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Dynamic SIF of interface crack between two dissimilar viscoelastic bodies under impact loading*

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Abstract. In this paper, the transient dynamic stress intensity factor (SIF) is determined for an interface crack between two dissimilar half-infinite isotropic viscoelastic bodies under impact loading. An anti-plane step loading is assumed to act suddenly on the surface of interface crack of finite length. The stress field incurred near the crack tip is analyzed. The integral transformation method and singular integral equation approach are used to get the solution. By virtue of the integral transformation method, the viscoelastic mixed boundary problem is reduced to a set of dual integral equations of crack open displacement function in the transformation domain. The dual integral equations can be further transformed into the first kind of Cauchy-type singular integral equation (SIE) by introduction of crack dislocation density function. A piecewise continuous function approach is adopted to get the numerical solution of SIE. Finally, numerical inverse integral transformation is performed and the dynamic SIF in transformation domain is recovered to that in time domain. The dynamic SIF during a small time-interval is evaluated, and the effects of the viscoelastic material parameters on dynamic SIF are analyzed.

Key words: Integral transformation, interface crack, singular integral equation, stress intensity factor, viscoelastic material.

1. Introduction

The fracture problems of interface cracks under static loading have been investigated extensively. It was revealed that the singular index of the stress field at crack-tip is not real but complex. The complex singular index of stress field for interface crack causes the oscillation of stress field and the overlapping of the crack surfaces near crack-tip. These phenomena which are assumed not rational in physics exist in the case of dynamic loads. In the case of dynamic loads, two loading cases are of interest, viz. Harmonic loading and impact loading. The former can be dealt with in frequent domain; and the latter is generally analyzed in time domain. The dynamic SIF plays an important role in dynamic fracture under both harmonic and impact loads, that predicts whether or not the fracture toughness of the material will be exceeded and catastrophic crack propagation will follow. The dynamic SIF of interface cracks between two dissimilar elastic materials has been studied widely. Kundu (1986, 1987, 1988) studied the dynamic SIF of interface crack under impact loading with the method based on Betti's reciprocal theorem. In his investigations one or two Griffith interface cracks that were located at one interface or different interfaces for multi-layered media were considered. Li (1991) studied the dynamic SIF of one Griffith interface crack in a four-layered composite by reducing the mixed boundary problem to the Cauchy-type singular integral equation. As for interface crack between two dissimilar orthotropic or anisotropic materials the investigation of dynamic SIF was given by Kuo (1984). Wang (1991) extended the investigation to modified Griffith interface cracks that has contact region at the crack-tip. As a result, the oscillation of stress field and the overlapping of the crack surfaces near crack-tip are removed. In light of investigation of purely elastic problem and the use of correspondence principle, Georgiadis and co-worker (1991, 1993, 1996) studied the dynamic SIF of Griffith crack, penny-shaped crack and half-infinite crack in homogenous viscoelastic material under impact loading. The time history of dynamic SIF during a small time-interval immediately after the sudden application of loading was obtained, and the influence of the viscoelastic material parameters was analyzed. It was indicated that dynamic SIF over-shoots are possible in viscoelastic materials, but retarded and not-too-sharp as compared to that in purely elastic materials due to the fact that viscoelastic wave-fronts are not sharp. In their works the integral transformation approach play an important role and the inverse integral transformation method proposed by Dubner and Abate (1968) and Crump (1976) (called as DAC method) was first introduced to carried out the numerical inverse Laplace integral transformations. It is interested to extend the investigation of dynamic response of crack to inhomogeneous viscoelastic material with crack, in particular, the interface crack between two dissimilar viscoelastic materials.

The objective of the present paper is to investigate the dynamic response of an interface crack between two dissimilar viscoelastic materials to anti-plane impact loading. Following the Georgiadis's work, the Laplace integral transformation and inverse integral transformation methods are used. Applying integral transformation to the viscoelastic mixed boundary problem leads to a set of dual integral equations. In view of the convenience of singular analysis of stress field, the dual integral equations are reduced to the Cauchy-type singular integral equation (SIE) of first kind by introduction of the crack dislocation density function. The numerical solutions of SIE were usually obtained by Jacobi or Chebyshev polynomial extension procedure (Erdogan F. et al. 1973). In the present paper, the piecewise continuous function method given by Kurtz (1994) is adopted. In contrast with polynomial extension method, the piecewise continuous function method doesn't need extrapolation at endpoints in view of its convenience of node disposition. The inverse integral transformation method called as DAC method is used to recover the results in the transformation domain to the time domain. The suitability and reliability of DAC method had been confirmed in Georgiadis's series works. In addition, the procedure above-mentioned can also be used to deal with in-plane impact loading which leads to two coupling singular integral equations representing the coupling behavior of stress field of interface crack. The details on diffraction of P or SV wave by interface crack under in-plane loads will be provided in another paper.

2. Problem statements

We consider an interface Griffith crack of length 2a between two homogeneous, isotropic, linear viscoelastic (HILV) half-spaces. A Cartesian coordinate system is assumed in such a way that the *x*-axis is along crack length direction and the *y*-axis is perpendicular to the crack as shown in Fig. 1. The crack surface has infinite width in the *z*-direction. Densities of the two materials are denoted by ρ_1, ρ_2 and shear relaxation function by $G_1(t)$, $G_2(t)$, respectively. A standard linear viscoelastic solid model is used whose shear relaxation function can be written as

$$G_i(t) = \mu_{\infty i}\left[1 + f_i \exp\left(-\frac{t}{\tau_i}\right)\right] \qquad (i = 1, 2), \tag{1}$$

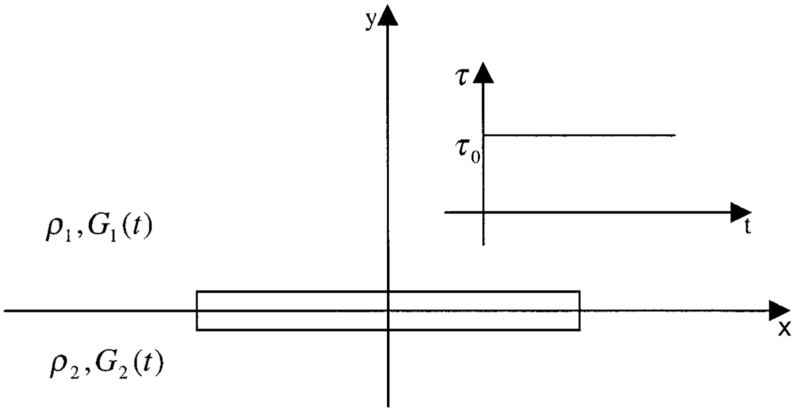

Figure 1. Interface crack between two dissimilar viscoelastic bodies under anti-plane impact loading.

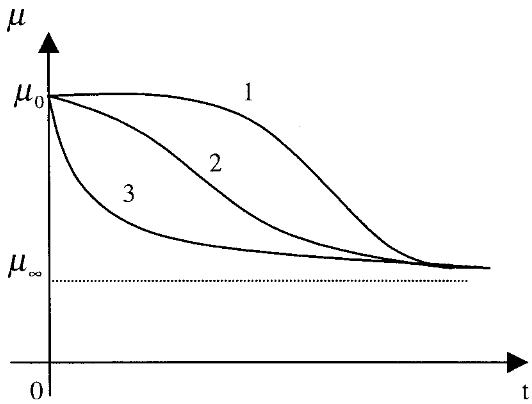

Figure 2. Relaxation modulus of viscoelastic material. (1) Large relaxation time. (2) Medium relaxation time. (3) Small relaxation time.

where $f_i = \mu_{0i}/\mu_{\infty i} - 1$ is a measure of the difference between the short-time, μ_{0i}, and the long-time, $\mu_{\infty i}$, shear modulus, and τ_i is the relaxation time of the viscoelastic material. The influence of relaxation time on $G(t)$ is shown in Fig. 2.

It is assumed that an anti-plane shear impact load $\tau_0 H(t)$ acts suddenly on the crack surface. The initial and boundary value conditions are written as

$$\sigma_{yz}(x, 0, t) = \tau_0 H(t) \qquad |x| < a, \tag{2a}$$

$$\sigma_{yz}(x, 0^+, t) = \sigma_{yz}(x, 0^-, t) \qquad -\infty < x < +\infty, \tag{2b}$$

$$w(x, 0^+, t) = w(x, 0^-, t) \qquad |x| > a, \tag{2c}$$

$$w(x, y, t) = 0 \qquad x, y \to \infty, \tag{2d}$$

$$\sigma_{yz}(x, y, 0) = w(x, y, 0) = \dot{w}(x, y, 0) = 0, \tag{2e}$$

where σ_{ij} denotes stress tensor, w and $\dot{w}$ denote displacement and velocity in z direction respectively, and $H(t)$ is the Heaviside function.

The objective of the present work is to determine dynamic SIF for the problems specified by (2).

3. Derivation and numerical solution of the singular integral equation

3.1. THE DUAL INTEGRAL EQUATION OF CRACK OPEN DISPLACEMENT Δw

Crack opening displacement is defined as:

$$w(x, 0^+, t) - w(x, 0^-, t) = \begin{cases} \Delta w(x, t) & |x| < a, \\ 0 & |x| > a. \end{cases} \quad (3)$$

The equation of motion for viscoelastic material can be written in integral form as:

$$G(t) * \nabla^2 dw(x, y, t) = \rho \ddot{w}(x, y, t) \quad (4)$$

where $*$ denotes the Stieltjes integration. Applying Laplace integral transformation to $*$ and Fourier integral transformation to x in Equation (4) in conjunction with the quiescent initial conditions from Equation (2e) and the radial condition from Equation (2d) yields:

$$\tilde{\tilde{w}}(s, y, p) \begin{cases} a(y)e^{-\sqrt{K_{T1}^2+s^2 y}} & y > 0, \\ b(y)e^{\sqrt{K_{T2}^2+s^2 y}} & y < 0. \end{cases} \quad (5)$$

where $\tilde{\tilde{w}}(s, y, p) = \{F[L[w(x, y, t)]], t \to p, x \to s\}$, L and F denote Laplace and Fourier integral transformation operator respectively. $K_{Ti}^2(p) = \rho p/\overline{G_i}(p)$, $\overline{G_i}(p) = L[G_i(t)]$. The factors $a(y)$ and $b(y)$ can be determined by boundary condition (2b).

$$a(y) = \frac{\beta}{\beta + 1}\Delta\tilde{\tilde{w}}, \quad b(y) = -\frac{1}{\beta - 1}\Delta\tilde{\tilde{w}}, \quad (6)$$

where

$$\beta(s, p) = \frac{\overline{G_2}(p)\sqrt{K_{T_2}^2 + s^2}}{\overline{G_1}(p)\sqrt{K_{T_1}^2 + s^2}}. \quad (7)$$

The remaining boundary conditions (2a) and (2c) constitute the dual integral equations

$$\begin{cases} F^{-1}\left[\Delta\tilde{\tilde{w}}\right] = 0 & |x| > a, \quad (8a) \\ F^{-1}\left[-p\overline{G_1}(p)\frac{\beta}{\beta+1}\sqrt{K_{T_1}^2 + s^2}\Delta\tilde{\tilde{w}}\right] = \frac{\tau_0}{p} & |x| < a, \quad (8b) \end{cases}$$

where F^{-1} denotes inverse Fourier transformation operator.

3.2. SINGULAR INTEGRAL EQUATION OF CRACK DISLOCATION DENSITY FUNCTION Φ

The crack dislocation density function is defined as

$$\Phi(x, t) = \frac{\partial}{\partial x}\Delta w(x, t). \quad (9)$$

Employing Fourier integral transformation on x and Laplace integral transformation on t in Equation (9) leads to

$$\tilde{\tilde{\Phi}}(s, p) = (is)\Delta\tilde{\tilde{w}}(s, p). \quad (10)$$

After applying Equation (10) to Equation (8), the Equation (8a) yields

$$\int_{-a}^{a} \Phi(x, t)\mathrm{d}x = 0. \tag{11}$$

Equation (8b) yields

$$-\frac{1}{2\pi i}\int_{-\infty}^{\infty} p\overline{G_1}(p)\frac{\beta}{\beta+1}\sqrt{K_{T_1}^2 + s^2}\frac{1}{s}\left[\int_{-a}^{a}\overline{\Phi}(u, p)e^{isu}\,\mathrm{d}u\right]e^{-isx}\,\mathrm{d}s = \frac{\tau_0}{p}. \tag{12}$$

Taking into account

$$\lim_{s\to\infty} p\overline{G_1}(p)\frac{\beta}{\beta+1}\sqrt{K_{T_1}^2 + s^2}\frac{1}{s} = \frac{p\overline{G_1}(p)\overline{G_2}(p)}{\overline{G_1}(p)+\overline{G_2}(p)}\mathrm{signs}(s), \tag{13}$$

and

$$\int_{-\infty}^{\infty} e^{is(u-x)}\mathrm{signs}(s)\,\mathrm{d}s = \frac{2i}{u-x} \tag{14}$$

the following Cauchy-type singular integral equation (SIE) of first kind is obtained

$$-\frac{p\overline{G_1}(p)\overline{G_2}(p)}{\overline{G_1}(p)+\overline{G_2}(p)}\frac{1}{\pi}\int_{-a}^{a}\frac{\overline{\Phi}(u, p)}{u-x}\mathrm{d}u + \int_{-a}^{a}\overline{K}(u, x, p)\overline{\Phi}(u, p)\,\mathrm{d}u = \frac{\tau_0}{p}. \tag{15}$$

where integral kernel has the following expression

$$\begin{aligned}\overline{K}(u, x, p) \;=\; -\frac{1}{2\pi i}\int_{-\infty}^{\infty}&\left[p\overline{G_1}(p)\frac{\beta}{\beta+1}\sqrt{K_{T_1}^2 + s^2}\frac{1}{s}\right.\\ &\left.-\frac{p\overline{G_1}(p)\overline{G_2}(p)}{\overline{G_1}(p)+\overline{G_2}(p)}\mathrm{signs}(s)\right]e^{-is(u-x)}\,\mathrm{d}s\end{aligned} \tag{16}$$

3.3. Numerical solution of singular integral equation

In light of the theory of singular integral equation (Muskhelishvili, 1953), the solution of Equation (15), viz., the dislocation density function in transformation domain can be written in the form as

$$\overline{\Phi}(x, p) = \overline{\varphi}(x, p)W(x). \tag{17}$$

where $W(x) = (a-x)^{-1/2}(a+x)^{-1/2}$ is the fundamental function of SIE (15). $\overline{\varphi}(x, p)$ is analytic function in complex plane.

Because of the complexity of the integral kernel $\overline{K}(u, x, p)$, the closed-form solutions to Equation (15) is not available, therefore much attention has been focused on numerical method of solution. In a singular integral equation with real or complex constants, the fundamental function turns out to be the weight function of some well-known orthogonal polynomials. For example, in the integral equation of first kind, the fundamental function is weight function of Chebyshev polynomials; and in the integral equation of second kind, the fundamental function is weight function of Jacobi polynomials. Thus, using the properties of related orthogonal polynomials, a numerical solution of the singular integral equation may be obtained in which the essential features of the singularity of the unknown function are preserved. More detailed applications using Chebyshev and Jacobi polynomials appear in Erdogan (1973). However, since the value of analytic function $\overline{\varphi}(x, p)$ at the points $x = \pm a$ are of special interest and the value of $\overline{\varphi}(x, p)$ is obtained at zeros of the chebyshev or Jacobi polynomials, extrapolation is needed. The alternative numerical method, which obviates the extrapolation, proposed by Kurtz (1994) is based on the piecewise continuous function. If impact load exerted on crack surface changes rapidly at local, the sampling points should be chosen so that the salient features of loads are captured. Thus requirement for flexibility of selection of nodes rises, and orthogonal polynomials fail to satisfy the requirement. In summary, the numerical method based on piecewise continuous function have the advantages both of eliminating the need for extrapolation at the endpoints and of making it possible to model with higher accuracy solutions with localized erratic behavior. In this paper, the piecewise continuous function method is used, and piecewise quadratic polynomials are adopted. The details of the method will not be repeated here. The authors of interest can make reference to Kurtz (1994).

4. The numerical inversion of Laplace integral transformation

After the crack dislocation density function $\overline{\Phi}(x, p)$ is obtained in the Laplace transformation domain, the shear stress field, $\overline{\tau}_{yz}(x, 0, p)$, and, furthermore, the dynamic SIF can be evaluated as

$$\overline{K}_{III}(p) = \lim_{x \to a} \sqrt{2\pi(x-a)}\,\overline{\tau}_{yz}(x, 0, p). \tag{18}$$

For convenience of analysis, dimensionless dynamic SIF is introduced as

$$\overline{m}(p) = \frac{\overline{K}_{III}}{\tau_0\sqrt{\pi a}}. \tag{19}$$

where $\overline{m}(p)$ represents the rate of dynamic SIF to static SIF.

In order to recover the dimensionless dynamic SIF in the Laplace transformation domain to that in time domain, viz., $\overline{m}(p)$. The inverse integral transformation is needed. The numerical inverse Laplace transformation method called as DAC method is firstly introduced to solve inertial viscoelastic problems by Georgiadis (1991, 1993, 1996). The suitability and reliability of DAC method had been confirmed, and was recommended to replace the old-fashioned orthogonal polynomial method like Miller/Guy technique (Miller and Guy, 1966). We adopted the DAC method here. By introducing the dimensionless time $t_d = t/t_0$, where $t_0 = \min\{a/\sqrt{\mu_{i0}/\rho_i}, i = 1, 2\}$, the inverse integral transformation formulation can be written as

$$\begin{aligned} m(t_d) \;=\; & \left(\frac{e^{ct_d}}{T}\right)\left\{\frac{\overline{m}(c/t_0)}{2t_0} + \sum_{K=1}^{N}\left[\mathrm{Re}\frac{\overline{m}((c+\frac{iK\pi}{T})/t_0)}{t_0}\cos\frac{K\pi t_d}{T}\right]\right. \\ & \left. - \sum_{K=1}^{N}\left[\mathrm{Im}\frac{\overline{m}((c+\frac{iK\pi}{T})/t_0)}{t_0}\sin\frac{K\pi t_d}{T}\right]\right\}, \end{aligned} \tag{20}$$

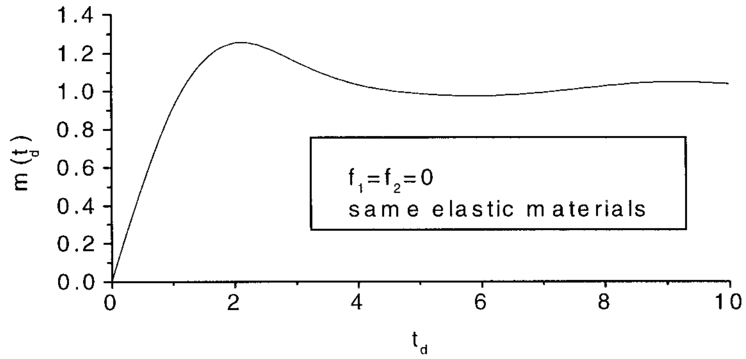

Figure 3. Time history of $m(t_d)$ of crack in homogenous elastic material.

where N, T, c are parameters properly chosen to satisfy required accuracy.

5. Numerical results

Numerical results for dynamic SIF of interface crack are obtained for two dissimilar viscoelastic materials with constants μ_{01}, $\mu_{\infty 1}$, τ_1, ρ_1, μ_{02}, $\mu_{\infty 2}$, τ_2, ρ_2 respectively. Equation (1) pertinent to the standard linear solid was considered. Clearly, the purely elastic case can be recovered from the material model by letting either $f \to 0$ or $\tau \to \infty$. In order to examine the effects of the viscoelastic behavior, following dimensionless parameters are introduced:

$$\lambda = \frac{\mu_{01}}{\mu_{02}}, \qquad f_1 = \frac{\mu_{01}}{\mu_{\infty 1}} - 1, \qquad f_2 = \frac{\mu_{02}}{\mu_{\infty 2}} - 1, \qquad \gamma = \frac{\tau_1}{\tau_2}.$$

The following material combination cases are considered

(1) When $\lambda = 1$, $f_1 = f_2 = 0$, the materials on both sides of interface are the same elastic materials.

(2) When $\lambda = 1$, $f_1 = f_2 \neq 0$, $\nu = 1$ the materials on both sides of interface are the same viscoelastic materials.

(3) When $f_1 \neq 0$, $f_2 = 0$ or $f_1 = 0$, $f_2 \neq 0$, the material on one side of interface is viscoelastic but elastic on the other side.

(4) When $f_1 \neq 0$, $f_2 \neq 0$, $\lambda \neq 1$, $\gamma \neq 1$, the materials on the both sides of interface are dissimilar viscoelastic materials.

For all material combination cases above-mentioned, $\rho_1 = \rho_2 = 1200$ kg m^{-3} $\mu_{01} =$ 1690 MN m^{-2}, $\tau_1 = 2t_0$. The values of material constants are from Georgiadis (1991). The dimensionless material parameters, viz., λ, f_1, f_2, γ, change in each case.

Figure 3 shows the time history of the dimensionless stress intensity factor $m(t_d)$ during a small time-interval for case (1), viz., purely elastic materials. It can be seen that dimensionless stress intensity factor $m(t_d)$ reaches to maximum at the vicinity of $t_d = 2$ and static limit $m(t_d) = 1$ with dimensionless time t_d goes to infinity. The phenomenon is well-known dynamic overshoot. For the present case of material constants, the dynamic over-shoots is about 25%.

Figures 4 and 5 show the time history of the dimensionless stress intensity factor $m(t_d)$ for case (2) and case (3), respectively. By comparison with Fig. 3, the peak values of the dimensionless stress intensity factor $m(t_d)$ of case (2) and (3) are lower than that in case (1),

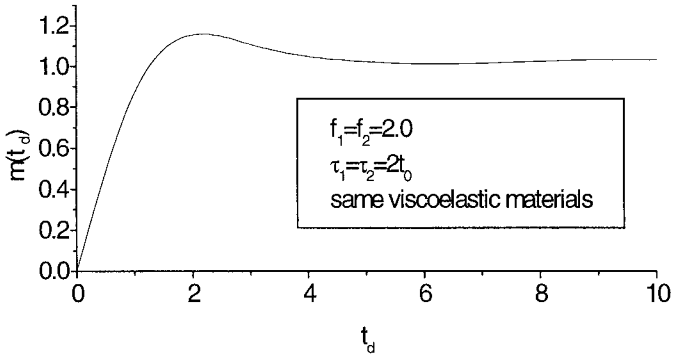

Figure 4. Time history of $m(t_d)$ of crack in homogenous viscoelastic material.

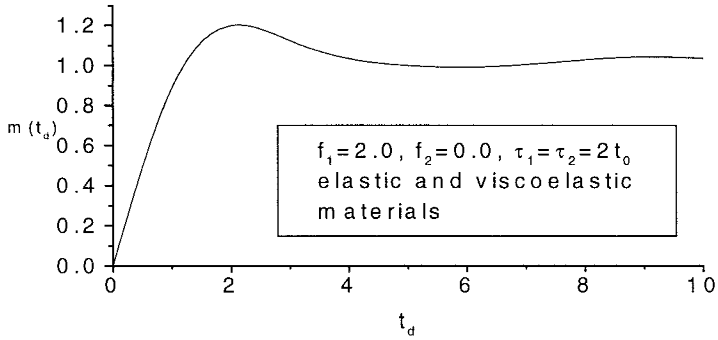

Figure 5. Time history of $m(t_d)$ of interface crack between elastic material and viscoelastic material.

and the oscillations around static limit alleviate slightly. Clearly, this is due to the continuous loss of energy when diffraction wave propagates in the viscoelastic materials.

To examine the effects of the material constants, we take various γ and f into account.

In Fig. 6, the time history of the dimensionless SIF is given for various γ with respect to $f_1 = f_2 = 2.0$ to show the influence of relaxation time. The peak value of dimensionless SIF $m(t_d)$ is the highest and the oscillation of curve is the most conspicuous when $\gamma = 0.1$; the peak value of dimensionless SIF $m(t_d)$ is the lowest and the oscillation of curve is least conspicuous when $\gamma = 10.0$. While the peak value of dimensionless SIF $m(t_d)$ and the

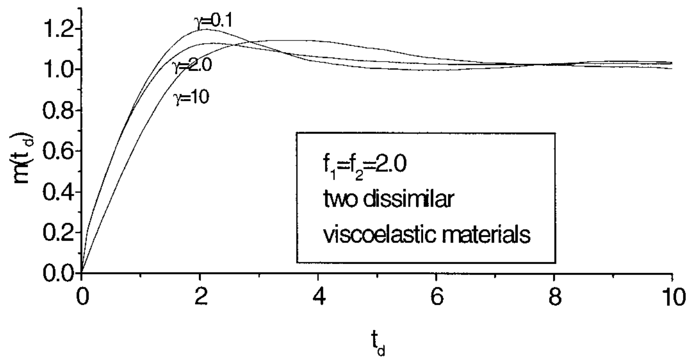

Figure 6. Time history of $m(t_d)$ of interface crack between two dissimilar viscoelastic materials with various γ and $f_1 = f_2 = 2.0$.

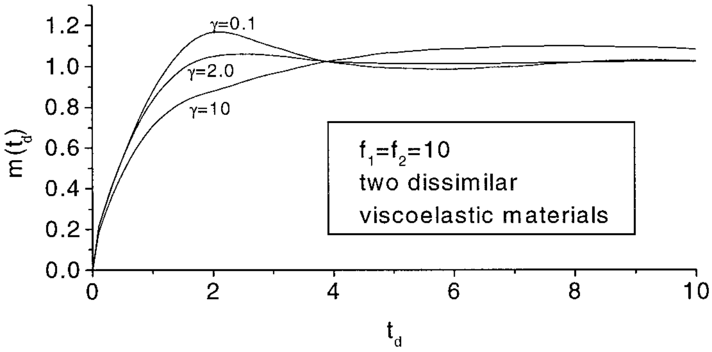

Figure 7. Time history of $m(t_d)$ of interface crack between two dissimilar viscoelastic materials with various γ and $f_1 = f_2 = 10$.

oscillation of curve are intermediate when $\gamma = 2.0$. The explanation of the phenomenon is as follows. A long relaxation time implies that viscoelastic material performs like an elastic material during short time interval while a short relaxation time will make viscoelastic material exhibiting strong viscosity effects at short time interval when difference between short-time shear modulus μ_0 and long-time shear modulus μ_∞, viz., f, keeps constant. The phenomena can be observed clearly from Fig. 2. We can also observe that the peak value of the dimensionless SIF $m(t_d)$ takes place at vicinity of $t_d = 2$ which corresponds to the moment when the viscoelastic wave from one crack-tip reaches another, and delays with the increase of viscosity of materials.

Figure 7 shows the time history of the dimensionless SIF $m(t_d)$ for the same group of γ but with a larger value of f, viz., $f_1 = f_2 = 10$. The influence of modulus difference f on $m(t_d)$ can be noted by comparison of Fig. 7 with Fig. 6. It is observed clearly that the peak value of the dimensionless SIF $m(t_d)$ decreases and the location of t_d corresponding to the peak value shifts backward when the modulus difference f increases with the relaxation time τ keeping fixed. When the modulus difference f is very large and the relaxation time τ is very small, the viscosity is so strong that the phenomenon of dynamic overshoot disappears.

These phenomena observed for interface crack and the interpretations are very close to ones for crack in homogeneous viscoelastic material given by Georgiadis (1991, 1993).

In summarizing our numerical study, the following conclusions are drawn:

(1) The time history of the dimensionless SIF $m(t_d)$ for the interface crack between two dissimilar viscoelastic materials shows dynamic overshoot at short time like that happens in elastic material. The peak value of the dimensionless SIF $m(t_d)$ happens at the vicinity of $t_d = 2$ that corresponds to the moment when reflection wave from one crack-tip arrives at another.

(2) By comparison with elastic material, the peak value of the dimensionless SIF $m(t_d)$ decreases, and the location of peak value of the dimensionless SIF $m(t_d)$ shifts backward. This phenomenon is more conspicuous when the viscosity effects of viscoelastic material increase. The answer to the delay of dynamic overshoot is the delayed elasticity of the viscoelastic material model.

(3) The viscoelastic material parameters, viz., f and τ, affect strongly the time history of the dimensionless SIF $m(t_d)$. The peak value of the dimensionless SIF $m(t_d)$ decreases and delays for large modulus difference f and small relaxation time τ. When the viscosity

effects of the viscoelastic material are enough strong, the dynamic overshoot phenomenon even disappears

(4) For bimaterials interface crack, suitable combination of viscoelastic material parameters, viz., f_1, τ_1, f_2, τ_2, will alleviate the dynamic overshoot. Consequently, the unstable propagation of crack under transient load can be restrained in some sense.

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