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The evolution of shear bands of saturated soil[1]

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Abstract

The development of the shear bands of saturated soil in coupling-rate- and pore-pressure-dependent simple shear has been discussed, using a simple model and a matching technique at the moving boundary of a shear band. It is shown that the development of shear bands are dominated by the coupling-rate and pore-pressure effect of the material. The strength of the soil acts as a destabilizer, whilst pore pressure diffusion makes the band expand. The theory is discussed and some computational solutions have been presented.

Keywords: Shear load; Saturated soil; Matching technique

1. Introduction

The shear band is closely related with the failure in saturated soils and it is assumed by analysis that there is a critical state, beyond which instability may develop and it is usually taken to be the condition for the emergence of a shear softening area. However, the analysis gives only a condition for the instability, and no indication of the emergence of a softening area. It is desirable to know the dynamics of the shear band of saturated soil and the factors governing the process. In this paper an analytical model is developed to achieve this aim.

2. The control equations

2.1. Two assumptions

(1) The shear volume strain and the increment of pore pressure are adopted as follows [1]:

$$\begin{cases}\Delta\varepsilon_{v1} = C_1 \cdot \tau\Delta\gamma_{xy}, \\ \Delta p = E_r \cdot \Delta\varepsilon_{v2},\end{cases} \tag{1}$$

where C_1 is a parameter and $C_1 = C_1(p)$, Δp is the increment of pore pressure, E_r the resilience module, $\Delta\varepsilon_{v2}$, $\Delta\varepsilon_{v2}$ the volume strains caused by shear and that recovered by water.

(2) The deformation can only occur in one direction but may have a gradient in the other directions.

The geometrical configuration and the deformation can be expressed as follows:

$x_1 = X_1,$

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$$x_2 = u(X_1, X_2) + X_2,$$

$$x_3 = X_3. \tag{2}$$

Since water cannot be compressed which means $\varepsilon_{wv} = 0$. At the same time, It is obvious that the volume caused by shear is the sum of that caused by of drainage $\Delta\varepsilon_{sv}$ and that of resilience, which is [2]

$$C_1 \tau \Delta\gamma = \frac{\Delta p}{E_r} + \Delta\varepsilon_{sv} \tag{3}$$

and $\Delta\varepsilon_{sv}$ is decided by Darcy law:

$$\frac{\partial \varepsilon_{sv}}{\partial t} = -\frac{1}{K}\frac{\partial^2 p}{\partial x^2}, \tag{4}$$

where p is the pore pressure, n the pore ratio, K the obstraction coefficient and $K = \mu/k$, where k the penetration ratio and μ the visco-coefficient.

Substituting Eq. (4) into Eq. (3), we will get the first control equation:

$$\frac{\partial p}{\partial t} - \frac{E_r}{K}\frac{\partial^2 p}{\partial x^2} = C_1 E_r \tau \frac{\partial \gamma}{\partial t}. \tag{5}$$

The equations of motion imply

$$\rho\frac{\partial^2 \gamma_{xy}}{\partial t^2} - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\tau_{xy} = \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y}. \tag{6}$$

Considering the assumption (2), Eq. (6) becomes

$$\rho\frac{\partial^2 \gamma_{xy}}{\partial t^2} - \frac{\partial^2 \tau_{xy}}{\partial x^2} = 0. \tag{7}$$

Now, the control equations can be rewritten as

$$\frac{\partial p}{\partial t} - \frac{E_r}{K}\frac{\partial^2 p}{\partial x^2} = C_1 E_r \tau \frac{\partial \gamma}{\partial t},$$

$$\rho\frac{\partial^2 \gamma_{xy}}{\partial t^2} = \frac{\partial^2 \tau_{xy}}{\partial x^2}, \tag{8}$$

where C_1 and E_r are both functions of p.

Now give the dimensionless form of the control equations, and they may be simplified to

$$\frac{E_r \dot{\gamma}_k}{\tau_k}\frac{\partial \bar{\gamma}}{\partial t} = \frac{\partial^2 \bar{\tau}}{\partial x^2},$$

$$\frac{\partial \bar{p}}{\partial t} - \frac{\partial^2 \bar{p}}{\partial x^2} = \frac{\tau \dot{\bar{\gamma}}}{2}, \tag{9}$$

where $\bar{p} = p/\sigma_{e0}$, $\dot{\bar{\gamma}} = \dot{\gamma}/\dot{\gamma}_k$, $\bar{\tau} = \tau/\tau_k$, $t = t/t_k$, $\bar{y} = y/\delta_k$, both material constants, t_k and δ_k^2 are taken to be $\sigma_{e0}/2C_1 E_r \tau_k \dot{\gamma}_k$ and $E_r t_k/K$, the latter quantity being related to t_h. Because $\tau_k \sim 10^5$Pa, $K \sim 10^6$ kg/m^3 s, $\dot{\gamma}_k \sim 10^0$/s, $\rho_s \sim 10^3$ kg/m^3, $n \sim 10^0$, the smallness of $E_r\dot{\gamma}_k/\tau_k$ reduces Eq. (9) to Eqs. (10a)–(10g). From now on the over-bar used to indicate a dimensionless quantity will be omitted.

Therefore the approximate model for shear bands is as follows:

$$\frac{\partial^2 \tau}{\partial x^2} = 0, \tag{10a}$$

$$\frac{\partial p}{\partial t} - \frac{\partial^2 p}{\partial x^2} = \frac{\tau\dot{\gamma}}{2}, \tag{10b}$$

$$y = 0, \quad \frac{\partial p}{\partial y} = 0, \tag{10c}$$

$$y = \delta(t), \quad p_{\delta_-} = p_{\delta_+}, \tag{10d}$$

$$\left.\frac{\partial p}{\partial y}\right|_{\delta_-} = \left.\frac{\partial p}{\partial y}\right|_{\delta_+}, \tag{10e}$$

$$v|_{\delta_-} = RV(t), \tag{10f}$$

$$t = 0, \quad p = p_0(y), \tag{10g}$$

where $R = v_0/\delta_k\dot{\gamma}_k$, $p_0(y)$ and v_0 are the initial disturbances of pore pressure and velocity, respectively, $V(t)$ is the dimensionless boundary velocity at $V = 1$ and $t = 0$. Here, the constitutive equation is assumed to be the one for pore-pressure-dependence. Strain ratio, provided the effect of strain can be neglected compared with strain ratio, is given by

$$\tau = \tau(\dot{\gamma}, p). \tag{11}$$

Outside the band, the material is assumed to remain rigid, no matter how high the pore pressure is. The governing equation here is the one for homogeneous diffusion with

$$p(0, y) = B, \tag{12}$$

where B is the assumed uniform initial pore pressure.

3. Analytical solution

Since the aim of this paper is to understand the mechanics of shear band formation, and the factors controlling the process, an analytical solution is more preferable, although, in order to obtain one, some approximations need to be made.

One of the simplifications is a linear version of the constitutive equation:

$$\tau = \dot{\gamma} + 1 - p. \tag{13}$$

The linear relation of τ and $\dot{\gamma}$ is consistent with observations at some stage of deformation. Moreover, linear softening approximates the behaviour of a variety of soils between statistic pore pressure and liquefactional pore pressure.

Substitution of Eq. (13) into Eqs. (10a)–(10g) leads to an inhomogeneous equation in p. The solution to it then can be expressed as [3]

$$p = \exp(H(t))\left[p_1 + \int_0^t \mathrm{e}^{-H(t)}\tau\frac{\tau-1}{2}\,\mathrm{d}t\right], \tag{14}$$

where

$$H(t) = \int_0^t -\frac{\tau}{2}\,\mathrm{d}\eta,$$

η is a variable in integration, and

$$\tau(t) = 1 - p_\delta(t). \tag{15}$$

The pore pressure p_1 satisfies the equation

$$\frac{\partial p_1}{\partial t} - \frac{\partial^2 p_1}{\partial x^2} = \frac{\tau p_1}{2}. \tag{16}$$

To deal with the moving boundary $\delta(t)$, we consider the initial boundary conditions

$$x = 0, \quad \frac{\partial p_1}{\partial x} = 0, \tag{17a}$$

$$x = \delta_s, \quad \frac{\partial p_1}{\partial x} = S(t), \tag{17b}$$

$$t = 0, \quad p_1 = p_0(x), \tag{17c}$$

where δ_s is an imaginary fixed boundary chosen to be greater than $\delta(t)$, and $S(t)$ is an arbitrary function determined by matching the condition at $y = \delta(t)$, i.e. Eqs. (10d) and (10e).

The solution p to the problem, but with initial boundary values (10f), can be expressed as Fourier cosine series:

$$\begin{aligned} p = {} & \mathrm{e}^{H(t)}\frac{1}{\delta_s}\left\{C_0 + \int_0^t S(\eta)\mathrm{e}^{-H(\eta)}\,\mathrm{d}\eta \right. \\ & \left. - \delta_s \int_0^t \tau\frac{1-\tau}{2}\mathrm{e}^{-H(t)}\,\mathrm{d}\eta\right\} \\ & + \frac{2}{\delta_s}\sum_1^\infty \exp(H(t) - \alpha_n^2 t) \\ & \times\left\{C_n + (-1)^n\int_0^t S(\eta)\exp(\alpha_n^2\eta \right. \\ & \left. - H(\eta))\,\mathrm{d}\eta\right\}\cos\alpha_n x, \end{aligned} \tag{18}$$

where $\alpha_n = n\pi/\delta_s$, $0 < x < \delta_s$, C_n are constants determined by the initial condition (10g) as

$$C_n = \int_0^{\delta_s} p_0(\xi)\cos(\alpha_n\xi)\,\mathrm{d}\xi, \tag{19}$$

where ξ is an integration variable, and Eq. (10f) requires that

$$\begin{aligned} \mathrm{RV}(t) &= v|_{\delta-} \\ &= \int_0^\delta \dot{\gamma}\,\mathrm{d}x = \frac{2}{\delta_s}\sum_1^\infty \exp(H(t) - \alpha_n^2 t) \\ & \quad \times\left\{C_n + (-1)^n\int_0^t S(\eta)\exp(\alpha_n^2\eta - H(\eta))\,\mathrm{d}\eta\right\} \\ & \quad \times\frac{\delta_s}{n\pi}\left\{\sin\frac{\alpha_n\delta(t)}{\delta_s} - \frac{\alpha_n\delta(t)}{\delta_s}\cos\frac{\alpha_n\delta(t)}{\delta_s}\right\}. \end{aligned} \tag{20}$$

The solution for the rigid material outside the band can be easily obtained as

$$p = \int_0^t [p^*(t-\eta) - B]\frac{x}{2\sqrt{\pi\eta^3}}\exp\left(-\frac{x^2}{4\pi}\right)\mathrm{d}\eta. \tag{21}$$

If it is assumed that $B =$ constant and the edge effect is neglected; $p^*(t)$ here is the pore pressure at an imaginary boundary $y = 0$.

In all, there are four unknown functions: $p^*(t)$, $S(t)$, $\delta(t)$ and $\tau(t)$. They can be determined by solving Eqs. (15), (18), (20) and (21), simultaneously.

4. Mechanics of shear band

Eq. (18) shows that three factors control a non-uniform shear field, namely $S(t)$, $H(t)$ and $\alpha_n^2 t$. The first one is related to the pore pressure diffusing out of the band, the second represents a cumulative effect of the strength of soil, and the third concerns the decaying mode of pore pressure diffusion within the band. The second and the third are both exponential, and so are much more important than the first. Even at the early stage of shear band development, the pore pressure diffusion (accounted for by $S(t)$) to the surrounding soil appears to be negligibly small, because $\partial p/\partial x|_{\delta_0} = 0$, where $\delta_0 = \delta(0)$. Therefore, $H(t)$ and $\alpha_n^2 t$ are bound to be the governing factors in shear band formation.

With the two assumptions, namely $S(t) = 0$ and $c_n = 0$ $(n \neq 1)$, which represent the most influential part of pore pressure diffusion and the simpler case, Eq. (20) becomes

$$\sin(\alpha_1 \delta(t)) - L = \alpha_1 \delta(t) \cos(\alpha_1 \delta(t)),$$

$$L = \frac{\pi RV}{2C_1} \exp(\alpha_1^2 t - H(t)), \tag{22}$$

provided $\delta_0 = \delta_s$ and a constant-velocity boundary condition is introduced. It is clear from Eq. (22) that shrinkage of the shear deformation field requires a decreasing value of L, i.e.

$$\frac{d}{dt}(H(t) - \alpha_1^2 t) > 0 \tag{23}$$

or

$$\tau(t)/2 > (\pi/\delta(0))^2. \tag{24}$$

Therefore the shrinkage is because of the strength τ, whereas pore pressure diffusion tends to smooth shearing.

For a material governed by the pore-pressure-independent, constitutive relation $\tau = \tau(\dot{\gamma})$, the solution to Eqs. (10a)–(10g) is

$$p = p_3(t, y) + \int_0^t \frac{\tau\dot{\gamma}}{2} d\eta, \tag{25}$$

$$\frac{\partial p_3}{\partial t} - \frac{\partial^2 p_3}{\partial x^2} = 0. \tag{26}$$

Unlike the solution (14), this solution shows that the strength τ will not be incorporated into a non-uniform shear field. This case corresponds to simple pore-pressure diffusion, whereas $\tau = \tau(p)$ would lead to a trivial solution. Therefore, one can conclude that only in the soil governed by a coupling-rate- and pore-pressure-dependent constitutive relation there can be a shrinking shear zone, with the strength of the soil acting as a destabilizer.

With decreasing $\tau(t)$, which usually happens in the saturated soil under vibration load, a narrowing shear band will be transformed into an expanding one at a certain time, because the right-hand side of the inequality (24) is constant. This means that there is a stable phase deformation dominated by pore pressure diffusion.

Fig. 1, based on the inequality (23), shows that long-wave disturbances are more liable to cause shrinkage than short-wave ones.

By introducing Eq. (15) and the condition $\dot{\gamma}|_{y=\delta} = 0$ into Eq. (20), Eq. (20) becomes

$$\mathrm{RV} = \int_0^\delta (p - p_\delta(t))\, dx. \tag{27}$$

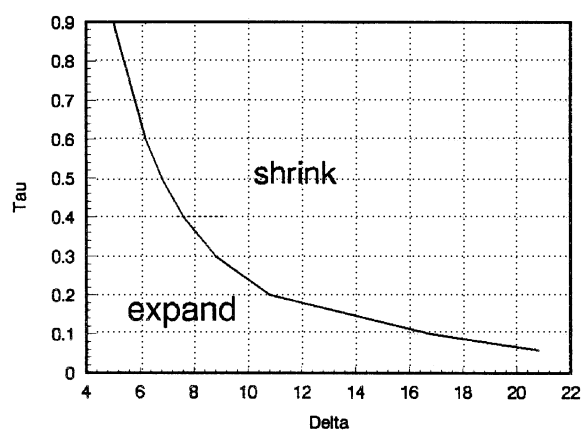

Fig. 1. The changes of shear band δ/δ_0 with shear stress τ/τ_0.

Differentiation of Eq. (23) with respect to time t under the constant-velocity boundary condition leads to

$$\left.\frac{\partial p}{\partial x}\right|_\delta \delta(t)\frac{\partial \delta}{\partial t} = \frac{\mathrm{RV}}{2} + \left.\frac{\partial p}{\partial x}\right|_\delta - \delta(t)\frac{\partial p_\delta}{\partial x}. \tag{28}$$

For $\partial p/\partial y \neq 0$ and $\delta(t) \neq 0$, Eq. (24) becomes an expression for shear band development:

$$\frac{\partial \delta}{\partial t} = \left\{\frac{\tau\bar{\dot{\gamma}}(t)}{2} + \frac{\partial^2 \bar{P}}{\partial x^2} - \left.\frac{\partial p}{\partial t}\right|_\delta\right\} \Big/ \left.\frac{\partial p}{\partial x}\right|_\delta. \tag{29}$$

When $\dot{\gamma} > 0$, within the shear band, $\partial p/\partial y|_\delta$ must be negative. Moreover, $\partial p/\partial t|_\delta = \partial^2 p/\partial x^2|_\delta$. Then Eq. (29) becomes

$$\frac{\partial \delta}{\partial t} = \left(\frac{\tau\bar{\dot{\gamma}}}{2} + \frac{\partial^2 \bar{p}}{\partial x^2} - \left.\frac{\partial^2 p}{\partial x^2}\right|_\delta\right) \Big/ \left.\frac{\partial p}{\partial x}\right|_\delta. \tag{30}$$

It is obvious that the term $\tau\bar{\dot{\gamma}}/2$ is always positive and therefore governs shear band contraction. However, there is usually a simple monotonically decreasing pore pressure distribution,

$$\frac{\partial^2 \bar{p}}{\partial x^2} - \left.\frac{\partial^2 p}{\partial x^2}\right|_\delta < 0 \tag{31}$$

from which it is seen that pore pressure diffusion in the shear band tends to expand the band.

5. Some computational examples and conclusions

Calculations are carried out for some situations. The initial disturbance is supposed to be a pore-pressure dependent one. For simplicity, p_0 is assumed to consist of only the basic mode:

$$p_0(x) = A\left(\cos\frac{\pi x}{\delta_0} + 1\right) + B, \quad x < \delta_0.$$

We have seen that the terms $H(t)$ and $\alpha_1^2 t$ play a more significant role than others; two additional assumptions are therefore introduced:

$$p_* \approx p_\delta - \delta\left.\frac{\partial p}{\partial y}\right|_\delta,$$

$$S(t) = (p_{\delta 0} - p_\delta)/(\delta_0 - \delta)$$

in their corresponding extension region, instead of accurate calculation $\partial p/\partial x|_{\delta -}$ and $S(t)|_{\delta_0}$.

Fig. 2 shows the rising of the pore pressure with time at different boundary velocities. It is obvious that pore pressure first rises slowly and then becomes fast under vibration load and the bigger the value of R, the smaller the changes of pore pressure between different R. It is shown also that the initial disturbances expand only if the amplitude of the disturbance exceeds some limit. Fig. 3 shows the

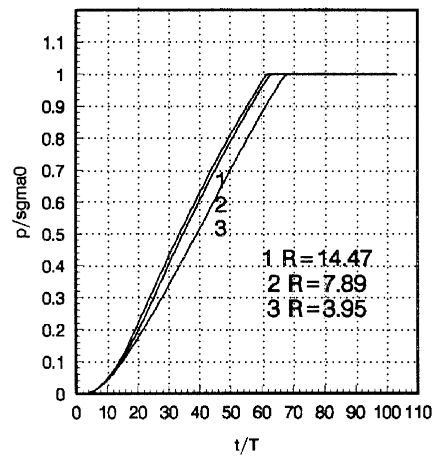

Fig. 2. The rising of pore pressure.

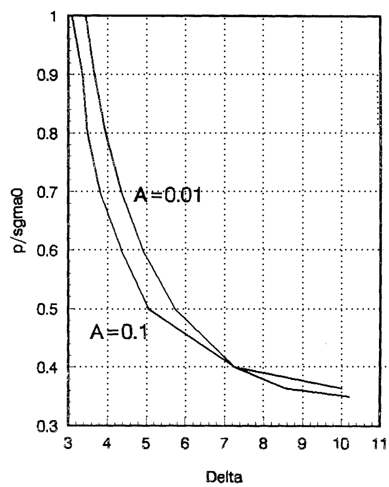

Fig. 3. The band width and the pore pressure at the center of band.

relationship of pore pressure and the width of band, the amplitudes span about one order but the curves remain quite close, which indicates that the shear bands have a strong intrinsic structure.

The analytical solution has been obtained in this paper to understand the mechanism of the shear band of saturated soil. It is shown that the pore pressure diffusion generally caused the shear band to expand while the shear strain rate acts on the contrary.

References

[1] W.D.L. Finn, K.W. Lee, G.R. Martin, Dynamic effective stress analysis of sands, in: Proc. 9th Int. Conf. on Soil Mech. and Foundation Eng., Session 4, 1981, p. 231.

[2] W.D. Finn, P.M. Byrne, G.R. Martin, Seismic response and liquefaction of sands, J. Geotech Eng. Div. ASCE 102 (1976) 841.

[3] Y.-L. Bai, C.-M. Cheng, Y.-S. Ding, Development of thermoplastic shear bands in simple shear, Res. Mechanica 22 (1987) 313.