Saltation in wind blown sand *, **

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Abstract The Boltzmann equation of the sand particle velocity distribution function in wind-blown sand two-phase flow is established based on the motion equation of single particle in air. And then, the generalized balance law of particle property in single phase granular flow is extended to gas-particle two-phase flow. The velocity distribution function of particle phase is expanded into an infinite series by means of Grad's method and the Gauss distribution is used to replace Maxwell distribution. In the case of truncation at the third-order terms, a closed third-order moment dynamical equation system is constructed. The theory is further simplified according to the measurement results obtained by stroboscopic photography in wind tunnel tests.

Keywords: saltation, blown sand, turbulent boundary flow.

The wind-blown sand transport is a two-phase turbulent boundary layer flow. It is divided into three types, i.e. suspension, saltation and creep[1]. Saltation plays a predominant role in sand transportation and most of the transported sand moves in saltation at a fast speed. Most theories about wind-blown sand transport are experiential or semi-experiential theories. Many primary properties in blown sand two-phase turbulent boundary layer were found by Owen[2] through the single trajectory assumption. In refs. [3,4] and other literature launch velocity distribution is studied by means of experimental measurements and statistical methods. Recently a numerical simulation method, the so-called self-regular method is developed[5—7]. It is necessary to associate macroscopic research with microscopic research to establish a dual fluid dynamic model for blown sand two-phase flow. In this paper, the closed third-order moment theory of particle-gas two-phase flow and its simplified form, which is useful and convenient for engineering, are obtained.

1 Velocity distribution function and generalized balance law of particle phase

It is found that the collision process of sand on the bed plays a predominant role in blown sand transport possesses[8], so do the blown sand characteristics of granular flow. The motion of individual particle is fully random. According to the characteristics of blown sand it is supposed that: (i) all the particles are spheres and have an identical diameter and mass; (ii) in microscopic representative volume element of saltation layer, the location and velocity of a sand particle is

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random, so the macroscopic movement state of particle phase can be described by means of a velocity distribution function; (iii) the rotation of a particle can be neglected; (iv) the fluctuation of gas is not affected by particle motion, and the drag force on particles is only dependent on relative velocity between a particle and the mean speed of gas. In the present paper, only saltation is taken into account and the Rouse number $Ro = \tau_v g/(\kappa u_*) \gg 1$. In this case, the particle motion has no effect on the high-frequency fluctuation of gas, and the random force caused by high frequency fluctuation of gas can be neglected.

Let ρ_a, μ_a, u, denote density, viscosity coefficient and velocity of air, respectively. Let $\mathrm{r}(x, y, z)$ D, ρ_p, $\boldsymbol{v}$ denote position, diameter, density and velocity of a sphere particle at time t. Assume that the force acting on it is $F(\boldsymbol{v}, \mathrm{r}, t)$. Then its motion equation is

$$m\frac{d\boldsymbol{v}}{dt} = \mathrm{F}, \quad \mathrm{F} = \mathrm{F}_D + m\mathrm{g}, \tag{1}$$

where

$$\mathrm{F}_D = m\beta(\mathrm{u} - \boldsymbol{v}), \quad m = \pi D^3 \rho_p/6, \tag{2}$$

$$\beta = f_D(Re)/\tau_v, \quad \tau_v = \rho_p D^2/18\mu_a. \tag{3}$$

According to ref. [9], f_D can be approximately expressed as $f_D(Re) \approx 1 + Re^{2/3}/6$, where $Re = \rho_a D|\mathrm{u} - \boldsymbol{v}|/\mu_a = D|\mathrm{u} - \boldsymbol{v}|/\nu$ is Reynold's number of a particle. Therefore β can be expressed as

$$\begin{cases} \beta = [\beta_0 + \beta_1 |\mathrm{u} - \boldsymbol{v}|^{2/3}], \\ \beta_0 = 18\mu_a/(\rho_p D^2), \quad \beta_1 = 3\rho_a \nu^{1/3}/(\rho_p D^{4/3}), \end{cases} \tag{4}$$

where τ_v is the relaxation time of particle motion. Let $f(\boldsymbol{v}, \mathrm{r}, t)$ be the velocity distribution function of particle phase. Then at time t, the probable number of particles in the volume element $\mathrm{d}\Omega_r (= \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z)$ centered at the point r with velocity in the range $(\boldsymbol{v}, \boldsymbol{v} + \mathrm{d}\boldsymbol{v})$ is $f(\boldsymbol{v}, \mathrm{r}, t)\,\mathrm{d}\Omega_r \mathrm{d}\Omega_v$ (where $\mathrm{d}\Omega_v = \mathrm{d}v_x\,\mathrm{d}v_y\,\mathrm{d}v_z$). So the Boltzmann equation to determine the particle velocity distribution function is

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \mathrm{r}} + \frac{1}{m}\mathrm{F} \cdot \frac{\partial f}{\partial \boldsymbol{v}} + \frac{1}{m} f \frac{\partial}{\partial \boldsymbol{v}} \cdot \mathrm{F} = \mathbf{D}_C f, \tag{5}$$

where "·" means inner product, $(\partial/\partial\boldsymbol{v}) \cdot \mathrm{F}$ is caused by viscous drag of interstitial fluid, $\mathbf{D}_C f$ is the mean collision rate of change of f. In the saltation layer of blown sand, the mean free length between particles is one order larger than their diameter, and the particle volume fraction is smaller then 10^{-4}; therefore the collision effects of particle can be neglected[10,11]. So in the following, we neglect all the collision terms.

Let n be the number density of particles. Then

$$n = \int f(\boldsymbol{v}, \mathrm{r}, t)\,\mathrm{d}\Omega_v = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\boldsymbol{v}, \mathrm{r}, t)\,\mathrm{d}v_x\,\mathrm{d}v_y\,\mathrm{d}v_z. \tag{6}$$

Given any particle property $\psi = \psi(\boldsymbol{v}, \mathrm{r}, t)$, its mean value $\langle\psi\rangle$ is determined by

$$\langle\psi\rangle = \frac{1}{n}\int \psi(\boldsymbol{v}, \mathrm{r}, t) f(\boldsymbol{v}, \mathrm{r}, t)\,\mathrm{d}\Omega_v, \tag{7}$$

especially when $\psi = \boldsymbol{v}$. The mean velocity of particle phase is $\bar{\boldsymbol{v}} = \langle\boldsymbol{v}\rangle$. Let C be the fluctuation of particle velocity. Then

$$\boldsymbol{v} = \bar{\boldsymbol{v}} + \mathrm{C}. \tag{8}$$

So any particle property ψ can also be expressed as a function of C, and (7) can be rewritten as

$$\langle\psi\rangle = \frac{1}{n}\int\psi(\mathrm{C}, \mathrm{r}, t) f(\mathrm{C}, \mathrm{r}, t)\, \mathrm{d}\Omega_C. \tag{9}$$

Let ψ be some property of a particle, from (5) it can be confirmed that the balance law of particle property in particle-gas two-phase flow is

$$\frac{\partial}{\partial t}\langle n\psi\rangle + \nabla \cdot \langle n\boldsymbol{v}\psi\rangle = n\langle \mathbf{D}\psi\rangle, \tag{10}$$

where

$$\mathbf{D}\psi = \frac{\partial\psi}{\partial t} + \boldsymbol{v} \cdot \frac{\partial \psi}{\partial \mathrm{r}} + \frac{1}{m}\mathrm{F} \cdot \frac{\partial\psi}{\partial\boldsymbol{v}}. \tag{11}$$

Let $\psi = m$. Then $\mathbf{D}\psi = 0$. From (10), we obtain

$$\frac{D\rho}{Dt} + \rho\nabla \cdot \bar{\boldsymbol{v}} = 0; \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + \bar{\boldsymbol{v}} \cdot \frac{\partial}{\partial \mathrm{r}}, \tag{12}$$

where $\rho = nm$. In the foregoing statement, the independent micro-variables are $(\boldsymbol{v}, \mathrm{r}, t)$, but it is more convenient to express ψ as a function of $(\mathrm{C}, \mathrm{r}, t)$. Let

$$\psi(\boldsymbol{v}, \mathrm{r}, t) = \psi(\bar{\boldsymbol{v}}(\mathrm{r}, t) + \mathrm{C}, \mathrm{r}, t) \equiv \psi(\mathrm{C}, \mathrm{r}, t). \tag{13}$$

Except special claim in the following statement we use C instead of $\boldsymbol{v}$ as an independent variable for any physical quantity. From (11) we obtain

$$\mathbf{D}\psi = \frac{D\psi}{Dt} + \left[\frac{1}{m}\mathrm{F} - \frac{D\bar{\boldsymbol{v}}}{\partial t}\right] \cdot \frac{\partial\psi}{\partial \mathrm{C}} + \mathrm{C} \cdot \frac{\partial\psi}{\partial \mathrm{r}} - \mathrm{C} \cdot \left(\frac{\partial\psi}{\partial \mathrm{C}} \cdot \frac{\partial\bar{\boldsymbol{v}}}{\partial \mathrm{r}}\right). \tag{14}$$

Then the balance law of particle property (10) can be expressed by the fluctuation velocity as follows:

$$\rho\frac{D\langle\psi\rangle}{Dt} + \nabla \cdot \langle\rho\mathrm{C}\psi\rangle = \rho\langle\mathbf{D}\psi\rangle, \tag{15}$$

where $\mathbf{D}\psi$ is determined by (14). Let $\psi = \mathrm{C}$ in (15). Then we obtain the momentum balance equations

$$\begin{cases} \rho\dfrac{E\bar{\boldsymbol{v}}}{Dt} = -\nabla \cdot \boldsymbol{P} + \mathrm{b}, \\ \mathrm{P} = \langle\rho\mathrm{CC}\rangle, \mathrm{b} = \int \mathrm{F} f(\mathrm{C}, \mathrm{r}, t)\, \mathrm{d}\Omega_c. \end{cases} \tag{16}$$

In the case of $\psi = \psi(\mathrm{C})$ (i.e. ψ is not dependent on r and t), by means of (16), the balance law of particle property (15) can be simplified into

$$\rho\frac{D\langle\psi\rangle}{Dt} = \nabla \cdot \mathrm{P} \cdot \left\langle\frac{\partial\psi}{\partial \mathrm{C}}\right\rangle - \nabla \cdot \langle\rho\mathrm{C}\psi\rangle - \rho\left\langle\frac{\partial\psi}{\partial \mathrm{C}}\mathrm{C}\right\rangle : \frac{\partial\bar{\boldsymbol{v}}}{\partial \mathrm{r}}$$

$$+ \tag{17}$$

2 Closed third-order moment theory and its simplified form in fully developed 2-dimensional static boundary layer

Define the Nth-order moment:

$$M_{i_1 i_2 \cdots i_N} = \langle C_{i_1} C_{i_2} \cdots C_{i_N}\rangle. \tag{18}$$

Taking $\psi = C_k C_l$, in (17), one obtains the second-order moment balance equations:

$$\rho\frac{DM_{kl}}{Dt} + \frac{\partial}{\partial r_i}(\rho M_{ikl}) + 2P_{il}\frac{\partial\bar{v}_k}{\partial r_i} = 2\frac{\rho}{m}\langle F(_k C_l)\rangle. \tag{19}$$

Taking $\psi = C_k C_l C_m$ and substituting it in (17), one obtains the third-order moment balance equations:

$$\rho \frac{DM_{klm}}{Dt} + \frac{\partial}{\partial r_i}(\rho M_{iklm}) + \rho\left(M_{ilm}\frac{\partial \overline{v_k}}{\partial r_i} + M_{ikm}\frac{\partial \overline{v_l}}{\partial r_i} + M_{ikl}\frac{\partial \overline{v_m}}{\partial r_i}\right)$$
$$= \frac{\partial P_{ik}}{\partial r_i}M_{lm} + \frac{\partial P_{il}}{\partial r_i}M_{km} + \frac{\partial P_{im}}{\partial r_i}M_{kl} + \frac{\rho}{m}(\langle F_k C_l C_m\rangle$$
$$+ \langle F_l C_k C_m\rangle + \langle F_m C_k C_l\rangle - \langle F_k\rangle M_{lm} - \langle F_l\rangle M_{km} - \langle F_m\rangle M_{kl}). \quad (20)$$

It is noticed that in general diffusion terms in the Nth-order moment balance equations are dependent on $(N+1)$th-order moments. The same as the classical turbulent flow theory, truncation is needed to obtain closure simultaneous equations. Following Grad[12] the traditional truncation method in classical granular flow is expanding the velocity distribution function into an infinite series in the vicinity as Maxwell distribution[10]. In two-phase turbulent boundary layer flow of blown sand, because the velocity distribution function of particle phase is strongly anisotropic in three directions, which has been confirmed by experimental results in wind tunnel, we introduce the Gauss distribution function f_0 instead of Maxwell distribution in wind-blown sand two-phase flow:

$$f_0(\mathrm{C}, \mathrm{r}, t) = \frac{n}{[2\pi]^{3/2}[\det M]^{1/2}}\exp\left\{-\frac{1}{2}\mathrm{C}\cdot \mathrm{M}^{-1}\cdot \mathrm{C}^{\mathrm{T}}\right\}, \quad (21)$$

where M is a 3-dimensional 2nd-order symmetric tensor, the components of which are determined by (18). det M is the determinant of M, and M^{-1} is the reverse of M. For any particle property $\psi(\mathrm{C}, \mathrm{r}, t)$, its another mean value is defined as follows:

$$\langle\psi\rangle^0 = \frac{1}{n}\int \psi(\mathrm{C}, \mathrm{r}, t) f_0(\mathrm{C}, \mathrm{r}, t)\,\mathrm{d}\Omega_{\mathrm{c}}, \quad (22)$$

Define the characteristic function of $f_0(\mathrm{C}, \mathrm{r}, t)$ as $\Phi(\eta, \mathrm{r}, t)$

$$\Phi(\eta, \mathrm{r}, t) = \langle\exp\{\sqrt{-1}\,\eta\cdot \mathrm{C}\}\rangle^0 = \exp\left\{-\frac{1}{2}\eta \mathrm{M}\eta^{\mathrm{T}}\right\}. \quad (23)$$

Using formula $\mathrm{e}^x = 1 + x + \frac{1}{2!}x^2 + \cdots$ and the basic property of characteristic function it is easy to show that

$$\langle C_{i_1} C_{i_2}\cdots C_{i_N}\rangle^0 = \left(\sqrt{-1}\right)^n \frac{\partial^n \Phi}{\partial\eta_{i_1}\partial\eta_{i_2}\cdots\partial\eta_{i_n}}\Big|_{\eta=0} = 0, \quad \text{when } N \text{ is odd}, \quad (24)$$

$$\langle C_k C_l\rangle^0 = M_{kl}. \quad (25)$$

Following Grad[12], we expand the velocity distribution function $f(\mathrm{C}, \mathrm{r}, t)$ into an infinite series at the vicinity of $f_0(\mathrm{C}, \mathrm{r}, t)$:

$$f(\mathrm{C}, \mathrm{r}, t) = \left[1 + \sum_{P=1}^{+\infty}(-1)^P \frac{a_{i_1 i_2\cdots i_P}}{P!}\frac{\partial^P}{\partial C_{i_1}\partial C_{i_2}\cdots\partial C_{i_P}}\right] f_0(\mathrm{C}, \mathrm{r}, t). \quad (26)$$

For any particle property $\psi(\mathrm{C}, \mathrm{r}, t)$ we have

$$\langle\psi\rangle = \langle\psi\rangle^0 + \sum_{P=1}^{+\infty}\frac{a_{i_1 i_2\cdots i_P}}{P!}\left\langle\frac{\partial^P\psi}{\partial C_{i_1}\partial C_{i_2}\cdots\partial C_{i_P}}\right\rangle^0. \quad (27)$$

From (24)—(26) and the definitions of number density n, mean velocity $\overline{v}$, and the second-order moment M_{kl}, it is easy to obtain $a_i = 0$, $a_{ij} = 0$; therefore (26) can be rewritten as

$$f(\mathrm{C},\mathrm{r},t) = \left[1 + \sum_{P=3}^{+\infty}(-1)^P \frac{a_{i_1 i_2 \cdots i_P}}{P!} \frac{\partial^P}{\partial C_{i_1}\partial C_{i_2}\cdots\partial C_{i_P}}\right] f_0(\mathrm{C},\mathrm{r},t). \qquad (28)$$

If only the second-order moments are considered (i.e. $a_{i_1 i_2 \cdots i_P} \equiv 0$, for all $P \geq 3$), then $f(\mathrm{C},\mathrm{r},t) = f_0(\mathrm{C},\mathrm{r},t)$. From the definition of the third-order moment and (27) one can obtain

$$M_{lmn} = \langle C_l C_m C_n \rangle = \langle C_l C_m C_n \rangle^0 + \frac{a_{ijk}}{3!}\left\langle \frac{\partial^3 (C_l C_m C_n)}{\partial C_i \partial C_j \partial C_k} \right\rangle^0.$$

Using (24) we get $M_{lmn} = a_{lmn}$. Therefore

$$f(\mathrm{C},\mathrm{r},t) = \left[1 - \frac{M_{ijk}}{3!}\frac{\partial^3}{\partial C_i \partial C_j \partial C_k} + \sum_{P=4}^{+\infty}(-1)^P \frac{a_{i_1 i_2 \cdots i_P}}{P!} \frac{\partial^P}{\partial C_{i_1}\partial C_{i_2}\cdots\partial C_{i_P}}\right] f_0(\mathrm{C},\mathrm{r},t). \qquad (29)$$

From the difinition of the fourth-order moment and (27) one obtains

$$M_{lmnp} = \langle C_l C_m C_n C_p \rangle^0 + a_{lmnp} = \frac{\partial^4 \Phi}{\partial \eta_k \partial \eta_l \partial \eta_m \partial \eta_n}\Big|_{\eta=0} + a_{lmnp}. \qquad (30)$$

Expanding (23) into the fourth-order terms and substituting them into the above equation one obtains

$$M_{klmn} = M_{kl}M_{mn} + M_{km}M_{ln} + M_{kn}M_{lm} + a_{lmnp}. \qquad (31)$$

Using (31), we can rewrite (21) as

$$\rho \frac{DM_{klm}}{Dt} + \frac{\partial}{\partial r_i}(\rho a_{iklm}) + P_{ik}\frac{\partial M_{lm}}{\partial r_i} + P_{il}\frac{\partial M_{km}}{\partial r_i} + P_{im}\frac{\partial M_{kl}}{\partial r_i} + \rho\left(M_{ilm}\frac{\partial \overline{v}_k}{\partial r_i} + M_{ikm}\frac{\partial \overline{v}_l}{\partial r_i} + M_{ikl}\frac{\partial \overline{v}_m}{\partial r_i}\right)$$
$$= \frac{\rho}{m}(\langle F_k C_l C_m \rangle + \langle F_l C_k C_m \rangle + \langle F_m C_k C_l \rangle - \langle F_k \rangle M_{lm} - \langle F_l \rangle M_{km} - \langle F_m \rangle M_{kl}). \qquad (32)$$

For the closed third-order moment theory, it is assumed that the system state variables of particle phase are n, $\overline{v}_i$, M_{ij}, M_{ijk}. Any coefficient of higher-order terms $a_{i_1 i_2 \cdots i_P}$ ($P \geq 4$) in (28) should be expressed with the system state variables based on experiments, which has been done in turbulent flow theory. For lack of experience, following Jekens and Richman[10], we truncate it to the third-order terms, i.e. let $a_{i_1 i_2 \cdots i_P} \equiv 0$ for all $P \geq 4$. In the present paper only this case is to be considered; therefore (12), (16), (19), (32) and the motion equation of gas phase form the complete equation system of the closed third-order moment theory.

In order to simplify the equations obtained in the above section, in this section we consider the fully developed 2-dimensional static boundary layer problem. It is assumed that: (i) $v_z = 0$ and (ii) all the macroscopic quantities such as v_x and v_y are only dependent on y. From assumption (i), the mass conservation equation (12) can be simplified into $\rho \overline{v}_y \equiv$ constant. Becanuse of $\rho \overline{v}_y = 0$ at infinity, in fully developed 2-dimensional static boundary layer problem only the x-component of the mean velocity of particle phase is non-zero, and we denote it by $\overline{v}(y)$. In this case, when any macroscopic quantity is acted on by an operator $\frac{D}{Dt} = \frac{\partial}{\partial t} + \overline{v}_i \frac{\partial}{\partial r_i}$, that is equal to zero. For the same reason, the gas velocity has only one component in x-direction, $u(y)$. Because the motion of particle does not affect the gas fluctuation, Prandtl's mixing length theory is also true. The effect of particle on gas can be simplified into a reaction force. Let τ_a denote the shear stress of particle phase, τ_s shear stress of gas phase. Then the momentum balance equation in x-direction of the mixture becomes $\partial(\tau_a + \tau_s)/\partial y = 0$. Integrating it we obtain

$$\tau_a + \tau_s = \tau_0, \tag{33}$$

where τ_0 is a constant denoting the total shear stress at boundary, $\tau_s = -\rho M_{12}$. According to Prandtl's mixing length theory we have $\tau_a = \rho_a(\kappa y \cdot \mathrm{d}u/\mathrm{d}y)^2$. Then the motion equation of gas phase can be expressed as

$$\rho_a(\kappa y \cdot \mathrm{d}u/\mathrm{d}y)^2 = \tau_0 + \rho M_{12}, \tag{34}$$

where κ is the Karman constant. From assumption (ii) it is known that any moment containing the subscript 3 of particle phase vanishes. Then in the fully developed 2-dimensional static boundary layer problem, the system state variables of particle phase for the closed third-order moment theory are

$$u, \rho, \bar{v}, M_{11}, M_{12}, M_{22}, M_{111}, M_{122}, M_{112}, M_{222}.$$

Then (21) becomes

$$f_0(\mathrm{C}, \mathrm{r}, t) = \frac{n^*}{2\pi\sqrt{M_{11}M_{22} - M_{12}^2}} \cdot \exp\left\{-\frac{M_{11}M_{22}}{(M_{11}M_{22} - M_{12}^2)}\left[\frac{C_1^2}{M_{11}} - \frac{2M_{12}C_1C_2}{M_{11}M_{22}} + \frac{C_2^2}{M_{22}}\right]\right\}. \tag{35}$$

The complete equation system of the closed third-order moment theory of two-dimensional problem can be written as

$$\left\{\begin{aligned}
&\rho_a\left(\kappa y\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2 = \tau_0 + \rho M_{12},\\
&\frac{\mathrm{d}}{\mathrm{d}y}(\rho M_{12}) = \rho[\langle\beta\rangle(u - \bar{v}) - \langle\beta C_1\rangle],\\
&\frac{\mathrm{d}}{\mathrm{d}y}(\rho M_{22}) = -\rho g - \rho\langle\beta C_2\rangle,\\
&\frac{\mathrm{d}}{\mathrm{d}y}(\rho M_{222}) = -2\rho\langle\beta C_2C_2\rangle,\\
&\frac{\mathrm{d}}{\mathrm{d}y}(\rho M_{112}) + 2\rho M_{12}\frac{\mathrm{d}\bar{v}}{\mathrm{d}y} = 2\rho[\langle\beta C_1\rangle(u - \bar{v}) - \langle\beta C_1C_1\rangle],\\
&\frac{\mathrm{d}}{\mathrm{d}y}(\rho M_{122}) + \rho M_{22}\frac{\mathrm{d}\bar{v}}{\mathrm{d}y} = \rho[\langle\beta C_2\rangle(u - \bar{v}) - 2\langle\beta C_1C_2\rangle],\\
&M_{12}\frac{\mathrm{d}M_{11}}{\mathrm{d}y} + M_{111}\frac{\mathrm{d}\bar{v}}{\mathrm{d}y} = [(\langle\beta C_1C_1\rangle - \langle\beta\rangle M_{11})(u - \bar{v})\\
&\qquad + \langle\beta C_1\rangle M_{11} - \langle\beta C_1C_1C_1\rangle],\\
&M_{12}\frac{\mathrm{d}M_{22}}{\mathrm{d}y} + 2M_{22}\frac{\mathrm{d}M_{12}}{\mathrm{d}y} + M_{222}\frac{\mathrm{d}\bar{v}}{\mathrm{d}y}\\
&\qquad = [(\langle\beta C_2C_2\rangle - \langle\beta\rangle M_{22})(u - \bar{v}) + \langle\beta C_1\rangle M_{22}\\
&\qquad\quad + 2\langle\beta C_2\rangle M_{12} - 3\langle\beta C_1C_2C_2\rangle],\\
&M_{22}\frac{\mathrm{d}M_{11}}{\mathrm{d}y} + 2M_{12}\frac{\mathrm{d}M_{12}}{\mathrm{d}y} + 2M_{122}\frac{\mathrm{d}\bar{v}}{\mathrm{d}y}\\
&\qquad = [2(\langle\beta C_1C_2\rangle - \langle\beta\rangle M_{12})(u - \bar{v}) + \langle\beta C_2\rangle M_{11}\\
&\qquad\quad + 2\langle\beta C_1\rangle M_{12} - 3\langle\beta C_1C_1C_2\rangle],\\
&M_{22}\frac{\mathrm{d}M_{22}}{\mathrm{d}y} = [\langle\beta C_2\rangle M_{22} - \langle\beta C_2C_2C_2\rangle],
\end{aligned}\right. \tag{36}$$

where β is determined by (4). The product terms (i.e. all $\langle \cdots \rangle$ terms) in eq. (36) can be calculated by means of (29).

3 Simplified equations for engineering application

Experimental researches on wind tunnel for the developed wind-blown sand turbulent boundary layer flow were carried out (limited by space, experimental details are omitted here). Under the experimental condition of the present work, the average value of experimental results within 0.04 m can be given as

$$\begin{cases} M_{11} = 1.772\text{m}^2/\text{s}^2, M_{12} = -0.472\text{m}^2/\text{s}^2, \\ M_{22} = 0.179\text{m}^2/\text{s}^2. \end{cases} \quad (37)$$

Experimental results show that M_{222} is far smaller than M_{22}. Experimental results of the three-order moments are scattering, and no satisfactory results are obtained. We can only evaluate the order of magnitude from experimental results. Particle density is indirectly calculated by particle flux and average particle velocity: $\rho = q/\overline{v}$. Experimental results show that the particle density ρ approximately attenuates as negative exponential function with height y. It can be approximately expressed as

$$\rho = \exp\{-54.75y\}, \quad (38)$$

where the dimension of ρ is in kg/m^3, the dimension of y is in m. The fitting curve of ρ is compared with the experimental results in fig. 1. This conclusion is also given by Nalpanis et al.,[3] Wu[13] and others.

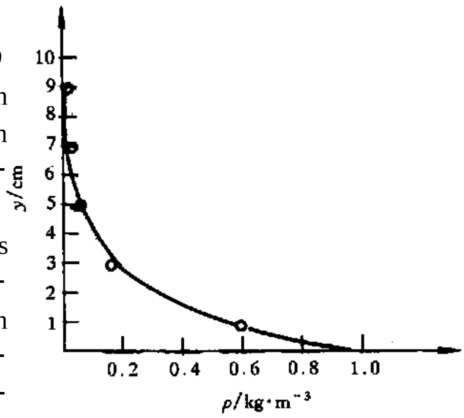

Fig. 1. Particle density and its fitting curve.

In the fully developed 2-dimensional particle-gas two-phase turbulent boundary layer flow, dynamic equations of closed third-order moment theory are given by (36). In general cases these equations are all coupled together. In our experiments the third-order moments cannot be measured accurately. All these situations make these simultaneous equations difficult in engineering. So we simplify eq. (36) according to the order of magnitude obtained in experiments. The following approximate solution can be obtained (the details are omitted here):

$$\begin{cases} \rho = \rho_0 e^{-\frac{gy}{M_{22}}}, \quad M_{222} = 0, \quad M_{22} = \text{const.}, \\ M_{12} = \text{const.}, \quad M_{11} = \text{const.}, \end{cases} \quad (39)$$

$$\begin{cases} \rho_a\left(\kappa y \dfrac{\mathrm{d}u}{\mathrm{d}y}\right) = \tau_0 + \rho_0 M_{12} e^{-\frac{gy}{M_{22}}}, \\ -\dfrac{gM_{12}}{M_{22}} = \beta_0(u - \overline{v}) + \beta_1(u - \overline{v})^{5/3} + \dfrac{2}{3}\beta_1(u - \overline{v})^{-1/3} M_{11}. \end{cases} \quad (40)$$

The density in eq. (39) is corresponding to eq. (38). The value of u and $\overline{v}$ can be obtained from eq. (40). In order to resolve eq. (40), two boundary conditions are required: (i) particle borne shear stress at $y = 0$, and (ii) wind speed at $y = D$. First we consider condition (1); although we can determine M_{12} by means of experiments, it is difficult to accurately determine density at $y = 0$; therefore we still use Owen's assumption[2]: suppose that air borne shear stress at $y = 0$ always has a threshold value for the initial saltation: $\tau_a = \rho_a u^2_{*t}$. From $\tau_0 = \rho_a u^2_*$ one can obtain

$$\tau_{s0} = \rho_0 M_{12} = - \rho_0 | M_{12} | = - \rho_a [u_*^2 - u_{*t}^2]. \qquad (41)$$

Substituting (41) into the first equation of (40), and noticing $\tau_0 = \rho_a u_*^2$, one can obtain

$$\frac{du}{dy} = \frac{u_*}{\kappa y}\left[1 - \left(1 - \frac{u_{*t}^2}{u_*^2}\right) e^{-\frac{gy}{M_{22}}}\right]^{1/2}. \qquad (42)$$

Assume that the second condition is

$$u = Bu_* \text{ at } y = D, \qquad (43)$$

where coefficient B is determined by experiments. In the present paper, B is a little larger than 8.5, so we can approximately take $B = 8.5$. From (42) and (43) we obtain

$$u = \frac{u_*}{\kappa}\int_D^y \left[1 - \left(1 - \frac{u_{*t}^2}{u_*^2}\right) e^{-\frac{gy_1}{M_{22}}}\right]^{1/2} \frac{dy}{y_1} + Bu_*. \qquad (44)$$

Substituting the relevant parameters into (44), and integrating it by numerical method one can obtain the theoretical profile of wind speed as shown in fig. 2. Below 0.02 m, the particle borne shear stress increases rapidly with the decreasing height, and air stress is no longer constant; therefore the wind speed deviates from the logarithmic profile. This situation causes the fitting curve to deviate from the practical wind speed below 0.02 m. From fig. 2. we can find that the experimental fitting curve of the wind speed is smaller than the theoretical one. From fig. 2 we also find that the roughness is increased due to particle shear stress. In fig. 2 curve 3 is $u = \frac{u_*}{\kappa}\ln(y/y_0)$, where $y_0 = D/30$, corresponding to the assumed case where sands are fixed on the bed, u_* is determined by regression analysis from the measured wind speed. Curve 1 is the curve fitting test data and curve 2 is a theoretical curve. In the simplified theory three second-order moments are constants, and particle density decays exponentially with height, and therefore the particle borne stress also decays exponentially with height:

$$\tau_s = \tau_{so} e^{-\frac{gy}{M_{22}}}, \quad \tau_{s0} = - \rho_o M_{12}, \qquad (45)$$

where ρ_0 is the particle density at $y = 0$, the value of M_{12} is negative. From (45) one can obtain

$$\tau_a = \tau_0 - \tau_{so} e^{-\frac{gy}{M_{22}}} = \rho_a u_*^2 + \rho_0 M_{12} e^{-\frac{gy}{M_{22}}}. \qquad (46)$$

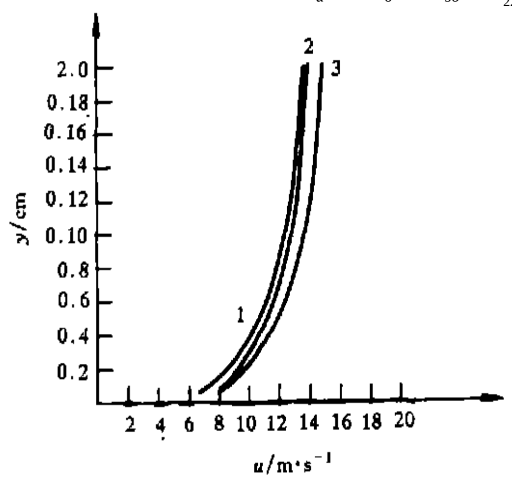

Fig. 2. Theoretical wind speed profile.

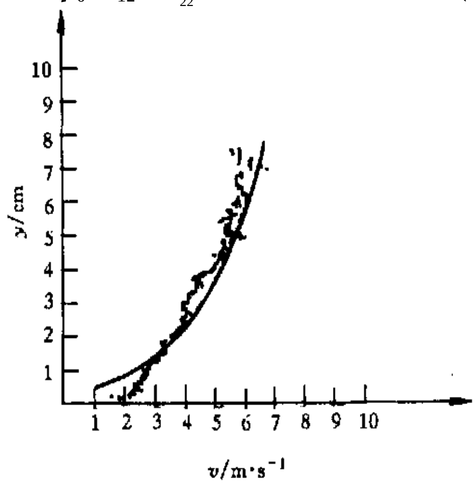

Fig. 3. Theoretical mean velocity of particle phase.

The variation in τ_s and τ_a with height is consistent well with the numerical results given in ref. [6]. From the second equation of (40) one can find that $(u - \overline{v})$ is approximately constant within an appropriate range, which has also been found in experiments. The particle average velocity determined from (40) and the experimental results are shown in fig. 3. Fig. 3 shows that they are well consistent with each other.

Appendix Balance law of particle property

$$\int \frac{\partial f}{\partial t}\psi \mathrm{d}\Omega_v = \int\left[\frac{\partial (f\psi)}{\partial t} - f\frac{\partial \psi}{\partial t}\right]\mathrm{d}\Omega_v = \frac{\partial}{\partial t}\int \psi f \mathrm{d}\Omega_v - \int \frac{\partial \psi}{\partial t} f \mathrm{d}\Omega_v$$
$$= \frac{\partial}{\partial t}(n\langle\psi\rangle) - n\left\langle\frac{\partial \psi}{\partial t}\right\rangle, \tag{A1}$$

$$\int \upsilon \cdot \frac{\partial f}{\partial \mathrm{r}}\psi \mathrm{d}\Omega_v = \int\left[\frac{\partial}{\partial \mathrm{r}}\cdot(\upsilon\psi f) - f\upsilon\cdot\frac{\partial \psi}{\partial \mathrm{r}}\right]\mathrm{d}\Omega_v = \frac{\partial}{\partial \mathrm{r}}\cdot\int \upsilon\psi f \mathrm{d}\Omega_v - \int \upsilon\cdot\frac{\partial \psi}{\partial \mathrm{r}} f \mathrm{d}\Omega_v$$
$$= \nabla\cdot\langle n\upsilon\psi\rangle - n\left\langle\upsilon\cdot\frac{\partial \psi}{\partial \mathrm{r}}\right\rangle, \tag{A2}$$

$$\int\left[\frac{1}{m}\mathrm{F}\cdot\frac{\partial f}{\partial \upsilon}\psi + \frac{1}{m}f\frac{\partial}{\partial \upsilon}\cdot\mathrm{F}\psi\right]\mathrm{d}\Omega_v = \int\left[\frac{\partial}{\partial \upsilon}\cdot\left(\frac{1}{m}\mathrm{F}f\psi\right) - \frac{1}{m}f\mathrm{F}\cdot\frac{\partial \psi}{\partial \upsilon}\right]\mathrm{d}\Omega_v$$
$$= -\int \frac{1}{m}\mathrm{F}\cdot\frac{\partial \psi}{\partial \upsilon} f \mathrm{d}\Omega_v = -n\left\langle\frac{1}{m}\mathrm{F}\cdot\frac{\partial \psi}{\partial \upsilon}\right\rangle, \tag{A3}$$

in which we have used $\int \frac{\partial}{\partial \upsilon}\cdot\left(\frac{1}{m}\mathrm{F}f\psi\right)\mathrm{d}\Omega_v = 0$. With (A1), (A2) and (A3), we obtain

$$\int {}^{*} f\psi \mathrm{d}\Omega_v = \frac{\partial}{\partial t}\langle n\psi\rangle + \nabla\cdot\langle n\upsilon\psi\rangle - n\langle \mathbf{D}\psi\rangle. \tag{A4}$$

From (13), (A4) and (5), (11) can be obtained.

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