Strain Regularity and Stiffness Tensor of Reinforcers in Dense Particle Reinforced Composites

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Abstract: Based on the study of strain distribution in composites reinforced with high-volume-fraction, randomly distributed particles, the quantitative relation between the average strains of matrix / reinforcers and microstructure parameters of the composite is investigated. It is revealed that besides the average strains, strain fluctuation is also an important factor affecting the stiffness modulus of the composites. The relation between the strain fluctuation and microstructure parameters of the composite is also obtained to derive the stiffness tensor. The theoretical prediction is favorably compared with experimental results.

Key words: particle reinforced composites; strain regularity; strain fluctuation; stiffness tensor
稠密颗粒增强复合材料应变分布规律和刚度张量. 王治国, 李敏, 吴炜, 梁乃刚. 中国航空学报(英文版), 2001, 14(4): 217—221.

摘　要: 针对高体积份数、随机分布、等轴状颗粒增强复合材料, 研究了材料的应变分布规律, 给出了基体和增强体应变平均值与材料微观结构参数之间的定量关系。结果表明, 除应变平均值外, 应变涨落是影响刚度张量的另一个重要因素, 研究了应变涨落与材料微观结构参数之间的关系, 并推导出了复合材料的刚度张量。与实验结果和以往的理论比较, 预测结果与实验结果吻合良好。

关键词: 颗粒增强复合材料; 应变分布规律; 应变涨落; 刚度张量
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Hill[1] pointed out that the prediction of elastic moduli of composites can be carried out if the relation of the local strain and macro strain are obtained. If the reinforcers are spherical/ellipsoidal, and their volume fraction is less than 10%, the elastic moduli of the composites can be predicted accurately by Eshelby equivalent inclusion method[2], the self-consistent method[3] and so on.

However, in actual engineering composites, the shape of the reinforcers is usually far much more complicated than a sphere or ellipsoid, the volume fraction of reinforcers is as high as 10%-40%, and they are irregularly distributed. It seems impossible to cut out a unit cell in which only a unique reinforcer is included, as done in previous theories. For composite in engineering, accurate prediction of effective elastic moduli is still an open problem until now.

In fact, macro mechanical properties of composites, particularly their elastic properties, are not sensitive to the details of the local stress and strain fields at the ends of the reinforcers. A good prediction of macro mechanical properties of composites can be obtained without the so-called exact micromechanical calculation.

Liu[4~6] studied short fiber/whisker reinforeced composites whose aspect ratio is larger than 3 by the network model. The effect of orientation of fiber/whisker on macro properties is considered, while the space location distributive randomicity of short fiber/whisker and the strain distributive detail in the end of reinforcers were neglected. The strain regularity of the matrix and reinforcers and the stiffness tenor are given. The stiffness tensor of the composite in engineering can be predicted in the condition of arbitrarily tropism distributing rein-

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forcers and arbitrarily deformed condition.

In the present paper, the transverse stiffness of composites reinforced with high-volume-fraction, randomly distributed, one-way fibers by the network model is studied. The precision of simulating composites reinforced with high-volume-fraction, randomly distributed particles by a two-dimensional model is discussed.

The quantitative relation between the average strain and fluctuation of matrix/reinforcers and microstructure parameters of the composite is investigated. Then an explicit stiffness tensor is derived, which is compared with the experimental measurement and previous theoretical results.

1 Strain Regularity

1.1 Average strain

The strain distribution regulation of particle reinforced composites is investigated with a two-dimensional network model[7]; a typical example is shown in Fig. 1. The reinforcers with the special volume fraction are distributed into the matrix. The stiffness of elements is decided according to the property ratio of the matrix to the reinforcers. Macro strain fields are given by controlling boundary displacement. According to the displacement of the vertex of triangles A, B and C, shown in Fig. 1(c), the strain ε_{ij} inside triangles can be calculated. The statistic regulation of the strain distribution is derived by a statistic analysis on the strain. The total number of triangles is 51543.

g represents reinforcement; t represents serial number of reinforcement element. For the t-th reinforcement element, the volume is denoted as $V_{t,g}$, the strain is denoted as $\varepsilon_{ij,t,g}$, and then the average strain of the reinforcer can be written as

$$\bar{\varepsilon}_{ij,g} = \frac{1}{V_g}\sum_t \varepsilon_{ij,t,g} V_{t,g} \tag{1}$$

By controlling boundary deformation, the first principal strain is unit value; the other principal strain is zero. The included angles of the principal strain and the X-axis are 0°, 5°, 10°, 15°, 20° and 25°, respectively. For every direction, eleven samples with reinforcers randomly distributed are studied; the strain is statistic by Eq. (1). The results show that to any first principal strain direction 'i', $\bar{\varepsilon}_{ij,g}$ is much greater than other strain components and almost keeps unchanged. It is denoted as reinforcers strain coefficient λ_g. The relation between the average strain of reinforcers and macro strain ε_{ij} is

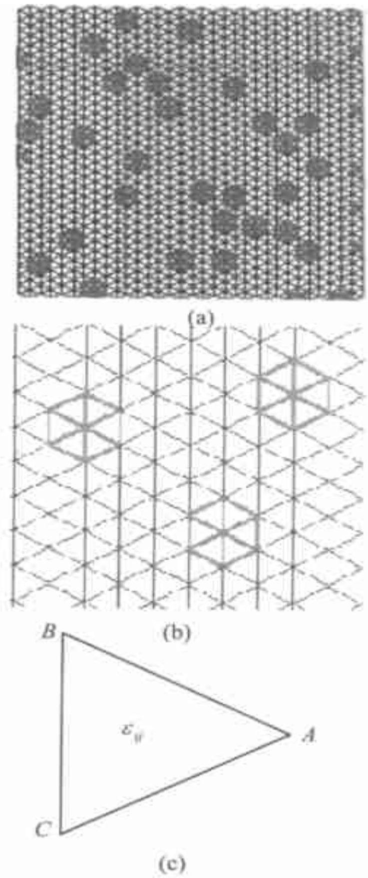

Fig. 1 Particle reinforced composites and network model
(a) particle reinforced composites;
(b) network model local enlarging;
(c) triangle cell

$$\bar{\varepsilon}_{ij,g} = \lambda_g \varepsilon_{ij} \tag{2}$$

λ_g is only dependent on the modulus ratio of the matrix to the reinforcers E_m/E_g and the volume of the reinforcers V_g. With fixed E_m/E_g and V_g, λ_g is a constant. So λ_g is an intrinsic parameter of composites. It is expressed as

$$\lambda_g = f\left(\frac{E_m}{E_g}, V_g\right) \tag{3}$$

According to the parameter combination listed in Table 1, the modulus ratio of the matrix to the particle E_m/E_g and the volume of the particle V_g

are changed. Based on the results, Eq. (3) can be expressed as

$$\lambda_g = (0.85 + 0.9 V_g)\left(\frac{E_m}{E_g}\right)^{0.9} \tag{4}$$

where $0 < E_m/E_g < 1/2$, $0 < V_g < 50\%$.

Table 1 Material parameter combinations

Parameters	Scales
E_m/E_g	1/2, 1/3, 1/4, 1/6, 1/8, 1/12, 1/16, 1/25
V_m	10%, 20%, 30%, 40%, 50%

Numerical results and the regressive function are shown in Fig. 2.

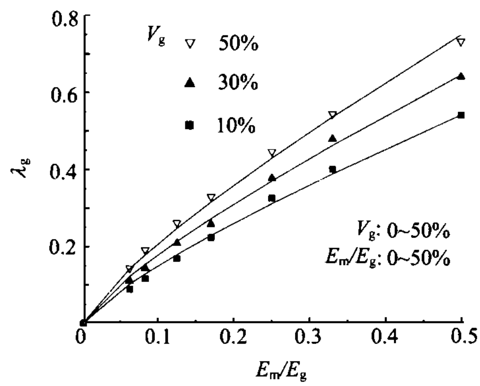

Fig. 2 λ_g versus E_m/E_g and V_g

Similar to reinforcers, g represents matrix, average strain in matrix is denoted as

$$\bar{\varepsilon}_{ij,m} = \frac{1}{V_m}\sum_t \varepsilon_{ij,t,m} V_{t,m} \tag{5}$$

Because the sum of deformation of the matrix and reinforcers equals macro deformation, matrix strain parameter λ_m and reinforcer strain parameter λ_g have a relation as

$$\lambda_g V_g + \lambda_m V_m = 1 \tag{6}$$

Numerical results prove Eq. (6) and

$$\bar{\varepsilon}_{ij,m} = \lambda_m \varepsilon_{ij} \tag{7}$$

1.2 Strain fluctuation

In short-fiber reinforced composites, the strain fluctuation is far less than that of the average strain. It is neglected[4,5]. But in particle reinforced composites, the strain fluctuation is not little compared with the average strain because of the shape. It is necessary to investigate strain fluctuation.

For the t-th reinforcement element, the strain difference is denoted as

$$\Delta_{ij,t,g} = \varepsilon_{ij,t,g} - \bar{\varepsilon}_{ij,g} \tag{8}$$

For the t-th matrix element, the strain difference is denoted as

$$\Delta_{ij,t,m} = \varepsilon_{ij,t,m} - \bar{\varepsilon}_{ij,m} \tag{9}$$

According to the parameter combinations and sample used in Section 1, statistic results indicate that though the strain difference of every element is distinct, the relation between the average value of the square of strain difference and macro strain can be expressed with a strain fluctuation coefficient. The strain fluctuation coefficients of reinforcers and matrix are expressed as Δ_g and Δ_m; one can get

$$\frac{1}{V_g}\sum_t \Delta_{ij,t,g}\Delta_{rs,t,g} V_{t,g} = \Delta_g^2 \varepsilon_{ij}\varepsilon_{rs} \tag{10}$$

and

$$\frac{1}{V_m}\sum_t \Delta_{ij,t,m}\Delta_{rs,t,m} V_{t,m} = \Delta_m^2 \varepsilon_{ij}\varepsilon_{rs} \tag{11}$$

Similar to a strain coefficient, strain fluctuation coefficients, Δ_g and Δ_m, are intrinsic parameters of the composite. They almost keep unchanged regardless of the loading direction and location of reinforcers. They are expressed asand

$$\Delta_g = f\left(\frac{E_m}{E_g}, V_g\right) \quad \text{and} \quad \Delta_m = g\left(\frac{E_m}{E_g}, V_m\right) \tag{12}$$

where $V_m = 1 - V_g$ is the volume fraction of the matrix.

Regressive functions of Δ_g and Δ_m are expressed as

$$\Delta_g = (0.01 + 0.15 V_g)\exp\left(-6.5\left(\frac{E_m}{E_g}\right)\right)$$

where

$$0 < V_g < 0.5,\ 0 < \frac{E_m}{E_g} < 0.5 \tag{13}$$

and

$$\Delta_m = (0.15 V_m)\exp\left(-5\left(\frac{E_m}{E_g}\right)\right)$$

where

$$0 < V_m < 0.5,\ 0 < \frac{E_m}{E_g} < 0.5 \tag{14}$$

Numerical results and regressive functions of Δ_g and Δ_m are shown in Fig. 3 and Fig. 4.

According to Eqs. (13) and (14), the stiffness of the composite over that of the matrix is calculated. The result is 2.38 without considering strain fluctuation, whereas the result is 3.12, considering strain fluctuation. The difference is 24%. It is shown that strain fluctuation is an important

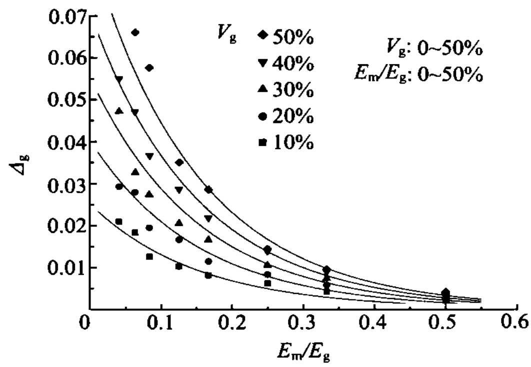

Fig. 3 Δ_m versus V_m and E_m/E_g

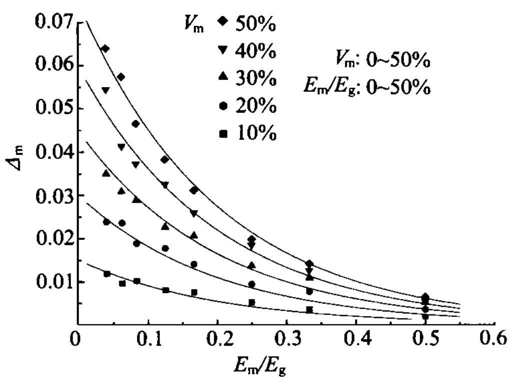

Fig. 4 Δ_g versus V_g and E_m/E_g

factor to stiffness.

2 Stiffness Tensor

2.1 Stiffness of reinforcers

Stiffness of reinforcers is denoted as $K_{ijrs,g}$, strain energy of the t-th reinforcement element is

$$W_{t,g} = \frac{1}{2} K_{ijrs,g} \varepsilon_{ij,t,g} \varepsilon_{rs,t,g} V_{t,g} \quad (15)$$

The sum of strain energy in reinforcers is

$$W_g = \frac{1}{2} K_{ijrs,g} \sum_t \varepsilon_{ij,t,g} \varepsilon_{rs,t,g} V_{t,g} \quad (16)$$

Substituting Eqs. (8) and (9) into Eq. (16) yields

$$W_g = \frac{1}{2} K_{ijrs,g} \sum (\bar{\varepsilon}_{ij,g} + \Delta_{ij,t,g})(\bar{\varepsilon}_{rs,g} + \Delta_{rs,t,g}) V_{t,g} = \frac{1}{2} K_{ijrs,g} \bar{\varepsilon}_{ij,g} \bar{\varepsilon}_{rs,g} V_g + \frac{1}{2} K_{ijrs,g} \sum \Delta_{ij,t,g} \Delta_{rs,g} V_{t,g} \quad (17)$$

Substituting Eqs. (2) and (10) into Eq. (17) yields

$$W_g = \frac{1}{2} K_{ijrs,g} \varepsilon_{ij} \varepsilon_{rs} + (\lambda_g^2 + \Delta_g^2) V_g \quad (18)$$

The second derivative of Eq. (18) yields the contribution of reinforcers to the stiffness of the composite

$$G_{ijrs,g} = (\lambda_g^2 + \Delta_g^2) V_g K_{ijrs,g} \quad (19)$$

2.2 Stiffness of matrix

Similar to reinforcers, the contribution of the matrix to the stiffness of the composite is

$$G_{ijrs,m} = (\lambda_m^2 + \Delta_m^2) V_m K_{ijrs,m} \quad (20)$$

2.3 Stiffness of composite

The stiffness tensor of the composite is yielded

$$K_{ijrs} = G_{ijrs,m} + G_{ijrs,g} = K_{ijrs,m}(\lambda_m^2 + \Delta_m^2) V_m + K_{ijrs,g}(\lambda_g^2 + \Delta_g^2) V_g \quad (21)$$

3 Comparison with Experiment

The numerical results are compared with experiment of short-fiber and particle composites.

With $V_g = 10\%$, $L/d = 4.5$, the verse stiffness moduli of SiCw/2024Al composite are calculated. The stiffness moduli of the whisker and matrix are equal to 480 GPa and 73 GPa, respectively. With Eqs. (1), (2) and (6), $\lambda_g = 0.233$, $\lambda_m = 1.125$, with Eqs. (8), (9) and (10), $\Delta_g = 0.031$, $\Delta_m = 0.067$. Combining Eq. (21), the verse moduli of composite is 86.05GPa. The experiment result is 86.5Gpa in Ref. [5]. The numerical result is favorably compared with the experimental one.

To a particle reinforced composite with $E_m/E_g = 1/25$, the experiment result is given in Ref. [8]. The numerical result in the present paper is shown in Fig. 5. The solid circles are experimental

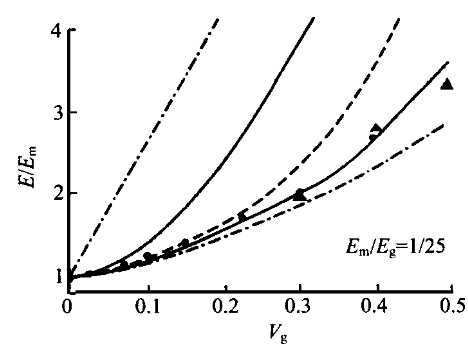

Fig. 5 Comparison between calculation and experiment

data; the broken line is self-consistent model; the chain-dotted lines are bounds of Hashin and Shtrikman; the solid line is theory in Ref. [8]. The triangle is the data calculated with the present method; V_g is 30%, 40%, 50%, respectively. It is obvious that the calculation result is in agreement with the experiment data.

4 Conclusions and Discussion

Based on the study of strain distribution in composites reinforced with high-volume-fraction, randomly distributed particles, it is revealed that the average strain and strain fluctuation are two important factors affecting the stiffness modulus of the composite. The relation between the average strain and strain fluctuation and microstructure parameters of the composite is obtained.

The present paper is aimed at the composite with spherical, randomly distributed, same-sized particles. To other conditions, it is necessary to specifically calculate to derive other formulas. It is necessary to further study the effect of the shape, array and size.

In the present paper, it is accurate to study the transverse stiffness of composites reinforced with one-way fibers; it is approximate to study particle reinforced composites. Three-dimensional effect will be discussed in other papers.

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