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压电材料裂纹顶端条状电饱和区模型的力学分析[1)]

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摘要 在线性压电本构方程框架下,对裂纹顶端条状电饱和区模型进行了严格的数学分析.完整地考虑了各向异性力电耦合效应.建立了电饱和区尺寸与外加电场的依赖关系.证实了当裂纹垂直极化轴时,压电材料的断裂应力随着外加正电场的增加而减小,随着外加负电场的增加而增加.当裂纹平行于极化轴时,与极化轴平行的外加电场对断裂应力无影响.

关键词 压电材料,裂纹,电饱和,极化

引 言

压电材料在智能元件中得到广泛应用.它既可用作具有感知功能的传感器,又可用作具有致动功能的执行器.由于电和力的耦合作用,压电材料的电致断裂、电致疲劳时有发生.对于压电材料断裂行为研究日益受到重视.

Parton[1], Deeg[2], Pak 和 Herrmann[3], McMeeking[4], Pak[5], Sosa[6], Suo, et al.[7], Suo[8], Zhang 和 Hack[9], Yang 和 Suo[10], Dunn[11], Zhang 和 Tong[12]等人对压电材料断裂进行了理论研究.但是理论与实验存在明显的差异[13~15].理论研究表明外加电场的存在总是阻碍裂纹扩展,而 Park 和 Sun[14]实验证实正电场促使裂纹扩展而负电场阻碍裂纹扩展. Gao 等人[16]针对理论与实验的明显差异,提出了一个电场条状饱和区模型.他们认为钛酸钡($BaTiO_3$)、锆钛酸铅(PZT)等铁电晶体可以看作是"理想"的弹性体,塑性屈服比较困难,而电屈服比较容易.为了考虑电屈服对压电材料断裂行为的影响,比较简单的是引入条状饱和区模型. Gao 等人[16]引入局部能量释放率准则,成功地解释了实验现象.

但是 Gao 等人[16]在分析中,对线性压电材料的本构方程及问题的提法作了一些简化假设.本文对条状饱和区模型进行严格的数学分析,在线性压电本构方程框架下,完整地考虑了各向异性力电耦合效应.证实了压电材料断裂应力与外加电场的线性依赖关系.

1 基本公式

线性压电材料的本构方程为

$$\left.\begin{aligned} \sigma_{ij} &= c_{ijkl}\gamma_{kl} - e_{kij}E_k \\ D_i &= e_{ikl}\gamma_{kl} + \varepsilon_{ik}E_k \end{aligned}\right\} \tag{1}$$

式中 σ_{ij}, γ_{ij} 分别是应力和应变张量, D_i, E_i 分别是电位移矢量和电场强度. c_{ijkl}, e_{ikl}, ε_{ij} 分别是压电材料的弹性模量张量,压电系数和介电系数.

场方程为

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$$\left.\begin{aligned}\sigma_{ij,i} &= 0\\ D_{i,i} &= 0\end{aligned}\right\}\tag{2}$$

应变 γ_{ij} ,电场强度 E_i 可表示为

$$\left.\begin{aligned}\gamma_{ij} &= \frac{1}{2}(u_{i,j} + u_{j,i})\\ E_i &= -\phi_{,i}\end{aligned}\right\}\tag{3}$$

式中 u_i 是位移, ϕ 是电势.

将公式(1),(3)代入(2)式得

$$\left.\begin{aligned}(c_{ijkl}u_k + e_{lij}\phi)_{,li} &= 0\\ (e_{ikl}u_k - \varepsilon_{il}\phi)_{,li} &= 0\end{aligned}\right\}\tag{4}$$

讨论二维问题,参照 Suo 等人[7]的工作,一般解可以用复势函数表示,

$$\{u_i, \phi\} = \mathrm{a}f(\zeta_1 x + \zeta_2 y)\tag{5}$$

式中 a 是含有 4 个元素的列阵, $\zeta_1 = 1$, $\zeta_2 = p$. 将(5)式代入(4)式得

$$\left.\begin{aligned}(c_{\alpha jk\beta}a_k + e_{\alpha\beta j}a_4)\zeta_\alpha\zeta_\beta &= 0\\ (e_{\alpha\beta k}a_k - \varepsilon_{\alpha\beta}a_4)\zeta_\alpha\zeta_\beta &= 0\end{aligned}\right\}\tag{6}$$

式中 α,β 取值 1, 2. j, k 取值 1, 2, 3. 这是关于列阵 a 的本征值问题. Suo 等人[7]证实了该问题的本征方程具有 8 个复根,它们构成 4 对共轭复数. 用 p_1, p_2, p_3, p_4 表示具有正的虚部的 4 个复根,则有

$$\{u_i, \phi\} = 2\mathrm{Re}\sum_{k=1}^{4}\mathrm{a}_k f_k(z_k)\tag{7}$$

式中 $z_k = x + p_k y$.

对于应力分量及电位移矢量有

$$\left.\begin{aligned}\{\sigma_{2j}, D_2\} &= 2\mathrm{Re}\sum_{k=1}^{4}\mathrm{b}_k f'_k(z_k)\\ \{\sigma_{1j}, D_1\} &= -2\mathrm{Re}\sum_{k=1}^{4}\mathrm{b}_k p_k f'_k(z_k)\end{aligned}\right\}\tag{8}$$

列阵 b 的分量为

$$b_j = (c_{2jk1}a_k + e_{12j}a_4) + (c_{2jk2}a_k + e_{22j}a_4)p\tag{9}$$

$$b_4 = (e_{21k}a_k - \varepsilon_{21}a_4) + (e_{22k}a_k - \varepsilon_{22}a_4)p\tag{10}$$

引入 4 ×4 矩阵 A 和 B

$$\left.\begin{aligned}\mathrm{A} &= [\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3, \mathrm{a}_4]\\ \mathrm{B} &= [\mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3, \mathrm{b}_4]\end{aligned}\right\}\tag{11}$$

定义单变量复函数矢量 $\mathrm{f}(z)$

$$\mathrm{f}(z) = \{f_1(z), f_2(z), f_3(z), f_4(z)\}\tag{12}$$

那么在实轴上广义的位移矢量及面力矢量可表示为

$$U(x) = \{u_j, \phi\} = Af(x) + \overline{A}\overline{f(x)} \tag{13}$$

$$t(x) = \{\sigma_{2j}, D_2\} = Bf'(x) + \overline{B}\overline{f'(x)} \tag{14}$$

2 条状电饱和区模型

2.1 力学分析

Gao 等人[16]在分析压电陶瓷的断裂行为时,引入了条状电饱和区模型.该模型可以看作是经典的 Dugdule 条状屈服区模型的一个新的发展.图 1 显示了无限大压电材料中一条长度为 $2a$ 的有限裂纹.无穷远处,受应力 σ_{11}^{∞}, σ_{22}^{∞} 和电位移 D_1^{∞}, D_2^{∞} 作用.在裂纹面上,满足广义面力为零的边条.

$$\sigma_{22} = 0, \quad \tau_{21} = 0, \quad \tau_{23} = 0, \quad D_2 = 0, \ |x| \le a \tag{15}$$

在条状电饱和区内,有

$$u_1^+ = u_1^-, \ u_2^+ = u_2^-, \ u_3^+ = u_3^-, \ \tau_{21}^+ = \tau_{21}, \ \sigma_{22}^+ = \sigma_{22}^-, \ \tau_{23}^+ = \tau_{23}^-, \ D_2 = D_s, \ a \le |x| \le c \tag{16}$$

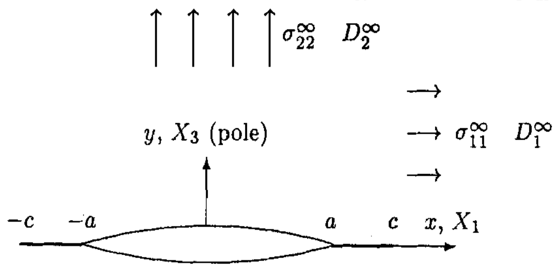

图 1　垂直于极化轴的电饱和裂纹

Fig. 1　An electric saturation crack perpendicular to the poling axis

外加应力场和电场所产生的均匀场为

$$\sigma_{11} = \sigma_{11}^{\infty}, \quad \sigma_{22} = \sigma_{22}^{\infty}, \quad \tau_{12} = \tau_{13} = \tau_{23} = 0 \tag{17}$$

$$D_1 = D_1^{\infty}, \quad D_2 = D_2^{\infty} \tag{18}$$

裂纹引起的非均匀扰动场,在裂纹面上需满足下述条件

$$t^+(x) = t^+(x) = -T, \quad y = 0, \quad |x| < a \tag{19}$$

$$T = \{0, \sigma_{22}^{\infty}, 0, D_2^{\infty}\} \tag{20}$$

在条状电饱和区内,有

$$t^+(x) = t^-(x), \quad a \le |x| \le c \tag{21}$$

$$u_i^+ = u_i^-, \quad y = 0, \quad a \le |x| \le c, \quad i = 1,2,3 \tag{22}$$

$$D_2 = D_s - D_2^{\infty}, \quad a \le |x| \le c \tag{23}$$

在整个实轴上广义面力 $t(x)$ 的连续条件必须得以满足,参照 Suo 等人[7]的工作不难证实

$$h(z) = Bf'(z) = \overline{B}\overline{f}'(z) \tag{24}$$

式中 $\mathbf{f}(z)$,是扰动场所对应的复函数矢量. 复函数矢量 $\mathbf{h}(z)$,满足裂纹面边界条件(19) ,

$$\mathbf{h}^{+}(x) + \mathbf{h}^{-}(x) = - \mathbf{T}, \quad |x| < a \tag{25}$$

另有

$$i\boldsymbol{\delta}'(x) = \mathbf{HBf}'^{+}(x) - \mathbf{HBf}'^{-}(x) \tag{26}$$

式中 $\boldsymbol{\delta}(x) = \{u_1^{+} - u_1^{-}, u_2^{+} - u_2^{-}, u_3^{+} - u_3^{-}, \phi^{+} - \phi^{-}\}$ 是广义张开位移. $\mathbf{H}$ 为实矩阵

$$\mathbf{H} = 2\mathrm{Re}\{i\mathbf{AB}^{-1}\} \tag{27}$$

引入复函数矢量

$$\mathbf{g}(z) = \mathbf{HBf}'(z) \tag{28}$$

从(22)式,不难看出,分量函数 $g_i(z)$, $i = 1,2,3$ 是全平面解函数,带有割痕裂纹 $(-a, a)$. 而分量函数 $g_4(z)$ 是全平面解析函数,带有实轴上割痕 $(-c, c)$. 由(28)式,得

$$\mathbf{h}(z) = \mathbf{H}^{-1}\mathbf{g}(z) = \boldsymbol{\Lambda}\mathbf{g}(z) \tag{29}$$

式中

$$\boldsymbol{\Lambda} = \mathbf{H}^{-1} \tag{30}$$

公式(25)写成分量形式,

$$\Lambda_{ik}\left(g_k^{+}(x) + g_k^{-}(x)\right) + \Lambda_{i4}\left(g_4^{+}(x) + g_4^{-}(x)\right) = -T_i, \quad i = 1,2,3, \quad |x| < a \tag{31}$$

$$\Lambda_{4k}\left(g_k^{+}(x) + g_k^{-}(x)\right) + \Lambda_{44}\left(g_4^{+}(x) + g_4^{-}(x)\right) = -T_4, \quad |x| < a \tag{32}$$

从(31), (32)式中消去 $g_4^{+}(x) + g_4^{-}(x)$,得到

$$\Lambda_{ik}^{*}\left(g_k^{+}(x) + g_k^{-}(x)\right) = -T_i^{*}, \quad |x| < a \tag{33}$$

式中

$$\left.\begin{aligned} \Lambda_{ik}^{*} &= \Lambda_{ik} - \Lambda_{i4}\Lambda_{4k}/\Lambda_{44}, \\ T_i^{*} &= T_i - T_4\Lambda_{i4}/\Lambda_{44}, \quad i,k = 1,2,3 \end{aligned}\right\} \tag{34}$$

(33)式可改写成矢量形式,

$$\boldsymbol{\Lambda}^{*}\mathbf{g}^{*+}(x) + \boldsymbol{\Lambda}^{*}\mathbf{g}^{*-}(x) = -\mathbf{T}^{*}, \quad |x| < a \tag{35}$$

式中 $\boldsymbol{\Lambda}^{*}$ 是 3 ×3 矩阵,它的元素是 Λ_{ij}^{*}. $\mathbf{g}^{*}$, $\mathbf{T}^{*}$ 是含有 3 个元素的列阵,

$$\mathbf{g}^{*}(z) = \{g_1(z), g_2(z), g_3(z)\} \tag{36}$$

$$\mathbf{T}^{*} = \{T_1^{*}, T_2^{*}, T_3^{*}\} \tag{37}$$

显然复函数矢量 $\mathbf{g}^{*}(z)$ 是全平面上解析的,除了割痕 $L(-a, a)$. 由(35)式,不难导出,

$$\boldsymbol{\Lambda}^{*}\mathbf{g}^{*}(z) = \mathbf{T}^{*}f_0'(z) \tag{38}$$

$$f_0'(z) = \frac{1}{2}\left\{\frac{z}{\sqrt{z^2 - a^2}} - 1\right\} \tag{39}$$

由公式(23),(29)和(32)不难推得,

$$\left.\begin{aligned} g_4^+(x)+g_4^-(x) &= -\{\Lambda_{4k}[g_k^+(x)+g_k^-(x)]+T_4\}/\Lambda_{44}, & |x|<a \\ g_4^+(x)+g_4^-(x) &= -\{\Lambda_{4k}[(g_k^+(x)+g_k^-(x)]+T_4\}/\Lambda_{44}+D_s/\Lambda_{44}, & a\le|x|\le c \end{aligned}\right\} \tag{40}$$

由此得到

$$g_4(z) = \{-\Lambda_{4k}g_k(z)+T_4 f_c'(z)\}/\Lambda_{44}+D_s g_0(z)/\Lambda_{44} \tag{41}$$

式中

$$\left.\begin{aligned} f_c'(z) &= \frac{1}{2}\cdot\left[\frac{z}{\sqrt{z^2-c^2}}-1\right] \\ g_0(z) &= \frac{1}{\pi}\left\{\frac{\pi}{2}-\frac{1}{2\mathrm{i}}\log\frac{\dfrac{z}{a}\cdot\dfrac{\sqrt{c^2-a^2}}{\sqrt{z^2-c^2}}+\mathrm{i}}{\dfrac{z}{a}\cdot\dfrac{\sqrt{c^2-a^2}}{\sqrt{z^2-c^2}}-\mathrm{i}}-\frac{z}{\sqrt{z^2-c^2}}\arccos\left(\frac{a}{c}\right)\right\} \end{aligned}\right\} \tag{42}$$

不难证实复函数 $g_0(z)$ 是全平面解析函数,带有割痕 $(-c, c)$.在割痕上它满足如下边条,

$$\left.\begin{aligned} g_0^+(x)+g_0^-(x) &= 0, & |x|<a \\ g_0^+(x)+g_0^-(x) &= 1, & a\le|x|\le c \end{aligned}\right\} \tag{43}$$

此外在无穷远处,$g_0(z)$ 趋于零.图 2 显示,复函数 $\log\frac{\zeta+\mathrm{i}}{\zeta-\mathrm{i}}$ 的计算方法.

$$\log\frac{\zeta+\mathrm{i}}{\zeta-\mathrm{i}} = \log\frac{r_2}{r_1}+\mathrm{i}(\theta_2-\theta_1) \tag{44}$$

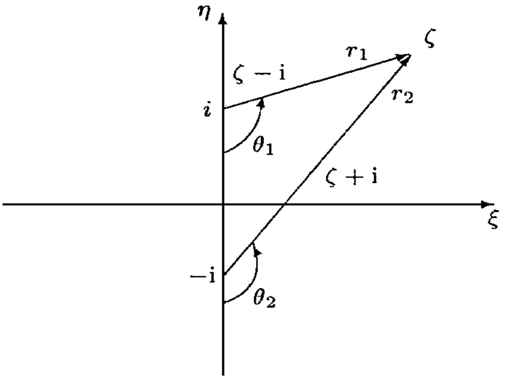

图 2　复变函数 $\log\frac{\zeta+\mathrm{i}}{\zeta-\mathrm{i}}$ 的计算方法示意图

Fig. 2　Calculation scheme of complex function $\log\frac{\zeta+\mathrm{i}}{\zeta-\mathrm{i}}$

公式(38),(41)提供 $g(z)$ 全部分量函数的解答.

为了求得由饱和区尺寸 c,我们来考察电饱和区前方的广义面力.我们有

$$t(x) = \{\sigma_{21},\sigma_{22},\sigma_{23},D_2\} = Bf'(x)+\bar{B}\bar{f}'(x) = \Lambda g^+(x)+\Lambda g^-(x), \quad |x|>c \tag{45}$$

$$D_2 = \Lambda_{4k}(g_k^+(x)+g_k^-(x))+\Lambda_{44}(g_4^+(x)+g_4^-(x)) = 2T_4 f_c'(x)+D_s(g_0^+(x)+g_0^-(x)) =$$

$$\left(D_2^\infty-\frac{2}{\pi}D_s\arccos\left(\frac{a}{c}\right)\right)\frac{x}{\sqrt{x^2-c^2}}-D_2^\infty+D_s\frac{2}{\pi}\left\{\frac{\pi}{2}-\frac{1}{2\mathrm{i}}\log\frac{\dfrac{x}{a}\cdot\dfrac{\sqrt{c^2-a^2}}{\sqrt{x^2-c^2}}+\mathrm{i}}{\dfrac{x}{a}\cdot\dfrac{\sqrt{c^2-a^2}}{\sqrt{x^2-c^2}}-\mathrm{i}}\right\}$$

$$|x|>c \tag{46}$$

从(16)式不难看出,为了确保电位移 $D_2(x)$ 的有限性,奇性项 $\frac{x}{\sqrt{x^2-c^2}}$ 前的系数必须为零.由此得到

$$\frac{a}{c} = \cos\left(\frac{\pi D_2^\infty}{2D_s}\right) \tag{47}$$

这个结果与 Dugdale 的经典结果惊人地相似. 尽管我们讨论的是任意各向异性的压电材料中的裂纹问题. 这样

$$D_2 = \frac{2}{\pi} D_s \left\{ \frac{\pi}{2} - \frac{1}{2\mathrm{i}} \log \frac{\frac{x}{a} \cdot \frac{\sqrt{c^2 - a^2}}{\sqrt{x^2 - c^2}} + \mathrm{i}}{\frac{x}{a} \cdot \frac{\sqrt{c^2 - a^2}}{\sqrt{x^2 - c^2}} - \mathrm{i}} \right\} - D_2^{\infty} =$$

$$\frac{2}{\pi} D_s \left\{ \frac{\pi}{2} - \frac{1}{2} (\theta_2 - \theta_1) \right\} - D_2^{\infty}, \quad x > c, \quad y = 0$$

总的电位移 $(D_2)_{\text{total}}$ 为

$$(D_2)_{\text{total}} = \frac{2}{\pi} D_s \arctan \left(\frac{x}{a} \cdot \frac{\sqrt{c^2 - a^2}}{\sqrt{x^2 - c^2}} \right), \quad x > c, \quad y = 0 \tag{48}$$

现在考察裂纹前方的应力场. 由(25), (29)和(38)式

$$\mathrm{t}(x) = \mathrm{h}^+(x) + \mathrm{h}^-(x) \tag{49}$$

$$\begin{aligned} t_i(x) = & \Lambda_{ik}(g_k^+(x) + g_k^-(x)) + \Lambda_{i4}(g_4^+(x) + g_4^-(x)) = \\ & \Lambda_{ik}^*(g_k^+(x) + g_k^-(x)) + \Lambda_{i4}\{D_2^{\infty}(f_c'^+(x) + f_c'^-(x)) + D_s[g_0^+(x) + g_0^-(x)]\}/\Lambda_{44} = \\ & 2T_i^* f_0'(x) + \Lambda_{i4}\{D_s - D_2^{\infty}\}/\Lambda_{44} = \\ & T_i^* \frac{x}{\sqrt{x^2 - a^2}} - T_i + \Lambda_{i4} D_s / \Lambda_{44}, \quad a < x < c \end{aligned} \tag{50}$$

总的面力为

$$(t_i(x))_{\text{total}} = (\sigma_{2i})_{\text{total}} = T_i^* \frac{x}{\sqrt{x^2 - a^2}} + \Lambda_{i4} D_s / \Lambda_{44}, \quad a < x < c \tag{51}$$

这说明裂纹前方的应力场仍然可以由应力强度因子来描述,相应的有

$$\left.\begin{aligned} K_{\mathrm{II}} &= \sqrt{\pi a}\left(\sigma_{12}^{\infty} - \frac{\Lambda_{14}}{\Lambda_{44}} D_2^{\infty}\right) \\ K_{\mathrm{I}} &= \sqrt{\pi a}\left(\sigma_{22}^{\infty} - \frac{\Lambda_{24}}{\Lambda_{44}} D_2^{\infty}\right) \\ K_{\mathrm{III}} &= \sqrt{\pi a}\left(\sigma_{23}^{\infty} - \frac{\Lambda_{34}}{\Lambda_{44}} D_2^{\infty}\right) \end{aligned}\right\} \tag{52}$$

2.2 极化轴垂直于裂纹与 y 轴一致

H 矩阵具有如下结构

$$\mathrm{H} = \begin{bmatrix} \frac{2}{C_L} & 0 & 0 & 0 \\ 0 & \frac{2}{C_T} & 0 & \frac{2}{e} \\ 0 & 0 & \frac{2}{C_A} & 0 \\ 0 & \frac{2}{e} & 0 & -\frac{2}{\varepsilon} \end{bmatrix} \tag{53}$$

其中各元素需要通过数值计算得到.

H 的逆矩阵 Λ 为

$$\Lambda = \begin{bmatrix} \frac{C_L}{2}e & & & \\ 0 & \frac{C_T}{2\rho_0} & 0 & \frac{C_T}{2\rho_0}\frac{\varepsilon}{e} \\ 0 & 0 & \frac{C_A}{2} & 0 \\ 0 & \frac{C_T}{2\rho_0}\frac{\varepsilon}{e} & 0 & -\frac{\varepsilon}{2\rho_0} \end{bmatrix} \tag{54}$$

式中

$$\rho_0 = 1 + \frac{C_T\varepsilon}{e^2} \tag{55}$$

代入(52)式得

$$\left.\begin{aligned} K_{\mathrm{II}} &= \sqrt{\pi a}\sigma_{12}^{\infty} \\ K_{\mathrm{I}} &= \sqrt{\pi a}\left(\sigma_{22}^{\infty} + \frac{C_T}{e}D_2^{\infty}\right) \\ K_{\mathrm{III}} &= \sqrt{\pi a}\sigma_{23}^{\infty} \end{aligned}\right\} \tag{56}$$

对于无穷远处只受 σ_{11}^{∞}, σ_{22}^{∞}, D_1^{∞}, D_2^{∞} 作用的情况,我们有

$$\left.\begin{aligned} K_{\mathrm{II}} &= K_{\mathrm{III}} = 0 \\ K_{\mathrm{I}} &= \sqrt{\pi a}\left(\sigma_{22}^{\infty} + \frac{C_T}{e}D_2^{\infty}\right) \end{aligned}\right\} \tag{57}$$

鉴于裂纹前方的电位移是有限的,而应力是有奇异性.因此,采用如下断裂准则是合理的

$$K_{\mathrm{I}} = K_{\mathrm{I}c} \tag{58}$$

由此得到临界的断裂应力

$$(\sigma_{22}^{\infty})_f = \sigma_{f0} - \frac{C_T}{e}D_2^{\infty} \tag{59}$$

式中

$$\sigma_{f0} = \frac{K_{\mathrm{I}c}}{\sqrt{\pi a}} \tag{60}$$

公式(56)表明断裂应力随着外界电场的增加而呈线性规律的减小.这与 Park 和 Sun[14]的实验及 Gao 等人[16]的分析是一致的.

2.3　极化轴与裂纹平行与 x 轴一致

如图 3 所示,裂纹与极化轴平行. H 矩阵与它的逆 Λ 分别为

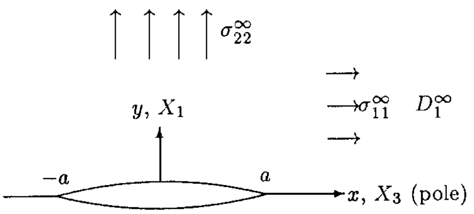

图 3　平行于极化轴的裂纹

Fig. 3　A crack parallel to the poling axis

$$
\mathrm{H} = \begin{bmatrix} \frac{2}{C_T} & 0 & 0 & \frac{2}{e} \\ 0 & \frac{2}{C_L} & 0 & 0 \\ 0 & 0 & \frac{2}{C_A} & 0 \\ \frac{2}{e} & 0 & 0 & -\frac{2}{\varepsilon} \end{bmatrix} \tag{61}
$$

$$
\boldsymbol{\Lambda} = \begin{bmatrix} \frac{C_T}{2\rho_0} & 0 & 0 & \frac{C_T}{2\rho_0}\frac{\varepsilon}{e} \\ 0 & \frac{C_L}{2} & 0 & 0 \\ 0 & 0 & \frac{C_A}{2} & 0 \\ \frac{C_T}{2\rho_0}\frac{\varepsilon}{e} & 0 & 0 & -\frac{\varepsilon}{2\rho_0} \end{bmatrix} \tag{62}
$$

式中 $\rho_0 = 1 + \frac{C_T\varepsilon}{e^2}$.

由此得到

$$
\left.\begin{aligned} K_{\mathrm{II}} &= \sqrt{\pi a}\left(\sigma_{12}^{\infty} + \frac{C_T}{e}D_2^{\infty}\right) \\ K_{\mathrm{I}} &= \sqrt{\pi a}\,\sigma_{22}^{\infty} \\ K_{\mathrm{III}} &= \sqrt{\pi a}\,\sigma_{23}^{\infty} \end{aligned}\right\} \tag{63}
$$

这说明平行于裂纹的外加电场 D_1^{∞} 对裂纹顶端的应力强度因子没有影响. 而垂直于裂纹的外加电场 D_2^{∞} ,只对 K_{II} 有影响. 对于无穷远处只受 σ_{22}^{∞}, D_1^{∞} 作用的情况 ,外加电场对应力强度因子不产生作用. 我们有

$$
K_{\mathrm{II}} = K_{\mathrm{III}} = 0, \quad K_{\mathrm{I}} = \sigma_{22}^{\infty}\sqrt{\pi a} \tag{64}
$$

因此 ,断裂载荷 σ_f 不受外加电场 D_1^{∞} 的影响.

参 考 文 献

1 Parton VZ. Fracture mechanics of piezoelectric materials. *Acta Astronautica*, 1976, 3: 671～683

2 Deeg WFJ. The analysis of dislocation, crack, and inclusion problems in piezoelectric solids. [PhD Thesis]. Stanford University, 1980

3 Pak YE, Herrmann G. Conservation laws and the material momentum tensor for the elastic dielectric. *Int J Engng Sci*, 1986, 24: 1365～1374

4 McMeeking RM. Electrostrictive forces near crack like flaws. *J Appl Phys*, 1989, 40: 615～627

5 Pak YE. Crack extension force in a piezoelectric material. *J Appl Mech*, 1990, 57: 647～653

6 Sosa H. On the fracture mechanics of piezoelectric solids. *Int J Solids Structures*, 1992, 29: 2613～2622

7 Suo Z, Kuo CM, Barnett DM, Willis JR. Fracture machanios for piezoelectric ceramics. *J Mech Phys Solids*, 1992, 40: 739～765

8 Suo Z. Models for breakdown - resistant dielectric and ferroelectric ceramics. *J Mech Phys Solids*, 1993, 41: 1155～1176

9 Zhang TY, Hack IE. Mode III cracks in piezoelectric materials. *J Appl Phys*, 1992, 71: 5865～5870

10 Yang W, Suo Z. Cracking in ceramic actuators caused by electrostriction. *J Mech Phys Solids*, 1994, 42: 649～663

11 Dunn ML. The effects of crack face boundary conditions on the fracture mechanics. *Eng Frac Mech*, 1994, 48: 25～39

12 Zhang TY, Tong P. Fracture mechanics for a mode III crack in a piegoelectric material. *Int J Solids & Struct*, 1996, 33: 343～359

13 Pak YE, Tobin A. On electric field effects in fracture of piezoelectric materials, Mechanics of Electromagnetic Materials and Structures. AMD - Vol 161/ MD - Vol 42, ASME, 1993

14 Park S, Sun CT. Fracture criteria for piezoelectric materials. *J Am Ceram Soc*, 1995, 78: 1475～1480

15 Kumar S, Singh RN. Crack propagation in piezoelectric materials under combined mechanical and electrical loadings. *Acta Mater*, 1995, 44: 173～200

16 Gao HJ, Zhang TY, Tong P. Local and global energy release rates for an electrically yielded crack in a piezoelecteic ceramic. *J Mech Phys Solids*, 1997, 45: 491～510

ANALYSIS OF STRIP ELECTRIC SATURATION MODEL OF CRACK PROBLEM IN PIEZOELECTRIC MATERIALS [1)]

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Abstract This paper presents a fully anisotropic analysis of strip electric saturation model proposed by Gao, Zhang and Tong[16] for piezoelectric materials. A complete exact solution is obtained for a single crack in an infinite pizoelectric media. The relationship between the size of the strip saturation zone ahead of a crack tip and the applied electric field is established. It is found that the size of the strip saturation zone ahead of a crack tip is independent of the applied stress. The explicit formula for the stress intensity factors is obtained. The stress intensity factors are not only dependent on the applied stresses but also on the applied electric field. The critical condition for crack propagation is discussed in detail. It is revealed that the critical fracture stresses for a crack perpendicular to the poling axis is linearly decreased with the increase of the positive applied electric field and increases linearly with the increase of the negative applied electric field. For a crack parallel to the poling axis, the failure stress is not effected by the parallel applied electric field.

Key words piezoelectric materials, crack, electric saturation, poling

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