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A LATTICE BOLTZMANN METHOD FOR KDV EQUATION*

Yan Guangwu (阎广武)[+,+++] Chen Yaosong (陈耀松)[++] Hu Shouxin (胡守信)[+++]
[+](*Laboratory for Nonlinear Mechanics of Continuous Media, Institute of Mechanics* Chinese Academy of Sciences, Beijing 100080, China)
[++](*Department of Mechanics, Peking University, Beijing* 100871, *China*)
[+++](*Department of Mathematics, Jilin University, Changchun* 130023, *China*)

ABSTRACT: We prepose a 5-bit lattice Boltzmann model for KdV equation. Using Chapman-Enskog expansion and multiscale technique, we obtained high order moments of equilibrium distribution function, and the 3rd dispersion coefficient and 4th order viscosity. The parameters of this scheme can be determined by analysing the energy dissipation.

KEY WORDS: lattice Boltzmann method, KdV equation, multiscale technique, 5-bit lattice, conservational law in time scale t_0

1 INTRODUCTION

In recent years, the lattice Boltzmann method (LBM) has attracted attention as an alternative numerical scheme for the simulation of fluid flows[1~3]. The main idea of lattice Boltzmann methods are to get available macroscopic physical equations by using the discreted BGK type Boltzmann equation. In general case, time, space and velocity are discreted on one lattice, and then, choose the equilibrium distribution function to fits some requirements which can be obtained with multiscale technique and Chapman-Enskog expansion. Recently, there are some studies about model equations by lattice Boltzmann method[4~6]. On the other hand, equation contained high order partial differential term, such as KdV, can be recovered by modifying the equilibrium distribution function with some high moments, and truncation error of the model controlled to high order.

We expand the distribution function to the third order by Chapman-Enskog expansion, use the conservational law in time scale t_0, get the error term of model equation of order $O(\varepsilon^3)$. ε is Knudsen number.

2 LATTICE BOLTZMANN MODEL

2.1 The Definition of Macroscopic Quantity

Consider a one dimensional model, we discrede the velocity of particles into four directions, a lattice with unit spacing is used in which each node has four nearest neighbors connected by four links. The distribution function f_α is the probability of finding a particle

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at time t, node x, with velocity e_α, here $\alpha = 0, 1, \cdots, 4$ ($\alpha = 0$ is rest particle). The particles velocity are $e_\alpha = (0, c, -c, kc, -kc), k = 2$ are given four neighbors node, see Fig.1. The macroscopic quantity $u(x,t)$ (particles number) was defined by

$$u(x,t) = \sum_\alpha f_\alpha(x,t) \tag{1}$$

The conservational condition was

$$\textstyle\sum_\alpha f_\alpha^{eq}(x,t) = u(x,t) \tag{2}$$

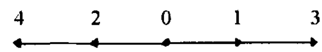

Fig.1 Schematic of a 1D lattice

The particle distribution function satisfy the lattice Boltzmann equations

$$f_\alpha(x + e_\alpha, t+1) - f_\alpha(x,t) = -\frac{1}{\tau}[f_\alpha(x,t) - f_\alpha^{eq}(x,t)] \tag{3}$$

where, $f_\alpha^{eq}(x,t)$ is the equilibrium distribution function at time t, x and τ is the single relaxation time factor.

2.2 A Series of Lattice Boltzmann Equations in Different Time Scales

Using ε as the small lattice unit in physical unit, ε can play the role of the Knudsen number[11], the lattice Boltzmann equation (3) in physical unites is

$$f_\alpha(x + \varepsilon e_\alpha, t + \varepsilon) - f_\alpha(x,t) = -\frac{1}{\tau}[f_\alpha - f_\alpha^{eq}] \tag{4}$$

Expanding Eq.(4)

$$f_\alpha(x + \varepsilon e_\alpha, t+\varepsilon) - f_\alpha(x,t) = \sum_{n=0}^{\infty} \frac{\varepsilon^n}{n!}\left[\frac{\partial}{\partial t} + e_\alpha \frac{\partial}{\partial x}\right]^n f_\alpha(x,t) \tag{5}$$

and retaining terms up to $O(\varepsilon^5)$ we obtain

$$\begin{aligned}\varepsilon\left[\frac{\partial}{\partial t} + e_\alpha\frac{\partial}{\partial x}\right] f_\alpha + \frac{\varepsilon^2}{2}\left[\frac{\partial}{\partial t} + e_\alpha\frac{\partial}{\partial x}\right]^2 f_\alpha + \frac{\varepsilon^3}{6}\left[\frac{\partial}{\partial t} + e_\alpha\frac{\partial}{\partial x}\right]^3 f_\alpha + \\ \frac{\varepsilon^4}{24}\left[\frac{\partial}{\partial t} + e_\alpha\frac{\partial}{\partial x}\right]^4 f_\alpha + O(\varepsilon^5)\end{aligned} \tag{6}$$

Next, the Chapman-Enskog expansion[7] is applied to f_α under the assumption that the mean free path is of the same order of ε. Expand f_α anout $f_\alpha^{(0)}$

$$f_\alpha = \sum_{n=0}^{\infty} \varepsilon^n f_\alpha^{(n)} = f_\alpha^{(0)} + \varepsilon f_\alpha^{(1)} + \varepsilon^2 f_\alpha^{(2)} + \varepsilon^3 f_\alpha^{(3)} + \varepsilon^4 f_\alpha^{(4)} +, \cdots \tag{7}$$

where, f_α^{eq} is $f_\alpha^{(0)}$.

To discuss changes in different time scales, we introduce $t_0, \cdots, t_3$; thus

$$t_0 = t, \quad t_1 = \varepsilon t, \quad t_2 = \varepsilon^2 t, \quad t_3 = \varepsilon^3 t$$

and

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \varepsilon\frac{\partial}{\partial t_1} + \varepsilon^2\frac{\partial}{\partial t_2} + \varepsilon^3\frac{\partial}{\partial t_3} + O(\varepsilon^4) \tag{8}$$

The equations to the order of ε is

$$\frac{\partial f_\alpha^{(0)}}{\partial t_0}+e_\alpha\frac{\partial f_\alpha^{(0)}}{\partial x}=-\frac{1}{\tau}f_\alpha^{(1)} \tag{9}$$

The equations to the order of ε^2 is

$$\frac{\partial f_\alpha^{(1)}}{\partial t_0}+\frac{\partial f_\alpha^{(0)}}{\partial t_1}+e_\alpha\frac{\partial f_\alpha^{(1)}}{\partial x}+\frac{1}{2}\Big[\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big]^2 f_\alpha^{(0)}=-\frac{1}{\tau}f_\alpha^{(2)} \tag{10}$$

The equations to the order of ε^3 is

$$\begin{aligned}&\frac{\partial f_\alpha^{(2)}}{\partial t_0}+\frac{\partial f_\alpha^{(1)}}{\partial t_1}+\frac{\partial f_\alpha^{(0)}}{\partial t_2}+e_\alpha\frac{\partial f_\alpha^{(2)}}{\partial x}+\frac{\partial^2 f_\alpha^{(0)}}{\partial t_0\partial t_1}+e_\alpha\frac{\partial^2 f_\alpha^{(0)}}{\partial t_1\partial x}+\\ &\quad\frac{1}{2}\Big[\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big]^2 f_\alpha^{(1)}+\frac{1}{6}\Big[\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big]^3 f_\alpha^{(0)}=-\frac{1}{\tau}f_\alpha^{(3)}\end{aligned} \tag{11}$$

The equations to the order of ε^4 is

$$\begin{aligned}&\frac{\partial f_\alpha^{(0)}}{\partial t_3}+\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)f_\alpha^{(3)}+\frac{\partial f_\alpha^{(2)}}{\partial t_1}+\frac{\partial f_\alpha^{(1)}}{\partial t_2}+\frac{1}{24}\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^4 f_\alpha^{(0)}+\\ &\quad\frac{1}{6}\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^3 f_\alpha^{(1)}+\frac{1}{2}\frac{\partial}{\partial t_1}\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^2 f_\alpha^{(0)}+\frac{1}{2}\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^2 f_\alpha^{(2)}+\\ &\quad\frac{\partial}{\partial t_1}\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)f_\alpha^{(1)}+\frac{\partial}{\partial t_2}\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)f_\alpha^{(0)}+\frac{1}{2}\frac{\partial^2 f_\alpha^{(0)}}{\partial t_1^2}=-\frac{1}{\tau}f_\alpha^{(4)}\end{aligned} \tag{11a}$$

From Eq.(9) it follows

$$\Big[\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big]^2 f_\alpha^{(0)}=-\frac{1}{\tau}\Big[\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big]f_\alpha^{(1)} \tag{12}$$

$$\Big[\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big]^3 f_\alpha^{(0)}=-\frac{1}{\tau}\Big[\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big]^2 f_\alpha^{(1)} \tag{13}$$

Substituting Eq.(10) into Eq.(12), we get

$$\frac{\partial f_\alpha^{(0)}}{\partial t_1}-\tau\Big(1-\frac{1}{2\tau}\Big)\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^2 f_\alpha^{(0)}=-\frac{1}{\tau}f_\alpha^{(2)} \tag{14}$$

Multiplying by operator $\dfrac{\partial}{\partial t_0}+e_\alpha\dfrac{\partial}{\partial x}$ in Eq.(14),

$$\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)\frac{\partial f_\alpha^{(0)}}{\partial t_1}+\Big(1-\frac{1}{2\tau}\Big)\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^2 f_\alpha^{(1)}=-\frac{1}{\tau}\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)f_\alpha^{(2)} \tag{15}$$

and eliminating $f_\alpha^{(2)}$ by using Eqs.(11), (11a), (14) and (15), we obtain

$$\frac{\partial f_\alpha^{(0)}}{\partial t_2}+(1-2\tau)\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)\frac{\partial f_\alpha^{(0)}}{\partial t_1}+\Big(\tau^2-\tau+\frac{1}{6}\Big)\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^3 f_\alpha^{(0)}=-\frac{1}{\tau}f_\alpha^{(3)} \tag{16}$$

$$\frac{\partial f_\alpha^{(0)}}{\partial t_3}+\Big(2\tau^2-\frac{5}{2}\tau+\frac{1}{2}\Big)\frac{\partial}{\partial t_1}\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^2 f_\alpha^{(0)}+$$
$$(1-2\tau)\frac{\partial}{\partial t_2}\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)f_\alpha^{(0)}+\Big(-\tau^3+\frac{3}{2}\tau^2-\frac{7}{12}\tau+\frac{1}{24}\Big)\cdot$$
$$\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^4 f_\alpha^{(0)}+\frac{\partial f_\alpha^{(2)}}{\partial t_1}+\frac{1}{2}\frac{\partial^2 f_\alpha^{(0)}}{\partial t_1^2}=-\frac{1}{\tau}f_\alpha^{(4)} \tag{17}$$

Eqs.(9), (14), (16) and (17) satisfy in all dimensional lattice Boltzmann equation generally, it is so-called a series lattice Boltzmann equations in different time scales. The coefficients in Eqs.(14)~(17) $\tau-1/2$, $\tau^2-\tau+1/6$ and $-\tau^3+3\tau^2/2-7\tau/12+1/24$ are needed in the derivation and may be used to give the feature of macroscopic equations.

2.3 KdV Equation

Taking the summation in Eqs.(9), (14), (16) and (17) about α, we obtain

$$\frac{\partial u}{\partial t_0}+\frac{\partial m^0}{\partial x}=0 \tag{18}$$

$$\frac{\partial u}{\partial t_1}+\Big(\frac{1}{2}-\tau\Big)\sum_\alpha\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^2 f_\alpha^{(0)}=0 \tag{19}$$

$$\frac{\partial u}{\partial t_2}+\Big(\tau^2-\tau+\frac{1}{6}\Big)\sum_\alpha\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^3 f_\alpha^{(0)}=0 \tag{20}$$

$$\frac{\partial u}{\partial t_2}+(1-2\tau)\sum_\alpha\frac{\partial}{\partial t_2}\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)f_\alpha^{(0)}+(1-\frac{1}{2\tau})\sum_\alpha\frac{\partial f_\alpha^{(2)}}{\partial t_1}+$$
$$\Big(2\tau^2-2\tau+\frac{1}{4}\Big)\sum_\alpha\frac{\partial}{\partial t_1}\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^2 f_\alpha^{(0)}+$$
$$\Big(-\tau^3+\frac{3}{2}\tau^2-\frac{7}{12}\tau+\frac{1}{24}\Big)\sum_\alpha\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^4 f_\alpha^{(0)}=0 \tag{21}$$

(18)+(19)×ε +(20)×ε^2 +(21)×ε^3 results in

$$\frac{\partial u}{\partial t}+\frac{\partial m^0}{\partial x}+\varepsilon\Big(\frac{1}{2}-\tau\Big)\sum_\alpha\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^2 f_\alpha^{(0)}+\varepsilon^2\mu\sum_\alpha\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^3 f_\alpha^{(0)}+$$
$$\varepsilon^3\Big[\frac{\partial u}{\partial t_2}+(1-2\tau)\sum_\alpha\frac{\partial}{\partial t_2}\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)f_\alpha^{(0)}+\Big(1-\frac{1}{2\tau}\Big)\sum_\alpha\frac{\partial f_\alpha^{(2)}}{\partial t_1}+$$
$$\Big(2\tau^2-2\tau+\frac{1}{4}\Big)\sum_\alpha\frac{\partial}{\partial t_1}\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^2 f_\alpha^{(0)}+$$
$$\Big(-\tau^3+\frac{3}{2}\tau^2-\frac{7}{12}\tau+\frac{1}{24}\Big)\sum_\alpha\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^4 f_\alpha^{(0)}\Big]=0 \tag{22}$$

Under the assumption that

$$m^0=\sum_\alpha f_\alpha^{(0)}e_\alpha=\frac{1}{2}u^2a \tag{23}$$

$$\pi^0=\sum_\alpha f_\alpha^{(0)}e_\alpha^2=\frac{1}{3}u^3a^2 \tag{24}$$

where a is constant number, we have

$$\sum_\alpha \Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)f_\alpha^{(0)}=0 \qquad \sum_\alpha \Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^2 f_\alpha^{(0)}=0 \tag{25}$$

In fact, Eq.(25) is the conservational law of u and m^0, this is so-called conservational law in time scale t_0. It plays an important role in the construction of lattice Boltzmann scheme of high order precision. Equation (25) had also been shown in references [5] and [6], but those are not conservational law in time scale t_0.

Equation (22) become

$$\frac{\partial u}{\partial t}+\frac{\partial}{\partial x}\Big(\frac{1}{2}au^2\Big)+\varepsilon^2\mu\sum_\alpha\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^3 f_\alpha^{(0)}+\varepsilon^3\eta\sum_\alpha\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^4 f_\alpha^{(0)}=0 \tag{26}$$

where $\mu=\tau^2-\tau+1/6, \eta=\Big(-\tau^3+\dfrac{3}{2}\tau^2-\dfrac{7}{12}\tau+\dfrac{1}{24}\Big)$. The third term of Eq.(26)'s left hand side is

$$\varepsilon^2\mu\sum_\alpha\Big(\frac{\partial}{\partial t_0}+e_\alpha\frac{\partial}{\partial x}\Big)^3 f_\alpha^{(0)}=\varepsilon^2\mu\frac{\partial^2}{\partial x^2}\Big[\frac{\partial \pi^0}{\partial t_0}+\frac{\partial P^0}{\partial x}\Big] \tag{27}$$

where $P^0=\sum_\alpha f_\alpha^{(0)}e_\alpha^3$. Denote $L^0=\sum_\alpha f_\alpha^{(0)}e_\alpha^4$, and choose

$$P^0=\xi_1 u+a^3\frac{1}{4}u^4 \qquad L^0=\xi_2\frac{1}{2}au^2+a^4\frac{1}{5}u^5 \tag{28}$$

thus

$$\frac{\partial \pi^0}{\partial t_0}+\frac{\partial P^0}{\partial x}=\xi_1\frac{\partial u}{\partial x} \tag{29}$$

$$\frac{\partial P^0}{\partial t_0}+\frac{\partial L^0}{\partial x}=\xi_2\frac{\partial\Big(\frac{1}{2}au^2\Big)}{\partial x} \tag{30}$$

Equation (26) becomes KdV equation[8~10], therefore

$$\frac{\partial u}{\partial t}+\frac{\partial}{\partial x}\Big(\frac{1}{2}au^2\Big)+\nu\frac{\partial^3 u}{\partial x^3}=O(\varepsilon^3) \tag{31}$$

The truncation error is

$$R=O(\varepsilon^3)=\lambda\frac{\partial^4}{\partial x^4}\Big(\frac{1}{2}au^2\Big)+O(\varepsilon^4)$$

Here the coefficent ν is given as

$$\nu=\xi_1\varepsilon^2\mu=\varepsilon^2\Big(\tau^2-\tau+\frac{1}{6}\Big)\xi_1 \tag{32}$$

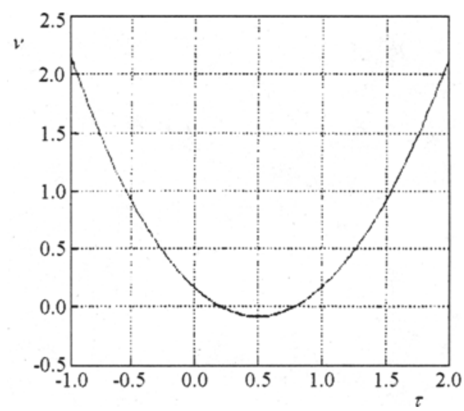

Fig.2 The curve of $\dfrac{\nu}{\xi_1\varepsilon^2}$ versus τ

From Eq.(32), the parameters ξ_1, ν and ε can be used to determine the relaxation factor τ, and ξ_1 is chosen to satisfy

$$\frac{1}{12}+\frac{\nu}{\varepsilon^2\xi_1}>0$$

The 4th viscosity is given by

$$\lambda = \varepsilon^3\Big(\tau^3 - \frac{3}{2}\tau^2 + \frac{7}{12}\tau - \frac{1}{24}\Big)(4\xi_1 - \xi_2) \tag{33}$$

and then

$$\lambda = \varepsilon^3(\tau - \frac{1}{2})\Big(\tau^2 - \tau + \frac{1}{12}\Big)(4\xi_1 - \xi_2) \tag{34}$$

Equation (34) is written as

$$\lambda = \varepsilon\Big(\frac{\nu}{\xi_1} - \frac{1}{12}\varepsilon^2\Big)\sqrt{\frac{1}{12} + \frac{\nu}{\varepsilon^2\xi_1}}(4\xi_1 - \xi_2) \tag{35}$$

where $\nu = 1/\sigma^2$. Choosing $\lambda > 0$, we get

$$\varepsilon\sigma < \sqrt{\frac{12}{\xi_1}} \tag{36}$$

It is an important conclusion that Eq.(36) become stability criterion for lattice Boltzmann equation. The parameters ξ_1 and ξ_2 are given by the model energy dissipation.

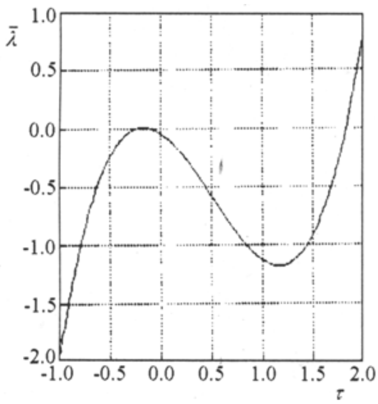

Fig.3 The curve of the 4th viscosity $\bar{\lambda} = \dfrac{\lambda(4\xi_1 - \xi_2)}{\varepsilon^3}$ versus τ

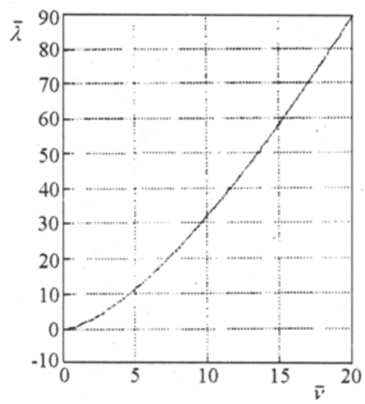

Fig.4 The relationship between the 3rd dispersion coefficent $\bar{\nu} = \dfrac{\nu}{\xi_1\varepsilon^2}$ and λ

2.4 The Energy Dissipation of the Lattice Boltzmann Model

Assuming that particles satisfy energy conservation in the form

$$m = \sum_\alpha f_\alpha e_\alpha = \sum_\alpha f_\alpha^{(0)} e_\alpha = m^0 \tag{37}$$

$$\sum_\alpha f_\alpha^{(k)} e_\alpha = 0 \qquad k \geq 1$$

Equations (9), (14) and (16) multiplying by e_α and taking the summation we get

$$\frac{\partial m}{\partial t_0} + \frac{\partial \pi^0}{\partial x} = 0 \tag{38}$$

$$\frac{\partial m}{\partial t_1} + \Big(\frac{1}{2} - \tau\Big)\xi_1\frac{\partial^2 u}{\partial x^2} = 0 \tag{39}$$

$$\frac{\partial m}{\partial t_2}+\Big(\tau^2-\tau+\frac{1}{6}\Big)(\xi_1-3\xi_2)\frac{\partial^3}{\partial x^3}\Big(\frac{1}{2}au^2\Big)=0 \tag{40}$$

Back to the time scale t, we have

$$\frac{\partial m}{\partial t}+\frac{\partial \pi^0}{\partial x}+\varepsilon\xi_1\Big(\frac{1}{2}-\tau\Big)\frac{\partial^2 u}{\partial x^2}+\varepsilon^2\Big(\xi_2-3\xi_1\Big)\Big(\tau^2-\tau+\frac{1}{6}\Big)\frac{\partial^3}{\partial x^3}\Big(\frac{1}{2}u^2\Big)+O(\varepsilon^3)=0 \tag{41}$$

Here, the remaining coefficients are determined by the positive condition of Eq.(41).

2.5 The Local Equilibrium Distribution

The moments of $f_\alpha^{(0)}$ can be expressed as

$$\sum_\alpha f_\alpha^{(0)}=u=B_1 \tag{42}$$

$$\sum_\alpha f_\alpha^{(0)}e_\alpha=\frac{1}{2}u^2a=cB_2 \tag{43}$$

$$\sum_\alpha f_\alpha^{(0)}e_\alpha^2=\frac{1}{3}u^3a^2=c^2B_3 \tag{44}$$

$$\sum_\alpha f_\alpha^{(0)}e_\alpha^3=\frac{1}{4}u^4a^3+\xi_1u=c^3B_4 \tag{45}$$

$$\sum_\alpha f_\alpha^{(0)}e_\alpha^4=\frac{1}{5}u^5a^4+\xi_2\frac{1}{2}u^2a=c^4B_5 \tag{46}$$

We get the equilibrium distribution as

$$f_1^{(0)}=\frac{1}{2}\frac{k^2B_3-B_5+k^2B_2-B_4}{k^2-1} \tag{47}$$

$$f_2^{(0)}=\frac{1}{2}\frac{k^2B_3-B_5-k^2B_2+B_4}{k^2-1} \tag{48}$$

$$f_3^{(0)}=\frac{1}{2}\Big(\frac{B_5-B_3}{k^4-k^2}+\frac{B_4-B_2}{k^3-k}\Big) \tag{49}$$

$$f_4^{(0)}=\frac{1}{2}\Big(\frac{B_5-B_3}{k^4-k^2}-\frac{B_4-B_2}{k^3-k}\Big) \tag{50}$$

$$f_0^{(0)}=B_1-(f_1^{(0)}+f_2^{(0)}+f_3^{(0)}+f_4^{(0)}) \tag{51}$$

3 NUMERICAL EXAMPLE

A test problem, the collison of two solitons[10], with initial and boundary function

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\mu\frac{\partial^3 u}{\partial x^3}=0$$

$$-\infty\le x\le\infty \qquad 0<t<T$$

$$u(x,0)=3c_1\mathrm{sech}^2(k_1x+d_1)+3c_2\mathrm{sech}^2(k_2x+d_2)$$

$$c_1=0.3 \quad c_2=0.1 \quad d_1=-6.0 \quad d_2=-6.0$$

$$k_1=\frac{1}{2}(c_1/\mu)^{1/2} \qquad k_2=\frac{1}{2}(c_2/\mu)^{1/2}$$

was simulated by using this model. It shows that bigger soliton should reach and collide with smaller soliton in the process. For the phenomenom of swallowing and spitting, see Fig.5. In the process of the collision, all particles number is conserved, see Fig.6, but the total particles energy has a bit dissipation see Fig.7. The Fig.5 shows the process of swallowing and spitting when two solitons collide.

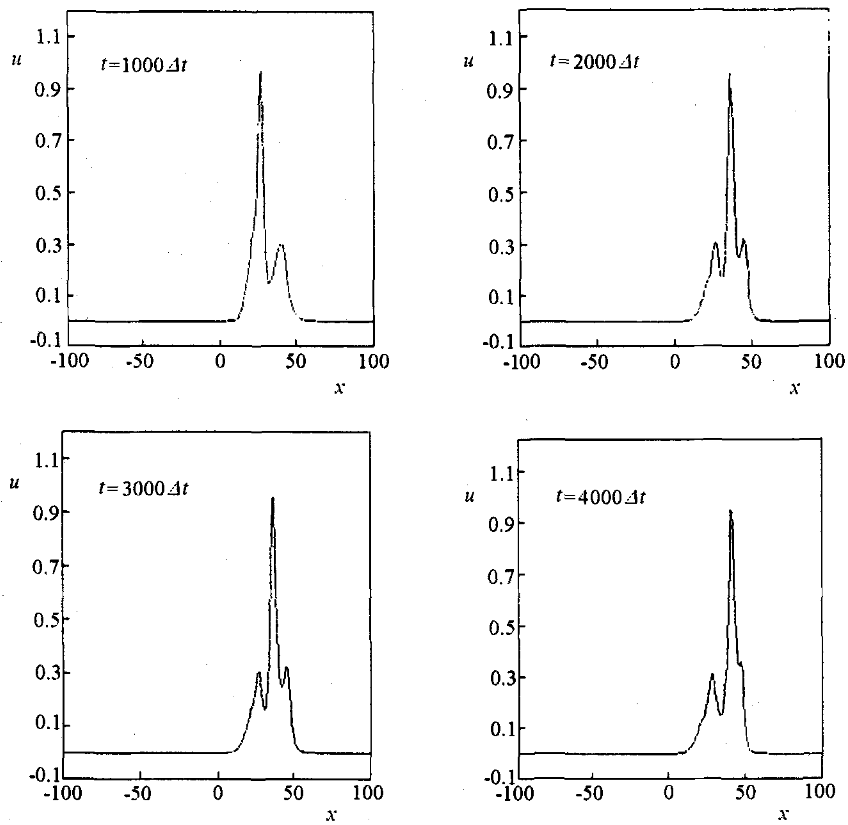

Fig.5 The process of two solitons collision, $c = 10.0$, $\mu = 1.0$

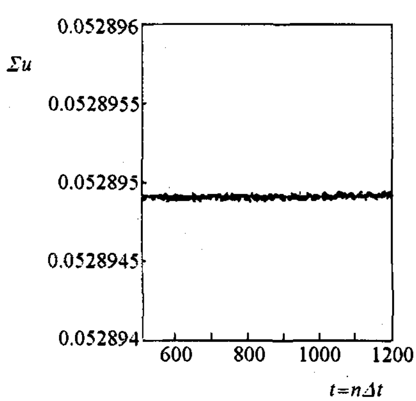

Fig.6 All particles number

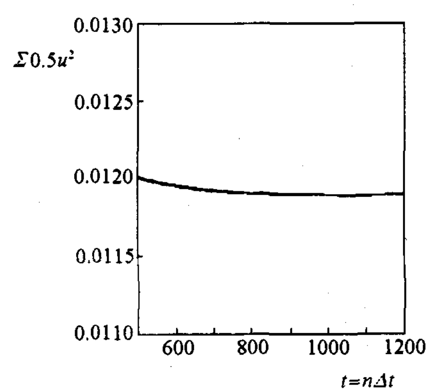

Fig.7 All particles energy, $c = 3.0$, $\mu = 1.0$

4 CONCLUSION

There was a famous method, Grad-13 moments equations in the area of gas dynamics, which was successful in the simulation of Navier-Stokes equations from Boltzmann equation. In 1991, U. Frisch pointed out the possibility that the requirments of higher moments may be used to construct lattice gas model for Navier-Stokes equation[12]. In this paper, we carried out Chapman-Enskog expanson and multiscale technique on the distribution function, obtained $f_\alpha^{(3)}$, and KdV equation with high order accuracy. The conservational law in time scale t_0, the equations of different time scales Eqs.(9), (14), (16) and (17) are important results in the lattice Boltzmann method.

The lattice Boltzmann model for KdV equation is simple, but Euler equations and Navier-Stokes are more complex which is the next step of the authors.

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