text{Re}(\eta) > -a$, 所有变换在重叠条带上 $|\text{Re}(\eta)| < a$ 解析.

引入一新函数 $S(\eta)$ 定义为

$$S(\eta) = \frac{R(\eta)}{k(c^2 - \eta^2)}, \quad k = 2(b^2 - a^2), \quad c = 1/c_R \tag{2.6}$$

这里 c_R 为瑞利波速.

式 (2.4) 变为

$$-\frac{\mu k}{b^2} \alpha^{-1} (c^2 - \eta^2) S(\eta) V = \gamma_- + e^{-s\eta} \gamma_+ + p_0 \frac{-1 + e^{-s(\eta + a \cos \alpha)}}{\eta + a \cos \alpha} \tag{2.7}$$

函数 $S(\eta)$ 可因子分解为

$$S_{\pm}(\eta) = \exp \left\{ -\frac{1}{\pi} \int_a^b \tan^{-1} \left[\frac{4\xi^2 \sqrt{b^2 - \xi^2} \sqrt{\xi^2 - a^2}}{(b^2 - 2\xi^2)} \right] \frac{\xi d\xi}{\xi \pm \eta} \right\} \tag{2.8}$$

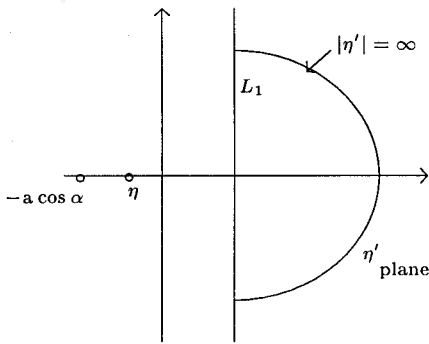


图 2 直线 L_1 和 η' 平面间的关系
Fig.2 The relation between the line L_1 and the η' plane

同样函数 $\alpha(\eta), D(\eta)$ 可因子分解为

$$\alpha(\eta) = [\sqrt{a + \eta}]_+ [\sqrt{a - \eta}]_- \tag{2.9}$$

$$D_{\pm}(\eta) = (a \pm \eta)^{-1/2} (c \pm \eta) S_{\pm}(\eta) \tag{2.10}$$

将 (2.7) 式中的 η 换为 η' , 两边同除以 $2\pi i D_-(\eta')(\eta' - \eta)$ 后, 沿 L_1 对 η' 积分, 设 L_1 为平行于虚轴的直线, 位于 $|\text{Re}(\eta')| < a$, 且假定 $\eta' = \eta, \eta' = -a \cos \alpha$ 位于 L_1 左边. 如图 2 所示. 左边积分在 L_1 右边是解析的, 且在 $x = 0, -l$ 处 $v = 0$ 的边界条件保证当 $|\eta'| \rightarrow \infty$ 时, $|V| \rightarrow 0$ 充分收敛, 因而沿 L_1 右边半无限圆积分为 0. 右边积分结果经变换可表示为

$$\begin{aligned} \gamma_-(\eta, s) &= \frac{p_0}{\eta + a \cos \alpha} \left[1 - \frac{D_-(\eta)}{D_+(a \cos \alpha)} \right] + \\ &\frac{D_-(\eta)}{2\pi i} \int_{L_1} \frac{e^{-s\eta'}}{D_-(\eta')(\eta' - \eta)} \left(\gamma_+ + \frac{p_0 e^{-sa \cos \alpha}}{\eta' + a \cos \alpha} \right) d\eta' \end{aligned} \tag{2.11a}$$

同样将 (2.7) 式中 η 换为 η' 两边同除以 $2\pi i D_+(\eta')(\eta' - \eta)$, 沿 L_2 积分. 设 L_2 为平行于虚轴的直线, 位于 $|\text{Re}(\eta')| < a$, 且假定 $\eta' = \eta, \eta' = -a \cos \alpha$ 位于 L_2 右边. 积分后结果变换为

$$\begin{aligned} \gamma_+(\eta, s) &= \frac{p_0 e^{-sa \cos \alpha}}{\eta + a \cos \alpha} \left[\frac{D_+(\eta)}{D_+(-a \cos \alpha)} - 1 \right] - \\ &\frac{D_+(\eta)}{2\pi i} \int_{L_2} \frac{e^{s\eta'}}{D_+(\eta')(\eta' - \eta)} \left(\gamma_- - \frac{p_0}{\eta' + a \cos \alpha} \right) d\eta' \end{aligned} \tag{2.11b}$$

这样，我们得到了一对耦合的奇异积分方程 (2.11)，下面我们用标准迭代法来求解。(2.11) 中第一项为零阶解 γ_+^0, γ_-^0 ，将 γ_+^0, γ_-^0 表达式代入 (2.11) 产生一阶解 γ_+^1, γ_-^1 ，通过这个过程类推 n 阶解可通过 $(n-1)$ 阶解加上 $(n-1)$ 阶解积分得到。每一次迭代对应一定时间内的精确解，准确解的有效时间随迭代次数的增加而增加。下面我们将讨论零阶解、一阶解和二阶解。

从方程 (2.11) 得到零阶解

$$\left. \begin{aligned} \gamma_+^0(\eta, s) &= \frac{p_0 e^{-sal \cos \alpha}}{\eta + a \cos \alpha} \left[\frac{D_+(\eta)}{D_+(-a \cos \alpha)} - 1 \right] \\ \gamma_-^0(\eta, s) &= \frac{p_0}{\eta + a \cos \alpha} \left[1 - \frac{D_-(\eta)}{D_+(a \cos \alpha)} \right] \end{aligned} \right\} \quad (2.12)$$

分别对应半无限裂纹 $x > l$ 和 $x < 0$ 的解，从物理上讲，这些结果在一次反射波到达考察点前是准确的。

将 (2.12) 代入 (2.11) 得到一阶结果即一次反射波到达后的解

$$\left. \begin{aligned} \gamma_+^1 &= \gamma_+^0 + \frac{D_+(\eta)p_0}{2\pi i D_+(a \cos \alpha)} \int_{L_2} \frac{D_-(\eta') e^{st\eta'}}{D_+(\eta')(\eta' - \eta)(\eta' + a \cos \alpha)} d\eta' \\ \gamma_-^1 &= \gamma_-^0 + \frac{D_-(\eta)p_0}{2\pi i D_+(-a \cos \alpha)} \int_{L_1} \frac{D_+(\eta') e^{-st\eta'} e^{-sal \cos \alpha}}{D_-(\eta')(\eta' - \eta)(\eta' + a \cos \alpha)} d\eta' \end{aligned} \right\} \quad (2.13)$$

同样得到二阶结果即二次反射波到达后的解

$$\left. \begin{aligned} \gamma_+^2 &= \gamma_+^1 - \frac{D_+(\eta)p_0}{2\pi i D_+(-a \cos \alpha)} \int_{L_2} \frac{D_-(\eta') e^{st\eta'}}{D_+(\eta')(\eta' - \eta)} \frac{1}{2\pi i} \\ &\quad \int_{L_1} \frac{D_+(\zeta) e^{-sl\zeta} e^{-sal \cos \alpha}}{D_-(\zeta)(\zeta - \eta')(\zeta + a \cos \alpha)} d\zeta d\eta' \\ \gamma_-^2 &= \gamma_-^1 + \frac{D_-(\eta)p_0}{2\pi i D_+(a \cos \alpha)} \int_{L_1} \frac{D_+(\eta') e^{-st\eta'}}{D_-(\eta')(\eta' - \eta)} \frac{1}{2\pi i} \\ &\quad \int_{L_2} \frac{D_-(\zeta) e^{sl\zeta}}{D_+(\zeta)(\zeta - \eta')(\zeta + a \cos \alpha)} d\zeta d\eta' \end{aligned} \right\} \quad (2.14)$$

3 动态应力强度因子

根据通常定义有

$$\sigma_{yy}(x, 0, t) \sim \frac{K_I(t)}{\sqrt{2\pi|x|}}, \quad x \rightarrow 0^-, \quad \sigma_{yy}(x, 0, t) \sim \frac{K_I(t)}{\sqrt{2\pi|x-l|}}, \quad x \rightarrow l^+ \quad (3.1)$$

K_I 为沿裂纹前缘的动态应力强度因子。

变换函数渐进特性的 Abel 定理给出

$$\overline{K_I}(0, s) = \lim_{\eta \rightarrow -\infty} \left[(2s|\eta|)^{\frac{1}{2}} \frac{\gamma_-(\eta, s)}{s^2} \right], \quad \overline{K_I}(l, s) = \lim_{\eta \rightarrow \infty} \left[(2s\eta)^{\frac{1}{2}} \frac{\gamma_+(\eta, s)}{s^2} \right] \quad (3.2)$$

$\overline{K}_I(0, s), \overline{K}_I(l, s)$ 分别表示 $x = 0, x = l$ 处的应力强度因子.

$$\overline{K}_I = \overline{K}_I^0 + \Delta\overline{K}_I^1 + \Delta\overline{K}_I^2 \tag{3.3}$$

$\overline{K}_I^0, \Delta\overline{K}_I^1, \Delta\overline{K}_I^2$ 分别表示入射波、一次反射波、二次反射波产生的应力强度因子.

利用方程 (2.12)~(2.14), (3.2), 得到 $\overline{K}_I^0(0, s), \Delta\overline{K}_I^1(0, s), \Delta\overline{K}_I^2(0, s)$, 其关于 s 的 Laplace 反变换即裂纹前缘 $x \rightarrow 0^-$ 的应力强度因子结果为

$$K_I^0(0, t) = \frac{4p_0 \cos(\alpha/2) T^{1/2} l^{1/2}}{\sqrt{\pi}(C + \cos \alpha) S_+(a \cos \alpha)} \tag{3.4a}$$

$$\Delta K_I^1(0, t) = \frac{4p_0 l^{1/2}}{\pi^{3/2}} \cdot \frac{\sin(\alpha/2)}{(C - \cos \alpha) S_-(a \cos \alpha)} \int_1^T \frac{(\lambda - 1)^{1/2} (C + \lambda) S_+^2(\lambda) \sqrt{T - \lambda - \cos \alpha}}{(\lambda + 1)^{1/2} (C - \lambda) S(\lambda) (\lambda + \cos \alpha)} d\lambda \tag{3.4b}$$

$$\Delta K_I^2(0, t) = -\frac{4p_0 l^{1/2} \cos(\alpha/2)}{\pi^{5/2} (C + \cos \alpha) S_+(a \cos \alpha)} \int_2^T \int_1^{T-1} F(\lambda, \tau) G(\lambda) d\lambda d\tau \tag{3.4c}$$

其中

$$\left. \begin{aligned} F(\lambda, \tau) &= \frac{(\tau - \lambda - 1)^{1/2} (C + \tau - \lambda) S_+^2(\tau - \lambda) \sqrt{T - \tau}}{(\tau - \lambda + 1)^{1/2} (C - \tau + \lambda) S(\tau - \lambda) \tau} \\ G(\lambda) &= \frac{(\lambda - 1)^{1/2} (C + \lambda) S_+^2(\lambda)}{(\lambda + 1)^{1/2} (C - \lambda) S(\lambda) (\lambda - \cos \alpha)} \end{aligned} \right\} \tag{3.4d}$$

$x = l$ 处的应力强度因子可同样得出

$$K_I^0(l, t) = \frac{4p_0 \sin(\alpha/2) T^{1/2} l^{1/2}}{\sqrt{\pi}(C - \cos \alpha) S_+(-a \cos \alpha)} \tag{3.5a}$$

$$\Delta K_I^1(l, t) = \frac{4p_0 l^{1/2}}{\pi^{3/2}} \cdot \frac{\cos(\alpha/2)}{(C + \cos \alpha) S_+(a \cos \alpha)} \int_1^T \frac{(\lambda - 1)^{1/2} (C + \lambda) S_+^2(\lambda) \sqrt{T - \lambda}}{(\lambda + 1)^{1/2} (C - \lambda) S(\lambda) (\lambda - \cos \alpha)} d\lambda \tag{3.5b}$$

$$\Delta K_I^2(l, t) = -\frac{4p_0 l^{1/2} \sin(\alpha/2)}{\pi^{5/2} (C - \cos \alpha) S_+(-a \cos \alpha)} \int_2^T \int_1^{T-1} F'(\lambda, \tau) G'(\lambda) d\lambda d\tau \tag{3.5c}$$

其中

$$\left. \begin{aligned} F'(\lambda, \tau) &= \frac{(\tau - \lambda - 1)^{1/2} (C + \tau - \lambda) S_+^2(\tau - \lambda) \sqrt{T - \tau - \cos \alpha}}{(\tau - \lambda + 1)^{1/2} (C - \tau + \lambda) S(\tau - \lambda) \tau} \\ G'(\lambda) &= \frac{(\lambda - 1)^{1/2} (C + \lambda) S_+^2(\lambda)}{(\lambda + 1)^{1/2} (C - \lambda) S(\lambda) (\lambda + \cos \alpha)} \end{aligned} \right\} \tag{3.5d}$$

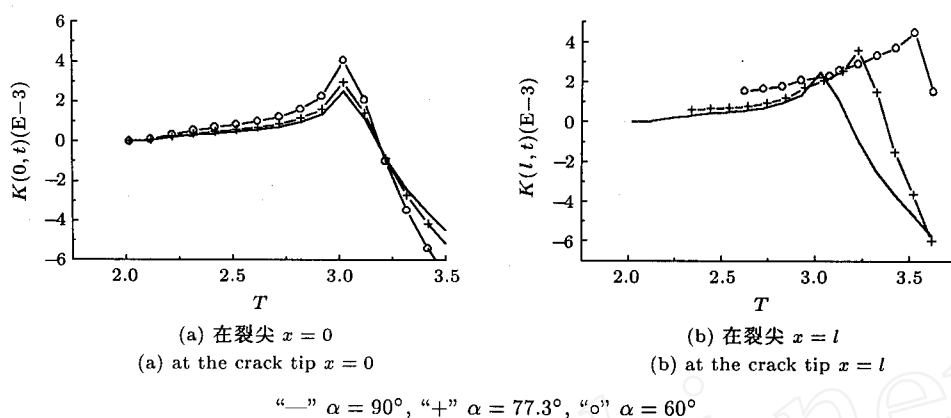
上面式中

$$C = c/a, \quad T = t/(al) = c_d t/l \tag{3.6}$$

至此, 我们得到了入射波、一次反射波、二次反射波分别产生的应力强度因子的解析解.

4 结果和讨论

对方程 (3.4), (3.5) 中积分, 关于不同的入射角 α , 本文进行了数值计算, 计算参数为 $b = 1.87a, c = 2.02a$, 将结果绘于图 3~图 5 中. 动态应力强度因子除以相应稳态值 $p_0 \sqrt{\pi l/2}$. 图中时



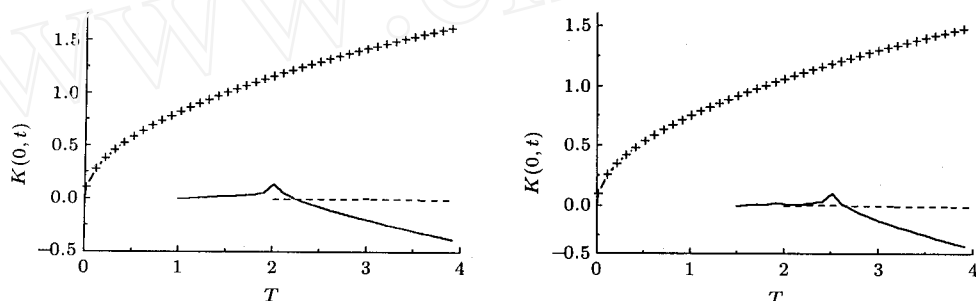
(a) 在裂尖 $x=0$
(a) at the crack tip $x=0$

(b) 在裂尖 $x=l$
(b) at the crack tip $x=l$

“—” $\alpha=90^\circ$, “+” $\alpha=77.3^\circ$, “o” $\alpha=60^\circ$

图 3 关于不同的入射角 α 二次反射波产生的应力强度因子随时间的变化

Fig.3 Stress intensity factor caused by the second scattered wave vs. time for various values of α



(a) 在裂尖 $x=0, \alpha=90^\circ$
(a) at the crack tip $x=0, \alpha=90^\circ$

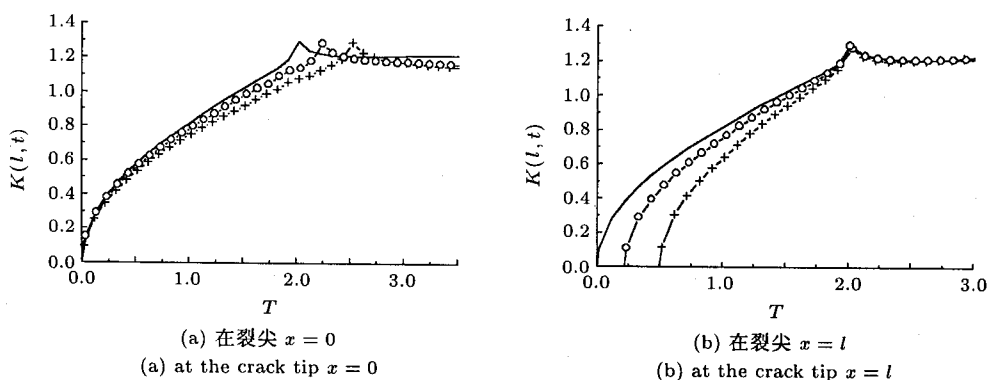
(b) 在裂尖 $x=0, \alpha=60^\circ$
(b) at the crack tip $x=0, \alpha=60^\circ$

“—” 一次反射波, “+” 一次波, “---” 二次反射波

“—” first scattered wave, “+” first wave, “---” second scattered wave

图 4 不同情形各次波产生应力强度因子随时间变化

Fig.4 Stress intensity factor vs. time for various condition



(a) 在裂尖 $x=0$
(a) at the crack tip $x=0$

(b) 在裂尖 $x=l$
(b) at the crack tip $x=l$

“—” $\alpha=90^\circ$, “+” $\alpha=77.3^\circ$, “o” $\alpha=60^\circ$

图 5 应力强度因子随时间变化关系

Fig.5 Stress intensity factor vs. time

间无量纲化, $T = c_d t / l$, 对于裂尖 $x = 0$, 时间 $T = 0, T = 1 + \cos \alpha, T = 2$ 分别相应于一次入射膨胀波、一次反射膨胀波、二次反射膨胀波到达考察点的时间, 所得的应力强度因子在三次反射波到达前即 $T < 3 + \cos \alpha$ 是准确的. 对于裂尖 $x = l$, 时间 $T = \cos \alpha, T = 1, T = 2 + \cos \alpha$ 分别相应于一次入射膨胀波、一次反射膨胀波、二次反射膨胀波到达考察点的时间, 所得的应力强度因子在三次反射波到达前即 $T < 3$ 是准确的. 图 3 为二次反射波产生的应力强度因子随时间的变化关系, 图 4 为各次波产生的应力强度因子的比较, 图 5 为各次波共同产生的应力强度因子随时间变化关系. 结果表明: 应力强度因子随二次反射膨胀波到达后使有所增加, 并随二次反射瑞利波的到达而下降. 二次反射波使应力强度因子的峰值增大约 5%, 一次反射波使应力强度因子的峰值增大约 10%, 相比起来影响非常小, 这一点从图 3, 图 4 可以看出. 可得出结论: 除一次反射波的影响较为显著外, 以后各次的反射波的影响非常小, 可以忽略不计.

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THE EFFECT OF THE SECOND SCATTERED WAVE FOR A FINITE CRACK IN AN ELASTIC CAUSED BY DILATATIONAL WAVE¹⁾

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Abstract In the dynamic fracture field, the problem with scattered stress wave is very difficult to analyze. At present only a few analytical results have been obtained. Stephen A Thau and Tsin-Hwei Lu (1970, 1971) treat the transient problem of diffraction of an a plane transient SH wave and an arbitrary plane dilatational wave by a finite crack in an infinite elastic solid. They employ the generalized Wiener-Hopf technique and the standard iteration method. Freund (1990) treat the analogous crack subjected to I load by the moving dislocation method. In all the work

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before only the effect of the first scattered wave is studied and that of the following scattered wave hasn't been considered.

In the paper the diffraction of transient arbitrary plane dilatational wave by a finite crack in an infinite elastic medium is analyzed. The general Wiener-Hopf technique and the standard iteration method are used to obtain the explicit expressions of the SIF after the second scattered wave reaches the investigated crack tip and numerical calculations are performed. The effect of the first wave, the first scattered and the second scattered wave is compared and shows that the SIF increases with the arrival of the second scattered longitudinal wave and decreases with the arrival of the second scattered Rayleigh wave, the peak SIF rises 10% with the arrival of the first scattered wave, but rises 5 % with the arrival of the second scattered wave, much less than the effect of the first scattered wave. So except the effect of the first scattered wave, the effect of the following scattered wave can be omitted.

Key words dynamic fracture, diffracted wave, iteration, dynamic stress intensity factor