Gas Exchange by Bubbles in Waves *

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A simple and feasible model for the calculation of the gas transfer by bubble clouds is proposed in this article. N_2, O_2, and CO_2 transfered by bubble clouds are obtained. At wind speed of 10 m/s, the calculated supersaturation of dissolved oxygen is 1.92–3.89% in agreement with the measurement.

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The gas exchange between air-sea considerably affects the ocean environment and climate. Most investigation in this area focused on the situation that wind speed is not too high and wave breaking seldom occurs. These studies did not include the effects of waves and bubbles on gas exchange. With wind speed increasing, ocean wave often breaks and a large amount of bubbles appears. Thorpe[1] mentioned the effect of bubble on gas exchange at first. Later Memery[2] studied the gas diffusion with a simple model that the bubble rose in still water. Woolf[3] considered the effect of Langmuir circulation and discussed the effect of bubble on the supersaturation of dissolved gas in ocean. Chanson[4] studied the gas transfer of bubble clouds based on the process of bubble entrainment by breaking-wave. For a long time, most researches about bubbles focused on the eigenfrequency of a bubble and the nonlinear effect on it. There are only a few work concerning about the translation of a bubble.[5,6] Liu *et al.*[7] discussed the bubble motion in waves.

In this letter, the effects of wave motion on gas diffusion from bubbles has been investigated. The simplified equation of gas diffusion is given together with the motion equation. On the basis of gas diffusion from a single bubble, the gas transfer by bubble clouds according the bubble distribution has been calculated as well.

Usually, bubbles in the ocean are very small, and their radii are rarely larger than the order of a millimeter. When the wind speed exceeds 11 m/s, the radii of most bubbles are within the range of 60–200 μm. So, we assume that the non-spherical deformation of a bubble can be ignored, and the bubble can be approximately considered as a sphere. It is also assumed that the fluid around the bubble is incompressible and the movement in it is potential. There is no effect of a bubble on waves because the wave length in the ocean is much larger than the diameter of a bubble. Furthermore, the bubble can be regarded as a particle in the wave field. The total velocity potential ϕ can be divided into two parts, one is the velocity potential of waves ϕ_w, the other is the velocity potential ϕ_b produced by the motion of a bubble, then $\phi = \phi_w + \phi_b$. The governing equation for bubble motion is

$$R\frac{d^2R}{dt} + \frac{2}{3}\left(\frac{dR}{dt}\right)^2 - \frac{U_b^2}{4} = \frac{1}{\rho_l}\left(p_g - p_w - \frac{2\sigma}{R}\right),$$

$$\frac{d}{dt}(R^3\boldsymbol{U}_b) = -2R^3\nabla p_w - \frac{3}{4}|U_b|\boldsymbol{U}_b R^2 C_d\ , \tag{1}$$

where R is the radius of the bubble, $p_w = p_a - \rho_l[\partial\phi_w/\partial t + (\nabla\phi_w)^2/2 + gz]_b$, p_a is the value of barometric pressure on the free surface, ρ_l is the density of fluid, p_g is the pressure inside the bubble, z is the depth of the bubble, C_d is the drag coefficient. $U_b = \nabla\phi_b$, where ϕ_b can be approximately written as

$$\phi_b = -\frac{U_b R^3\cos\theta}{2r^2} - \frac{R^2\dot{R}}{r} \tag{2}$$

in the coordinate that its origin is located at the center of the spherical bubble. If the period of waves in ocean is larger than one second, the radius of bubble is smaller than 1 mm, and the steady rising speed U_0 of a bubble in still water is within the range of 0.1–0.3 m/s. By neglecting small terms $\partial\boldsymbol{V}/\partial t$, $\partial c/\partial t$ and the acceleration $\boldsymbol{a}$ of the bubble, the process of gas diffusion satisfies the equations

$$\begin{aligned}\boldsymbol{V}\cdot\nabla\boldsymbol{V} &= \frac{1}{\rho_l}\nabla p + \nu\nabla^2\boldsymbol{V}\\ \boldsymbol{V}\cdot\nabla C &= D\nabla^2 C\,.\end{aligned} \tag{3}$$

Here $\boldsymbol{V}$ is the velocity of the fluid around the bubble, C is the concentration of dissolved gas, D is the coefficient of gas diffusion. Sherwood number is defined as $S_h = Q/[2rD(C - C_\infty)]$, where Q is the gas diffused from the bubble during the unit time, C and C_∞ are the concentration of dissolved gas at the bubble surface and infinity, respectively. There are a number of formulae for S_h number to model gas exchange process due to a bubble moving in the infinitely extensive fluid at steady velocity. The process of gas diffusion from a bubble can be described by the equation

$$\frac{dn_i}{dt} = -2rD_iS_{hi}S_i(p_{g_i} - p_{\infty_i})\,, \tag{4}$$

where subscript i means the i-th gas, n_i is the gas in the bubble, $p_{\infty_i} = C_{\infty_i}/S_i$, S_i is a constant. Equation (4) coupled with the motion equation (1) can be changed into the first-order equations and written in

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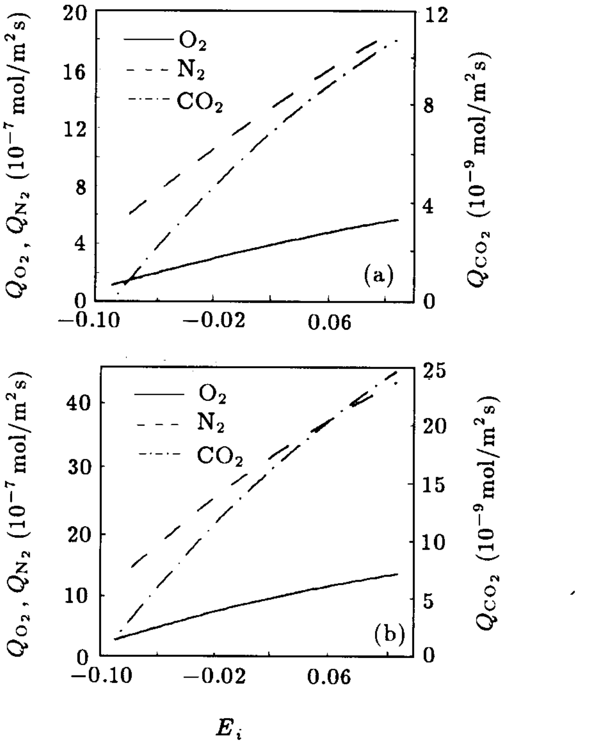

Fig. 1. Gas quantity transfered by bubble clouds with (a) wind spee: 10 m/s, phase: $\pi/2$ and (b) wind speed: 10 m/s, phase: $3\pi/2$.

dimensionless form as

$$\frac{\mathrm{d}r}{\mathrm{d}t} = H,$$

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{1}{R}\left(p_\mathrm{g} - p_\mathrm{w} - \frac{2}{RW} - \frac{3}{2}H^2 + \frac{U_\mathrm{b}^2}{4}\right),$$

$$\frac{\mathrm{d}U_{\mathrm{b}x}}{\mathrm{d}t} = -2\frac{\partial p_\mathrm{w}}{\partial x} - \frac{3}{4}\sqrt{U_{\mathrm{b}x}^2 + U_{\mathrm{b}z}^2}\frac{U_{\mathrm{b}x}C_\mathrm{d}}{R} - \frac{3HU_{\mathrm{b}x}}{R},$$

$$\frac{\mathrm{d}U_{\mathrm{b}z}}{\mathrm{d}t} = -2\frac{\partial p_\mathrm{w}}{\partial z} - \frac{3}{4}\sqrt{U_{\mathrm{b}x}^2 + U_{\mathrm{b}z}^2}\frac{U_{\mathrm{b}z}C_\mathrm{d}}{R} - \frac{3HU_{\mathrm{b}z}}{R},$$

$$\frac{\mathrm{d}X}{\mathrm{d}t} = U_x = U_{\mathrm{b}x} + \frac{\partial \phi_\mathrm{w}}{\partial X},$$

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = U_z = U_{\mathrm{b}z} + \frac{\partial \phi_\mathrm{w}}{\partial Z},$$

$$\frac{\mathrm{d}n_i}{\mathrm{d}t} = -\frac{3R}{2\pi}S_{h_i}\frac{K_i(p_{\mathrm{g}_i} - p_{\infty_i})}{p_{\mathrm{g0}_i}}. \quad (5)$$

The gas transfered by bubble clouds is determined based on the gas diffused from a single bubble, considering probability distribution of bubble clouds. The gas diffused from a single bubble can be obtained by solving equation (5). The probability distribution of bubble clouds is given by Chanson.[4] During a unit time the gas quantity transfered by bubble clouds from a unit area of ocean surface can be calculated by the formula

$$Q_{\mathrm{gas}_i} = \frac{1}{\lambda}\int_0^{H^*} p(H^*)\mathrm{d}H^* \cdot \int_{-d}^{0}\int_0^{\infty} S(R_*, z, H^*)Q_i(R_*, z)\mathrm{d}R_*\mathrm{d}z, \quad (6)$$

where λ is the mean wave length, R_* is dimensionless bubble radius, H^* is dimensionless wave height, $P(H^*)$ is the probability density of breaking waves, $Q_i(R_*, z)$ is a certain gas quantity diffused from a bubble, $S(R_*, z, H^*)$ is the distribution of bubble clouds. The change of gas quantities of N_2, O_2, and CO_2 transfered by bubble clouds with dimensionless number E_i and initial phase at wind speed of 10 m/s is shown in Fig. 1, where $E_i = (C_{\mathrm{sw}_i} - C_{\mathrm{w}_i})/C_{\mathrm{sw}_i}$, C_{sw_i} is saturated concentration, C_{w_i} is the concentration of disolved gas.

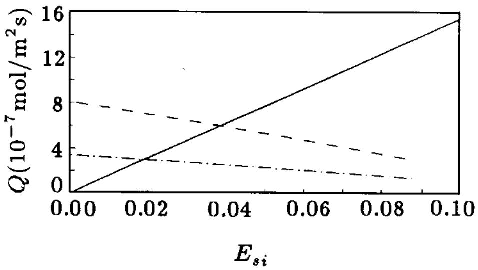

Fig. 2. Supersaturation of dissolved oxygen at wind speed of 10 m/s. The solid line indicates the gas quantity diffused into water through interface, the dashed line indicates the possible maximum gas quantity transferred by bubble clouds, and the dot-dashed line indicates the minimum gas quantity transferred by bubble clouds.

The calculated supersaturation of dissolved oxygen is demonstrated in Fig. 2, where solid line indicates the gas quantity diffused to atmosphere through the air-sea interface, dashed line and dash-dotted line indicate the possible maximum and minimum gas quantity transfered into the water by bubble clouds, respectively. The value of $E_{si}(= -E_i)$ at the point of intersection of solid line and dashed line (and dash-dotted line) is the possible maximum (minimum) supersaturation that the dissolved oxygen can reach. In Fig. 2 we can see that the dissolved oxygen can reach the supersaturation 1.92–3.89% at the wind speed of 10m/s. For the oceans all over the world, the supersaturation of the dissolved oxygen is 3% (Ref. 8) on average, and this level of supersaturation can rise to 8% after a storm[9] (the wind speed is greater than 10 m/s). So we believe that our model is simple and feasible.

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