# A numerical study of indentation using indenters of different geometry

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Finite element simulation of the Berkovich, Vickers, Knoop, and cone indenters was carried out for the indentation of elastic–plastic material. To fix the semiapex angle of the cone, several rules of equivalence were used and examined. Despite the asymmetry and differences in the stress and strain fields, it was established that for the Berkovich and Vickers indenters, the load–displacement relation can closely be simulated by a single cone indenter having a semiapex angle equal to 70.3° in accordance with the rule of the volume equivalence. On the other hand, none of the rules is applicable to the Knoop indenter owing to its great asymmetry. The finite element method developed here is also applicable to layered or gradient materials with slight modifications.

## I. INTRODUCTION

In recent years, microindentation and nanoindentation have become a standard test in investigating the mechanical properties of various new types of surface materials. As a principal tool, application of the finite element method (FEM) to simulate the indentation process plays an important role in interpreting the experimental phenomena and in providing much insight and detailed data for a better description of material properties. In their studies, Bhattacharya and Nix[1,2] and Laursen and Simo[3] had to replace the standard Vickers and Berkovich indenters by a conic indenter to reduce the problem from three-dimensional (3D) to more tractable two-dimensional (2D). Even so, due to the limited computer performance of the time, it still would take 1 to 2 days to solve such a finite element problem with 400 to 2000 four-node-rectangular elements on a supercomputer. Although this artifice greatly reduced the work load, certain new problems arise, such as (i) how well can the fine stress–strain field around the tip of a non-conic indenter be simulated by a cone, and (ii) how to determine the semiapex angle of the cone to ensure that the load–displacement curve of the 3D indenter is sufficiently well simulated. The clarification of problems such as these ultimately determines the usefulness of the 2D axial symmetrical simulation.

In the work of Bhattacharya and Nix[1,2] and Laursen and Simo,[3] the apex angle of the cone was taken to be 136°, which is the included angle of a Vickers indenter (see Fig. 3). This choice leads to a displaced volume (defined as the volume bounded by the lateral surface and the base area) versus depth of indentation relation that very closely emulates that of either the Vickers or the Berkovich indenters. Sun et al.[4] and Bolshakov et al.[5] substituted the Berkovich indenter by a conic indenter with 70.3° semiapex angle according to the rule that the relationship between the base area and indentation depth of the two indenters be identical. Recently, following the rapid development of the nanoindentation technique and the progress in processing technology, different values of the cone angle have been used in the literatures on FEM simulation. Generally speaking, four approaches have been used: (i) the volume equivalence based on identical volume versus indentation depth relation; (ii) the lateral surface area equivalence based on identical lateral surface area versus indentation depth relation; (iii) the base area equivalence based on identical base area versus indentation depth relation; and (iv) simply choose 68° as the semiapex angle. Each approach has its own emphasis. It should be noted that for all the indenters examined in the current paper, the equal volume equivalence and the equal base area equivalence rules are identical. Roughly, for the same depth of indentation, the volume equivalence requires that the volume of the material displaced by equivalent indenters be the same; surface area equivalence requires that the average pressure on the contact surface of material under different indenters be the same; base area equivalence requires that the definition of nanoindentation hardness remains unchanged; and the 68° rule reflects a characteristic of the Vickers indenter.

The 3D simulation of nanoindentation process was first aimed at investigating the 3D characteristics of indentation and the indentation of inhomogeneous materials.[6] The Vickers indenter and Knoop indenter are standard indenters in current microindentation equipment. The standard indenter of nanoindentation equipment is usually the Berkovich indenter. In the current study, the 3D indentation using different indenters were simulated by employing FEM, and conic indenters were constructed according to the different equivalence rules. The applicability of the 2D axial symmetrical cone model and the rules followed in determining the conic apex angle were discussed by comparing the stress–strain fields and the load–displacement curves.

## II. DESCRIPTION OF GEOMETRICAL MODEL

The basic geometry of the conic, Berkovich, Vickers, and Knoop indenters are shown in Figs. 1, 2, 3, and 4, respectively. Slight differences in indenter geometry may exist depending on the manufacturer. Because all four indenters are self-similar, the volume of each indenter is equal to one-third of the base area times the height measured along the axis of symmetry.

The volume of the cone and the pyramids truncated by the base is defined as the displaced volume mentioned in the previous section. Likewise, the contact surface area is defined as the lateral surface area shown in the figures.

In indentation, we may identify the depth of indentation with the height of the indenter and the intersection of the indenter with the original surface of the specimen being tested as the base area. Then we say that a conic indenter is equivalent to a nonconic but self-similar indenter when for the same depth of indentation the base areas are equal. This is known as the volume equivalence rule. This rule requires that the base area/(height)$^2$ ratio be equal for the cone to be equivalent.

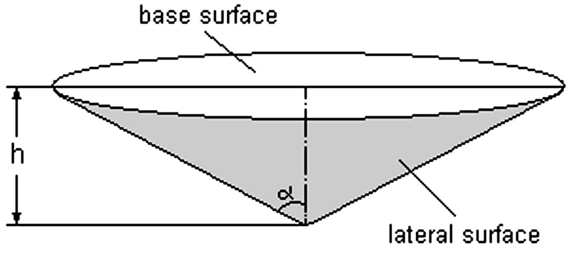

FIG. 1. Sketch of conic indenter.

From Figs. 2, 3, and 4, it can easily be shown that for the Berkovich, Vickers, and Knoop indenter, the base area/(height)$^2$ ratio is respectively equal to 24.57, 24.50, and 65.44. Consequently, a cone with a half apex angle equal to 70.3° having an area/(height)$^2$ ratio equal to 24.51 may be regarded as equivalent to both the Berkovich indenter and the Vickers indenter. According to the same rule, for the Knoop indenter the equivalent conic indenter has a half apex angle equal to 77.6°.

Our numerical simulation shows that for the Berkovich and Vickers indenters, the equivalent conic indenter does yield nearly identical load versus depth of indentations relations. On the other hand, the load versus depth of indentation relation of the Knoop indenter departs markedly from that of its equivalent conic indenter.

The application of the other equivalence rules mentioned in Sec. I will also be discussed.

## III. FEM MODEL AND RESULTS

A detailed description of the 3D FEM simulation of nanoindentation was presented in the literature,[6] in which

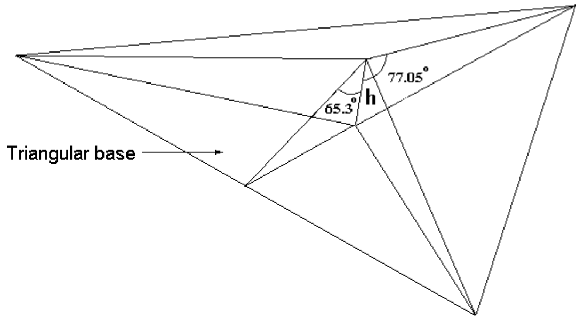

FIG. 2. Sketch of Berkovich indenter.

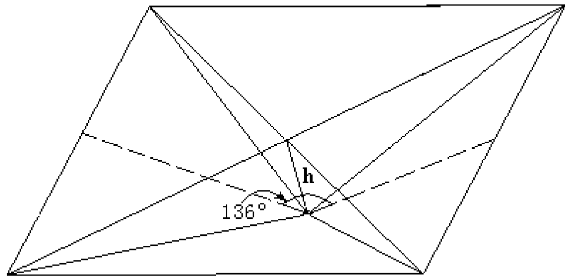

FIG. 3. Sketch of Vickers indenter.

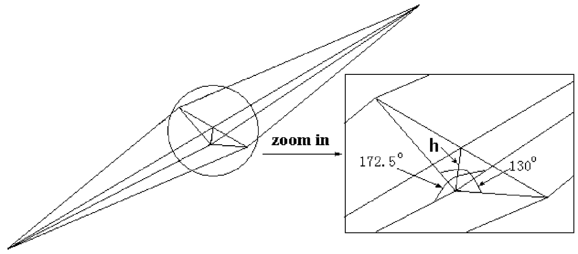

FIG. 4. Sketch of Knoop indenter.

the Berkovich indenter was considered. In the 3D FEM models of the other indenters, mesh density, boundary condition, load–unload method, and contact arithmetics are essentially the same.

From symmetry consideration, one-sixth of the Berkovich indenter is selected and the FEM model is shown in Fig. 5 (3D models for the other indenters are similarly set up). Because of rotational symmetry of the conic indenter, a 2D axial symmetrical element is adopted, and the FEM model is shown in Fig. 6. The 3D model used here had been validated by experiments.[6]

The indenter is made of diamond that is regarded as an elastic body with Young's modulus 1141 GPa and Poisson ratio 0.07. Considering the anisotropy of elastic and plastic cannot be simulated by a 2D model, only isotropic material was taken in this paper. The specimen is copper whose Young's modulus $E$, Poisson ratio $\nu$, yield strength $\sigma_y$, and hardening modulus $H$ are 130 GPa, 0.36, 100 MPa, and 2.652 GPa, respectively. As shown in Fig. 7, the specimen is considered to be an elastic–plastic Von Mises material with isotropic hardening. More calculations of aluminum and tungsten were taken, and the conclusions were consistent with that of copper. To be concise, only results about copper were presented.

The displacement field, stress–strain field, and load–displacement curve can be obtained through calculation. Figure 8 shows the displacement fields of the specimen and indenter under the action of maximum load for the 3D model, where a legend for the displacement is also shown.

Likewise, one-eight and one-fourth of the Vickers and Knoop indenters are selected, and the FEM models and numerical simulation are set up and carried out in a similar way.

## IV. DISCUSSION

The results of the indentation process obtained by FEM simulation can be categorizes as follows: (i) the displacement field, stress field, strain field, and plastic strain field; and (ii) the load–displacement curve of the indentation process. The former reflects the details of material deformation, which are the foundations for further analysis. It can hardly be obtained through experiment and represents a distinctive advantage of numerical simulation over experiment. The latter represents the overall effects of material deformation, which can be compared directly with experiment. In the following, these two kinds of results are separately discussed.

### A. Displacement, stress, and strain fields

Displacement, stress, and strain fields provide fundamental information for detailed study of the indentation process. They are sensitive to the geometry of the indenters. The displacement, stress, strain, and plastic strain fields all reflect the detailed effects induced by the indenter geometry. To be concise, only the plastic strain fields of the Berkovich, Vickers, and Knoop indenters under the same load are shown in Figs. 9, 10, and 11, respectively, where the lower left quadrant and the right lower quadrant represent two different symmetry planes. Figures 12 and 13 show the results for the conic indenter.

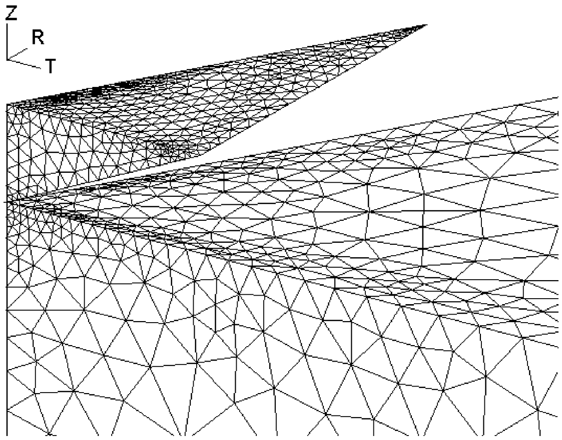

FIG. 5. Local meshes of 3D FEM model.

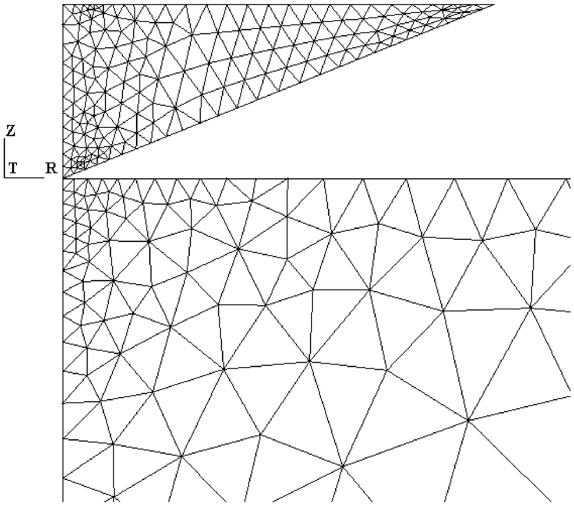

FIG. 6. Local meshes of 2D FEM model.

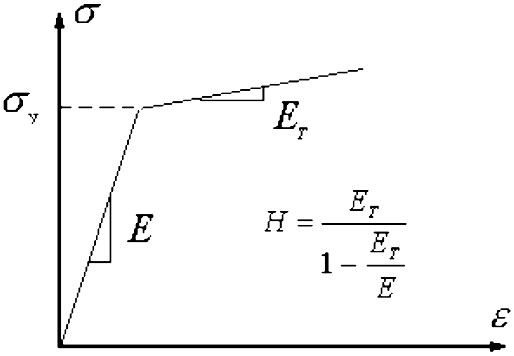

FIG. 7. Sketch of material parameters.

In Fig. 12, the conic semiapex angle is 70.3°, which is equivalent to the Berkovich and Vickers indenters by volume; and in Fig. 13, the conic semiapex angle is taken to be 77.6° to simulate the Knoop indenter by the same equivalence rule. Sixteen equal ranks were taken from the maximum value to minimum value in the contour map of plastic strain. The maximum value and minimum value were shown in the pictures.

In Figs. 9, 10, and 12, it can be seen that the plastic strain fields for the Berkovich indenter and the equivalent conic indenter with a conic semiapex angle of 70.3° are rather similar, and that of the Vickers indenter is even

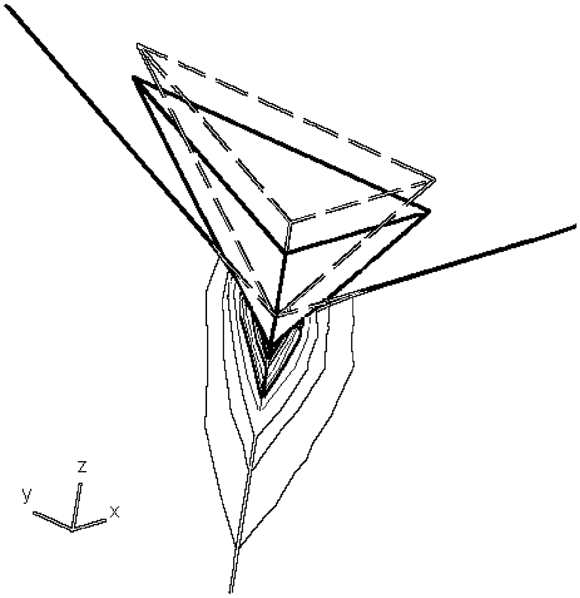

FIG. 8. Displacement field obtained with 3D model.

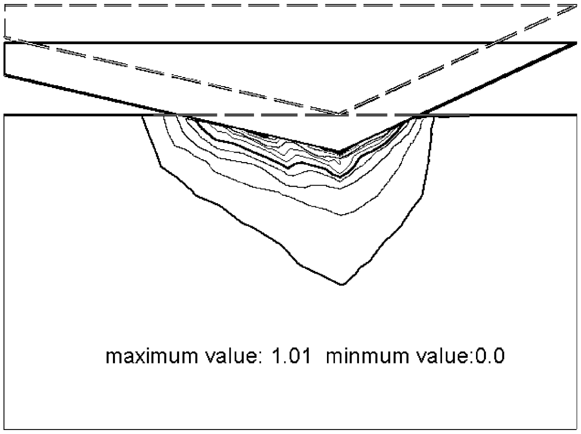

FIG. 9. Plastic strain field under the Berkovich indenter.

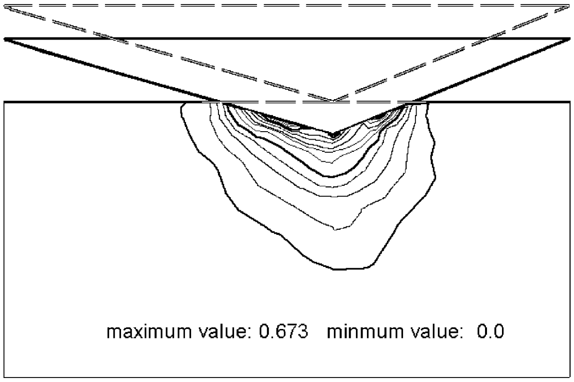

FIG. 10. Plastic strain field under the Vickers indenter.

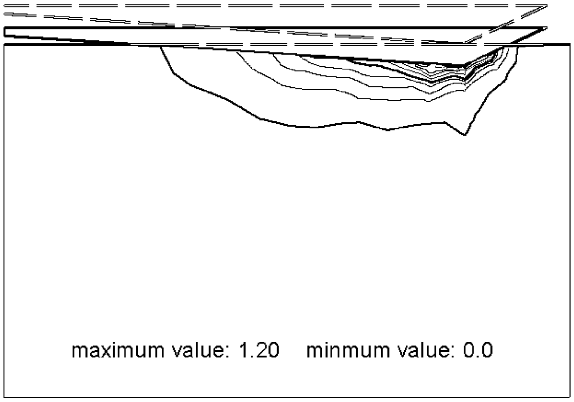

FIG. 11. Plastic strain field under the Knoop indenter.

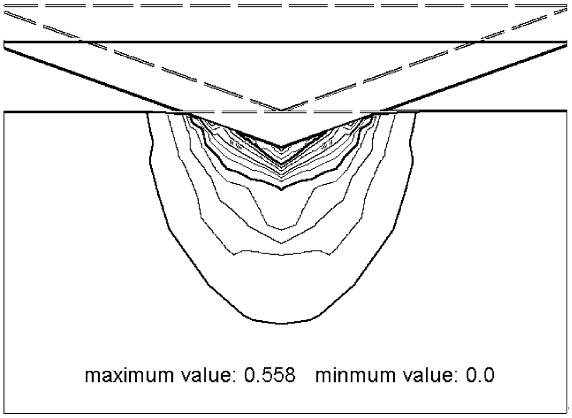

FIG. 12. Plastic strain field under the conic indenter (70.3°).

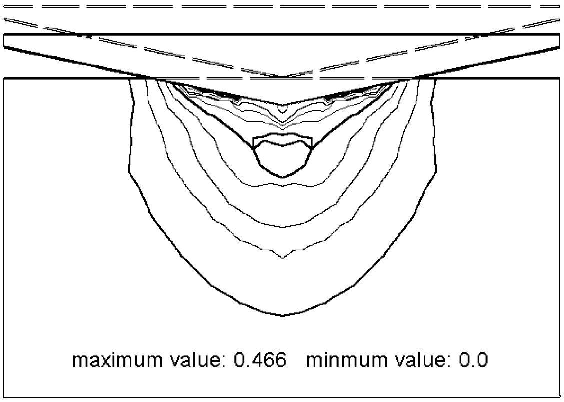

FIG. 13. Plastic strain field under the conic indenter (77.6°).

more so. On the other hand, Figs. 11 and 13 indicate that great difference exists between the plastic strain fields of the Knoop indenter and that of the equivalent conic indenter with a semiapex angle of 77.6°; that is, the plastic strain gradient of the former is clearly larger than that of the latter. For the Knoop indenter, there also exist distinct differences between the symmetrical planes containing the long edge and the short edge.

For the load–displacement curve that reflects the overall effects of indentation, it may be predicted, from the results and discussions above, that the Berkovich and Vickers indenters can be well simulated by the corresponding conic indenters, whereas the Knoop indenter cannot.

## B. Load–displacement curves

The load–displacement curve represents an overall material property, which can directly be compared with results of the indentation experiment. In the case that the research work is not mainly directed at the details of material deformation, some of the material properties and/or variations of them can be investigated by such curves. Figures 14, 15, and 16 compare the load–displacement curves for the Berkovich indenter, Vickers indenter, Knoop indenter, and the conic indenters at given maximum load 22.8 mN, 22.8 mN, and 60 mN. The open circle and open upper triangle line represents lateral area equivalence and volume equivalence, respectively. The percentage displacement errors of the equivalent cone relative respectively to the Berkovich, Vickers, and Knoop indenters at maximum load are shown in Table I. The calculation was shown that the displacement errors have less dependence on applied load.

For the Berkovich and Vickers indenters, irrespective of the equivalence rules adopted, it can be seen from Figs. 14 and 15 and Table I that the load–displacement curves based on the 2D model are in excellent agreement with those based on the 3D model, especially when the volume equivalence rule is adopted. When the equivalence rule of 68° is adopted, the relative displacement error is less than 10%.

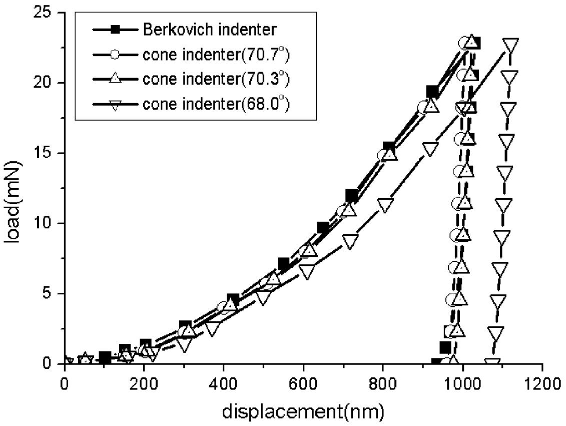

FIG. 14. Results for the Berkovich indenter.

For the Knoop indenter, neither the equivalence volume nor the equivalence contact area approaches can yield a proper 2D model. Figure 16 and Table I show the results obtained by using these 2D models. They would lead to calculated values of hardness far in excess of the experimental values provided by Riester et al.[7] For copper, a great deal of calculation has shown that the Knoop indenter can be well simulated by a conic indenter with a semiapex angle of 81.7°. This angle, however, is sensitive to the material tested; for example, for tungsten, the semiapex angle has to be 79.4° to simulate the load–

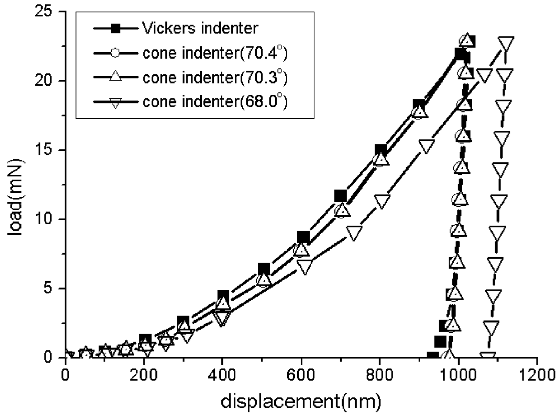

FIG. 15. Results for the Vickers indenter.

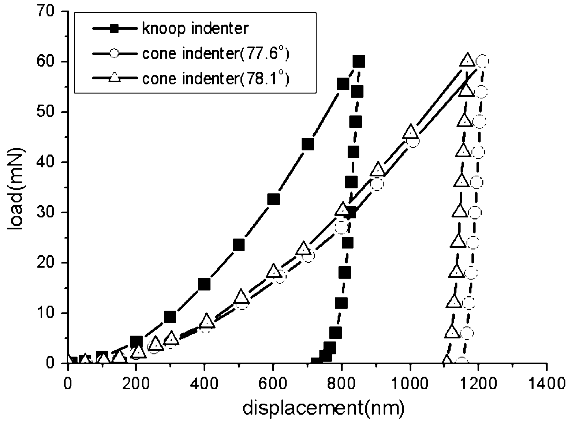

FIG. 16. Results for the Knoop indenter.

TABLE I. The percentage displacement error of several equivalent models.

| Type of indenter | Cone following volume equivalence | Cone following lateral area equivalence | 68° Cone |
|---|---|---|---|
| Berkovich | –0.7% | –2.3% | 8.8% |
| Vickers | –0.6% | –1.0% | 8.9% |
| Knoop | 42.5% | 37.1% | |

displacement curve well. The source of the above phenomenon lies in the large ratio of the two diagonals of the base causing the stress and strain fields under the Knoop indenter to differ greatly from that of the conic indenter.

## V. CONCLUSIONS

The influences of geometrical shape of several indenters such as the Berkovich, Vickers, Knoop, and cone indenters on the deformation field and the load–displacement curve are investigated by means of the FEM numerical simulation. The accuracy of substituting 3D model by an appropriate 2D model is studied. The results indicate that:

(1) Details of the stress–strain fields under different indenters are different. In studies where the details of the displacement or stress–strain fields are important, (e.g., interface stress of thin film materials, the "pile-up" and "sink-in" of the material surface, the strain gradient of the deformation field, and so on), then a 3D simulation is called for.

(2) When only the load–displacement curve is needed, the Berkovich and Vickers indenters can be well simulated by conic indenters.

(3) For the Berkovich and Vickers indenters, a 2D model based on either the volume (or base area) equivalence rule or the lateral surface equivalence rule would yield load–displacement relations in excellent agreement with those from the 3D model. This is especially true when the volume equivalence rule is adopted. When a 68° semiapex angle is adopted, the relative displacement error is less than 10%.

(4) For the Knoop indenter, irrespective of whether the volume equivalence or the lateral surface equivalence rule is adopted, the results based on 2D models differ greatly from those of the 3D model. Also, the choice of the 2D model is material sensitive. The cause lays in the large diagonal ratio of the Knoop indenter, which leads to great differences in the stress–strain fields between the 2D and 3D models.

## ACKNOWLEDGMENT

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