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Flow characteristics of liquid with pressure-dependent viscosities in microtubes

Received: 28 September 2004 / Revised: 11 July 2005 / Accepted: 2 November 2005 / Published online: 10 January 2006
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Abstract It is obvious that the pressure gradient along the axial direction in a pipe flow keeps constant according to the Hagen-Poiseuille equation. However, recent experiments indicated that the distribution of the pressure seemed no longer linear for liquid flows in microtubes driven by high pressure (1–30 MPa). Based on H-P equation with slip boundary condition and Bridgman's relation of viscosity vs. static pressure, the nonlinear distribution of pressure along the axial direction is analyzed in this paper. The revised standard Poiseuille number with the effect of pressure-dependent viscosity taken into account agrees well with the experimental results. Therefore, the dependence of the viscosity on the pressure is one of the dominating factors under high driven pressure, and is represented by an important property coefficient α of the liquid.

Keywords Microfluidic · Liquid · Microtube · Viscosity

1 Introduction

The characteristics of liquid flows in microchannels are important in designing microfluidic devices. Pfahler et al. [1] measured the friction factor of isopropanol in a microchannel with a hydraulic diameter of 39.7 μm , and found that its apparent viscosity became smaller than that in macro-

The project supported by the Chinese Academy of Sciences Major Innovation Project (KJ CX2-SW-L2) and the National Natural Science Foundation of China (10272107).
The English text was polished by Yunming Chen.

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scale cases, whereas Papautsky's [2] experiments with water (Reynolds number = 0.001–18 and hydraulic diameter = 57 μm) revealed that the experimental flow rate was smaller than the prediction by the classical theory. However, with more accurate measurement methods, the recent results of Sharp et al. [3], Li et al. [4, 5] and Judy et al. [6] all indicated that the behaviors of liquid flows in microchannels with diameters larger than tens micrometer are in good agreement with the continuum theory. In order to extend the experimental research to smaller microtubes, Cui et al. [7] measured the flow characteristics of liquids in microtubes with diameters from 3 μm to 10 μm . As the flow rate is proportional to the fourth power of the diameter, a higher pressure (1–30 MPa) must be applied to maintain a flow rate that can be accurately measured. From their experiments, they found that the viscosity of liquid along the axial direction was changed, and the pressure gradient of liquid flow seemed to be nonlinear. They gave a simplification formula of normalized friction coefficient to correlate their experimental results. However, the pressure and viscosity distributions are worth making further investigation.

In this paper, the pressure distribution along the axial direction in microtubes under a high pressure is analyzed, and the variation of viscosity along the flow direction is expressed as a function of pressure. Considering the high-pressure effect, a revised standard Poiseuille number is proposed and compared with the experimental results.

2 Theoretical analysis

2.1 Velocity and flow rate

For a steady, fully developed laminar flow in a circular cross-section tube, H-P equation in Δx element can be expressed as

$$\frac{\partial p}{\partial x} = \frac{\mu}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right), \quad (1)$$

where, p is the pressure, μ is the viscosity, u is the velocity, x and r are the axial and radial coordinates, respectively (Fig. 1).

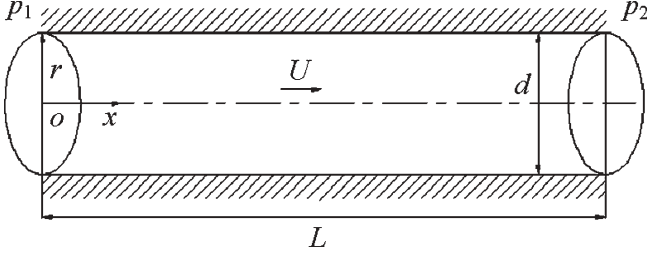


Fig. 1 A schematic depiction of the flow in microtubes

Based on the experiments of Choi et al. [8] and Tretheway et al. [9] in microchannels, the slip boundary condition should be considered,

$$\begin{aligned} r = \frac{d}{2}, \quad u = -L_s \dot{\gamma}_w = -L_s \left(\frac{\partial u}{\partial r} \right)_w; \\ r = 0, \quad \frac{\partial u}{\partial r} = 0, \end{aligned} \quad (2)$$

where L_s is the slip length, $\dot{\gamma}_w$ is the shear rate at wall, d is the diameter of the tube.

Considering μ is not a function of r , because there is no effect in this direction, we integrate Eq.(1) respect to r two times and get

$$u(r) = \frac{1}{4\mu} \left[r^2 - \frac{d^2}{4} \right] \frac{\partial p}{\partial x} - \frac{L_s d}{4\mu} \frac{\partial p}{\partial x}. \quad (3)$$

It is easy to determine the volume flow rate Q through the tube,

$$\begin{aligned} Q &= \int_0^{2\pi} \int_{-\frac{d}{2}}^{\frac{d}{2}} u(r) \cdot r d\theta dr \\ &= -\frac{\pi d^4}{128\mu} \frac{\partial p}{\partial x} - \frac{\pi d^3 L_s}{16\mu} \frac{\partial p}{\partial x} \\ &= -\frac{\pi d^4}{128\mu} \left(1 + \frac{8L_s}{d} \right) \frac{\partial p}{\partial x}. \end{aligned} \quad (4)$$

Under no slip condition, Eq.(3) and Eq.(4) are converted into the well-known H-P equation. $u(r)$ and Q_{th} are expressed as

$$u(r) = \frac{1}{4\mu} \left[r^2 - \frac{d^2}{4} \right] \frac{\partial p}{\partial x}, \quad (5)$$

$$Q_{th} = \frac{\pi d^4}{128\mu_0 L} \Delta p, \quad (6)$$

where $\Delta p = p_1 - p_2$ is the pressure drop between the entrance pressure p_1 and the exit pressure p_2 , μ_0 is the viscosity under the atmospheric temperature and L is the length of the tube.

2.2 Pressure and viscosity distributions at high pressure

According to Bridgman's results [10], the relationship of the viscosity vs. the static pressure can be expressed as an exponential function,

$$\mu = \mu_0 e^{\alpha(p-p_0)}, \quad (7)$$

where, μ is the real viscosity, p_0 is the atmospheric pressure and α is a property coefficient of the liquid. Substituting Eq.(7) into Eq.(4), we obtain

$$e^{-\alpha p} dp = -\frac{128\mu_0 Q}{\pi d^4 A} dx, \quad (8)$$

where $A = 1 + 8L_s/d$. Integrating Eq.(8) along x -direction, we get

$$\begin{aligned} \int_{p_1}^p e^{-\alpha p} dp &= -\int_0^x \frac{128\mu_0 Q}{\pi d^4 A} dx, \\ p(x) &= -\frac{1}{\alpha} \ln \left[\frac{128\mu_0 \alpha Q}{\pi d^4 A} x + e^{-\alpha p_1} \right]. \end{aligned} \quad (9)$$

Obviously, the distribution of $p(x)$ along x -direction is a logarithmic function, which depends on the inlet pressure p_1 and the position x . It's well known that the pressure gradient along the axial direction of tube keeps constant in a classical H-P flow, therefore, the flow driven by high pressure is no longer a H-P flow.

The pressure at the exit p_2 is equal to p_0 , Eq.(9) can be rewritten as

$$\frac{p(x)}{p_1} = -\frac{\ln \left((1 - e^{-\alpha p_1}) \frac{x}{L} + e^{-\alpha p_1} \right)}{\alpha p_1}. \quad (10)$$

Figure 2 shows the distribution of $p(x)/p_1$ along x -direction with $\alpha = 7.9 \times 10^{-3} \text{ MPa}^{-1}$ for isopropanol.

The difference between the nonlinear pressure distribution $p(x)$ and corresponding parameters of the classical H-P flow $p_{th}(x)$ is defined as ε ($\varepsilon = p(x) - p_{th}(x)$). Figure 3 shows the variation of ε with x/L . The maximum value of ε can reach 3%.

According to Taylor formula, Eq.(7) is expanded at the atmospheric pressure p_0 ,

$$\mu(p) = \mu_0(p_0) + (p - p_0) \frac{\partial \mu(p_0)}{\partial p} + \dots, \quad (11)$$

where $\frac{\partial \mu(p_0)}{\partial p} = \mu_0 \alpha e^{\alpha(p-p_0)} \Big|_{p=p_0} = \mu_0 \alpha$, we obtain,

$$\mu(p) \approx \mu_0(p_0) [1 + \alpha(p - p_0)] \approx \mu_0(p_0) [1 + \alpha p]. \quad (12)$$

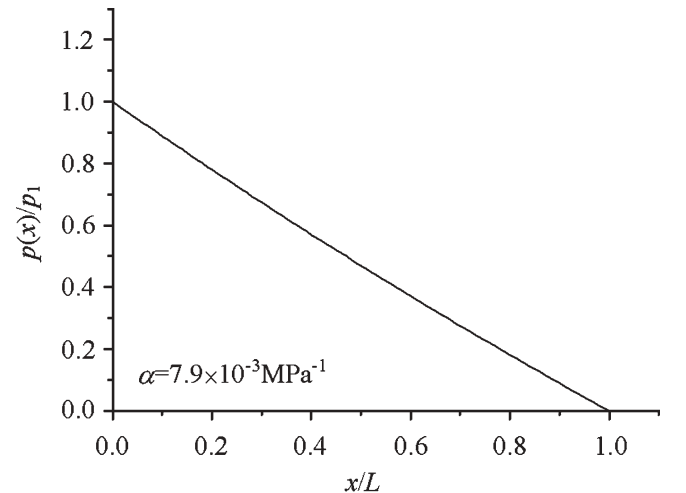


Fig. 2 Dimensionless pressure distribution along x -direction

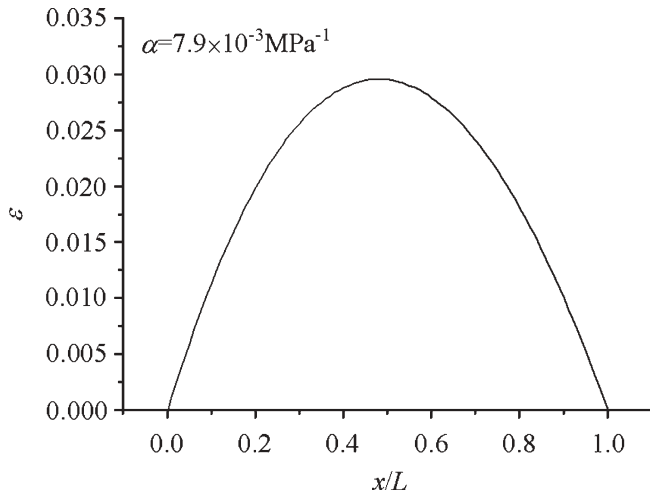


Fig. 3 Variation of ε with x/L

Substituting Eq.(9) into Eq.(12), we noted that $\mu(p)$ is a nonlinear function along the flow direction.

3 Comparison with experimental results

3.1 Experimental methods

The experiments have been carried out in LNM laboratory [7]. The experimental setup [4, 5] includes three units: a pressure source, a testing section and a flux-measuring unit (Fig.4). The high-pressure pump of 1-40MPa (147-5800Psi) is used to drive the working liquids. The testing section consists of a tee connector, a pressure transducer, a thermocouple and a microtube. The pressure transducer and the thermocouple

are connected to the tee connector, through which the inlet pressure of the microtube and the temperature of the working liquid can be measured. The microtube is long enough to neglect the inlet and outlet effects ($d = 3-10\mu\text{m}$, $L \approx 50\text{mm}$). One end of the microtube is linked to the tee connector with a mechanical seal and the other end is inserted into a glass capillary with glues. The working liquid passing through the microtube is gathered in the glass capillary. Through measuring the displacement s of liquid's column surface in the capillary and the interval t , the flow rate Q_{exp} can be calculated by the following equation:

$$Q_{\text{exp}} = \frac{\pi D^2 s}{4t}, \quad (13)$$

where D is the diameter of the capillary. The test liquid presented here is only isopropanol.

The inner surface roughness of microtube is measured with AFM. The average height Δh is about 70-7.12nm. The relative surface roughness is calculated to be less than 0.7%. So the microtube can be regarded as the hydraulic smooth tube. The influences of surface evaporation, viscous heating and liquid's compressibility etc. can be neglected in this experiment [7].

The experimental uncertainty consists of two parts: the uncertainty of the experimental system and a product of the standard deviation and ζ , where ζ is a coefficient related to the number of measurements. The uncertainty of experimental system mainly depends on the precision of the diameter measurement [7]. With images of SEM, the measurement precision is about $\pm 0.1\mu\text{m}$. Therefore, the measurement uncertainties are $\pm 1\%$ - $\pm 3.3\%$ for tubes of 3-10 μm diameters. Because the flow rate is proportional to the forth power of the diameter, the experimental uncertainties for three microtubes (10 μm , 5 μm 3 μm) are $\pm 4.8\%$, $\pm 8.6\%$ and $\pm 13.6\%$,

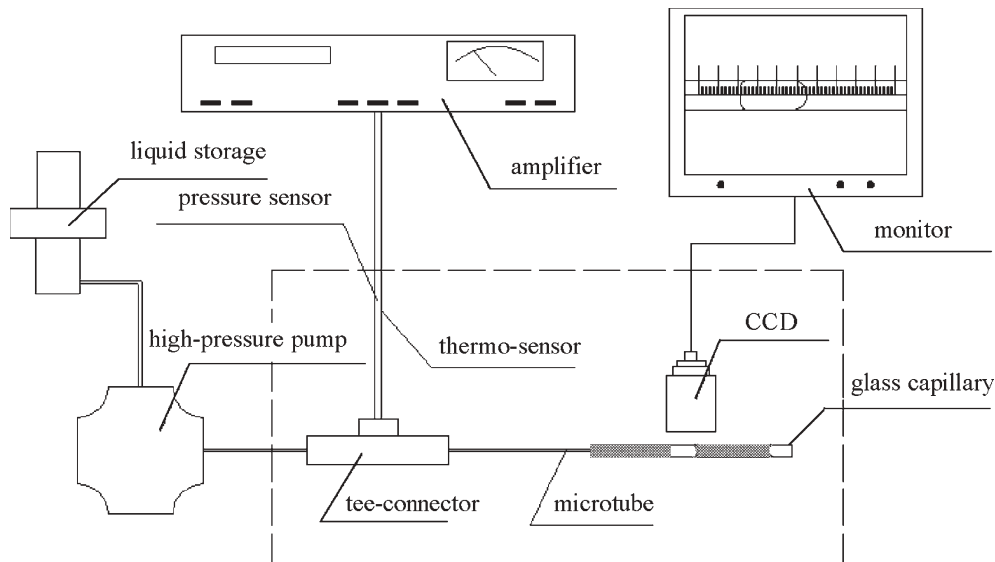


Fig. 4 A sketch of the high-pressure microflow setup

respectively. The experimental results are presented as individual arithmetic average values with corresponding experimental error bars. Each arithmetic average value is calculated from at least three measurements.

3.2 Experimental results

Poiseuille Number Po in a pipe flow is defined as $Po = f Re$, f is the Fanning friction coefficient [11]. $Po = 16$ for the flow in a tube with a circular cross-section. To compare the theoretical values with the experimental results, the standard Poiseuille Number Po^* is defined as

$$Po^* = \frac{Po_{\text{exp}}}{Po_{\text{th}}} = \frac{(Re \cdot f)_{\text{exp}}}{(Re \cdot f)_{\text{th}}} = \frac{Q_{\text{th}}}{Q_{\text{exp}}}, \quad (14)$$

where the subscripts th and exp present the theoretical and experimental values, respectively. If the value of Q_{exp} agrees with the prediction by H-P equation, $Po^* = 1$. However, from the experimental results, Po^* deviates from 1 and the values increase noticeably with the pressure (Fig.5-7). Even though the uncertainty is considered, experimental results are still 3%-5% larger than 1 at the pressure of 30MPa (Fig.5).

3.3 Revised standard poiseuille number $Po^{*'}$

Considering the dependence of the viscosity on the pressure, the revised $Po^{*'}$ is defined as

$$Po^{*' } = \frac{Po_{\text{th}-\mu}}{Po_{\text{th}}} = \frac{Q_{\text{th}}}{Q_{\text{th}-\mu}}, \quad (15)$$

where $Q_{\text{th}-\mu}$ is the flow rate when the revised viscosity is taken into account. From Eq.(9), $Q_{\text{th}-\mu}$ is given as

$$Q_{\text{th}-\mu} = \frac{\pi d^4 A}{128 \mu_0 \alpha L} e^{-\alpha p_1} (e^{\alpha \Delta p} - 1). \quad (16)$$

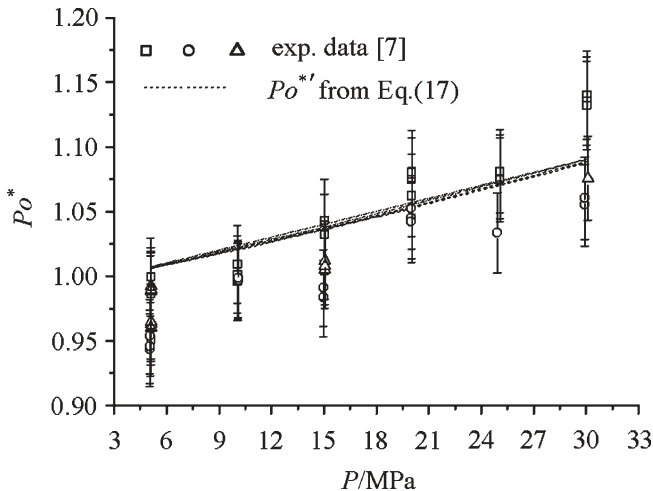


Fig. 5 Comparison of Po^* vs. the pressure drop of isopropanol in $10\mu\text{m}$ tube

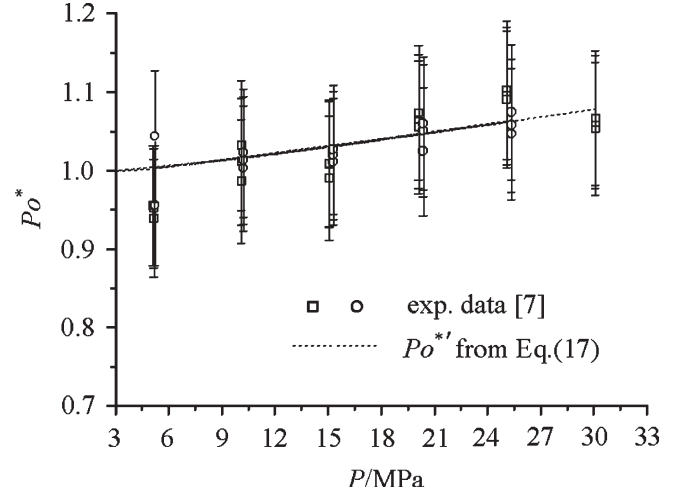


Fig. 6 Comparison of Po^* vs. the pressure drop of isopropanol in $5\mu\text{m}$ tube

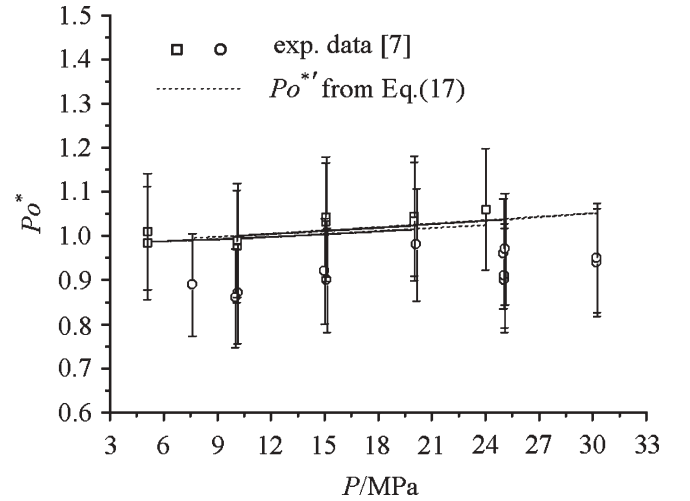


Fig. 7 Comparison of Po^* vs. the pressure drop of isopropanol in $3\mu\text{m}$ tube

The revised standard Poiseuille number $Po^{*'}$ is

$$\begin{aligned} Po^{*' } &= \frac{Q_{\text{th}}}{Q_{\text{th}-\mu}} \\ &= e^{\alpha p_1} \cdot \Delta p (e^{\alpha \Delta p} - 1)^{-1} \cdot \frac{\alpha}{A} = \frac{\alpha \Delta p e^{\alpha p_1}}{A \cdot e^{\alpha \Delta p} - 1}. \quad (17) \end{aligned}$$

The increment of the average flow rate resulted from the slip condition is smaller than 5% at 30MPa in the case of $10\mu\text{m}$ microtube [7], so A is equal to 1.05. In Fig.5, the revised standard Poiseuille number $Po^{*'}$ is presented as a dash line. It increases significantly with the pressure and shows the same trend as the experimental results. This reveals that the revised flow rate agrees well with the experimental value. It means that the dominant influence on the viscosity is the high pressure. The same trends appear in the microtubes with the diameter of $3\mu\text{m}$ and $5\mu\text{m}$, respectively.

3.4 Discussions

- (1) Based on Eq.(9), the nonlinear distribution of pressure depends on the property of the liquids, i.e. the coefficient α . For most liquids, except for water, the values of α are nearly 10^{-2} - 10^{-3} MPa $^{-1}$ [10]. When the driven pressure becomes higher than 10MPa, the effect of pressure-dependent viscosity should be considered. This is confirmed by experimental results. In Fig.5, Po^* is noticeably larger than 1 when $p > 10$ MPa;
- (2) For water, because of hydrogen bond, the value of α keeps near zero when the driven pressure is smaller than 100MPa [10]. The experimental results of Cui et al. [7] also demonstrated that the viscosity of water in microflows keeps constant for pressures ranging from 1-30MPa.

4 Conclusions

When the diameters of microtubes are smaller than 10 μ m, the driven pressure should be increased to 1-30MPa. The high pressure leads to variation in the viscosity of liquids. In this paper, we analyzed both the pressure distributions and the viscosity distributions along the flow direction, and introduced a revised standard Poiseuille number to correlate the experimental results. The main results are as follows:

- (1) The distributions of the pressure and the viscosity become nonlinear along the axial direction. The behavior of the nonlinearity is mainly related to the property coefficient of the liquid α . For example of isopropanol, the difference between the nonlinear pressure distribution and the corresponding parameters of classical H-P flow reaches 3%;
- (2) The revised standard Poiseuille number $Po^{*'}$ agrees well with the experimental results. It means that the dependence of the viscosity on the pressure is a dominant factor in microflows under high pressure;
- (3) The variation of viscosity under high pressure dominantly depends on the property coefficient α of the liquid. For most liquids, except for water, the effect of pressure-dependent viscosity should be considered if the driven pressure is higher than 10MPa.

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