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Study on Current-Random Wave Forces Acting on a Framework[1]

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Abstract — The forces of random wave plus current acting on a simplified offshore platform (jacket) model have been studied numerically and experimentally. The numerical results are in good agreement with experiments. The mean force can be approximated as a function of equivalent velocity parameter and the root-mean-square force as a function of equivalent significant wave height parameter.

Key words: *random wave*; *wave forces*; *current condition*; *jacket*; *framework*

1. Introduction

Studies on random wave forces acting on framework structures are very important to offshore platform design. As the cylinders of the structures are in different positions and different axial directions, the correlation coefficient between every two sectional forces is very difficult to obtain, so the usual method used to calculate the force spectrum of single vertical cylinder (Reid, 1957; Borgman, 1967; Chakrabati, 1980; Sarpkaya, 1981) does not work here.

Recently, Yeh and Tung (1996) studied the statistical properties of random forces on two vertical circular cylinders. Their theoretical method is too complicated to be used for such frameworks.

Model experiments also have difficult problems. We can obtain the spectra of random waves and random forces on the framework, but from the spectra we cannot obtain a reliable transfer function which is the most important, because small errors in the experimental spectra may cause great differences in transfer function evaluation, and data error in sampling and processing is unavoidable.

Current-random wave interacting forces are much more complicated. The cases of a horizontal and a vertical cylinder have been studied by Tung and Huang (1976), Moe and Crandall (1977) and Kang and Li (1990). They found that even a small current could cause great changes in the statistical properties of random forces. As for multi-cylinders little research has been done.

In this paper, a simple method is proposed for the calculation of the spectrum of current-wave interacting force and experiments are performed to verify the method.

2. Theoretical Considerations

Suppose that in a water field of finite depth a current flows from left to right at constant

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velocity U_0 and a 2D random wave train propagates in the same direction as that of the current, as shown in Fig. 1. The random wave surface displacement can be expressed as

$$\eta(x, t) = a\cos[kx - (\omega + kU_0)t + \varphi] \tag{1}$$

where a, k, ω, φ are wave height, wave number, circular frequency and phase angle respectively. They are all random variables which vary slowly.

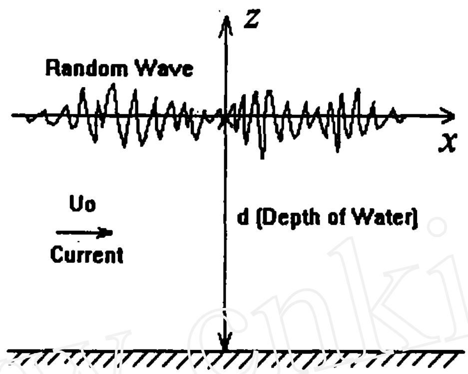

Fig. 1. Sketch of 2D current-random wave.

Now we introduce two other random variables as Tung and Huang (1984) did:

$$\left.\begin{aligned} X &= a\cos[\varphi - (\omega + kU_0)t] \\ Y &= a\sin[\varphi - (\omega + kU_0)t] \end{aligned}\right\} \tag{2}$$

then the water particle velocity and acceleration at a fixed space point (x, z) can be expressed as a function of (X, Y, k) by linear regular wave theory.

$$\left.\begin{aligned} V_x &= U_0 + \alpha(k, z)(X\cos kx - Y\sin kx) \\ V_z &= \beta(k, z)(X\sin kx - Y\cos kx) \\ A_x &= \omega\alpha(k, z)(Y\cos kx + X\sin kx) \\ A_z &= \omega\beta(k, z)(Y\sin kx - X\cos kx) \end{aligned}\right\} \tag{3}$$

where

$$\left.\begin{aligned} \alpha(k, z) &= \sqrt{\frac{gk}{\tanh kd}}\,\frac{\cosh k(z+d)}{\cosh kd} \\ \beta(k, z) &= \sqrt{\frac{gk}{\tanh kd}}\,\frac{\sinh k(z+d)}{\sinh kd} \end{aligned}\right\} \tag{4}$$

From the statistics of experimental data, the probability density function of wave height a and wave period T can be obtained. Using $\sqrt{X^2 + Y^2}$ instead of a, we have the probability density as follows:

$$\rho(X, Y, T) = f' \frac{\sqrt{X^2 + Y^2}}{(2\pi\sigma^2)^{\frac{3}{2}}} e^{\frac{1+f^2}{2\sigma^2}(x^2 + y^2)} \tag{5}$$

$$\left.\begin{aligned} f = f(T) &= \frac{P - \frac{1}{2}}{\sqrt{P - P^2}} \\ P = P(T) &= \int_0^T \rho(T)\, dT \end{aligned}\right\} \tag{6}$$

$$\rho(T) = \frac{0.5 f'(T)}{\left[1 + f^2(T)\right]^{\frac{3}{2}}} \tag{7}$$

where $\rho(T)$ is the probability density of wave period T, which can be easily obtained from experimental data statistics, and $f' = f'(T)$, wave period $T = 2\pi / \omega$, ω being related to wave number k by dispersion relation, and σ is root-mean-square of wave surface displacement fluctuation.

Considering the effects of free surface fluctuation (Yeh and Tung, 1996), the hydrodynamic force on one section of a circular cylinder is

$$\left.\begin{aligned} F_s &= F_d + F_m \\ F_d &= \left[C_d \frac{1}{2} \rho_w D \left|\vec{V}_n\right| V_{nx}\right] H(\eta - z) \\ F_m &= \left[C_m \rho_w \frac{\pi}{4} D^2 A_{nx}\right] H(\eta - z) \end{aligned}\right\} \tag{8}$$

where D — diameter of the cylinder; ρ_w — mass density of water; $H(\eta - z)$ — unit step function, which means that no dynamic force acts on the section when the random water surface does not cover it; $\vec{V}_n$ — water particle velocity normal to the cylinder axis; V_{nx} — the x direction component of $\vec{V}_n$; A_{nx} — x direction component of water particle acceleration normal to the cylinder axis, and C_d and C_m — local resistant and inertial force coefficients.

C_d and C_m are functions of random variables (X, Y, k). Given the wave height $\sqrt{X^2 + Y^2}$ and the wave number k, we can obtain the values of C_d and C_m from the known regular wave hydrodynamic coefficients charts (Sarpkaya and Isaacson, 1981).

The velocity and acceleration of water particle are

$$\left.\begin{aligned} V_{nx} &= V_x (1 - e_x^2) - V_z e_x e_z \\ \left|\vec{V}_n\right| &= \sqrt{V_x^2 + V_z^2 - (V_x e_x + V_z e_z)^2} \\ A_{nx} &= A_x (1 - e_x^2) - A_z^2 e_x e_z \end{aligned}\right\} \tag{9}$$

where e_x and e_z are x and z components of cylinder axial unit vector.

So the sectional force is a function of random variables X, Y and k, and it can be expanded into a power series of random variables X and Y according to probability distribution of X and Y. The coefficients of the power series are functions of wave number k. Integrating the sectional forces for all cylinders, we have the total random force $F(t)$.

A linearized approximation of the total force is

$$F = F_{dT} + F_{mT} \tag{10}$$

$$\left.\begin{aligned} F_{dT} &= A_1(k)X + B_1(k)Y + C_1(k) \\ F_{mT} &= A_2(k)X + B_2(k)Y \end{aligned}\right\} \tag{11}$$

$$\left.\begin{aligned} A_{1,2}(k) &= q_1 \sum_i \int_{-\infty}^{\infty} dY \int_{z_j/q}^{\infty} X\rho(X,Y) F_d F_m \, dX \\ B_{1,2}(k) &= q_1 \sum_i \int_{-\infty}^{\infty} dY \int_{z_i/q}^{\infty} Y\rho(X,Y) F_d F_m \, dX \\ C_{1,2}(k) &= \frac{2}{\pi^{\frac{3}{2}}} \sum_i \int_{-\infty}^{\infty} dY \int_{z_i/q}^{\infty} \rho(X,Y) F_d \, dX \end{aligned}\right\} \tag{12}$$

and

$$\left.\begin{aligned} q &= \frac{\sqrt{2}\,\sigma}{\sqrt{1+f^2(T)}}, \quad q_1 = \frac{8}{3q\pi^{\frac{3}{2}}} \\ \rho(X,Y) &= \sqrt{X^2+Y^2}\, e^{-(X^2+Y^2)} \\ F_d &= F_d(qX, qY, kx_i, kz_i, U_0) \\ F_m &= F_m(qX, qY, kx_i, kz_i, U_0) \end{aligned}\right\} \tag{13}$$

where x_i and z_i are the coordinates of circular cylinder section.

The spectrum of the total force is

$$S_{FF}(\omega) = \left[(A_1 + A_2)^2 + (B_1 + B_2)^2)\right] S_{\eta\eta}(\omega) \tag{14}$$

and the mean force is

$$\bar{F} = \int_0^{\infty} C_1(k)\,\rho(T)\,dT\,. \tag{15}$$

3. Laboratory Experiment

The experiment was carried out in a 2D wave basin at the State Ocean Engineering Key Laboratory, Shanghai Jiaotong University, P. R. China.

The basin is 40 m in length, 30 m in width and 6.0 m in depth. The water depth can be changed as required from 0 to 5.0 m by adjusting the position of the false bottom of the basin. In this experiment, the static water depth is 1.3 m. The test framework is a simplified model of a jacket platform in the southwest China sea.

As shown in Fig. 2, the framework is 1.5 m long. 0.75 m wide and 2 m high, consisting of 32 circular cylindrical elements. Their 6 main pile legs are 5 cm in diameter and all the others are 3.2 cm.

The measurement system includes a wave monitor to measure the water surface fluctuations, a force scale to record the variation of random acting on the framework and an accelerometer to trace the movements of the framework at a fixed point.

Fig. 2. Framework model.

4. Experimental and Calculated Results

4.1 Statistical Properties of Random Waves

Fig. 3 and Fig. 4 show the statistical probability density distributions of wave height $\rho(a)$ and wave period $\rho(T)$. The joint probability density $\rho(a, T)$ from experimental data statistics is shown in Fig. 5, which is in good agreement with that from Eq. (5) when $\sqrt{X^2 + Y^2}$ is replaced by a.

4.2 Expansion Coefficients of Random Force

Fig. 6 shows the expansion coefficients of $A_j(k)$. $B_l(k)$ and $C_1(k)$ from Eqs. (12) and (13). $A_1(k)$ plays the main role in drag force fluctuation, $B_2(k)$ is the main part of inertia force, and $C_1(k)$ represents the mean force.

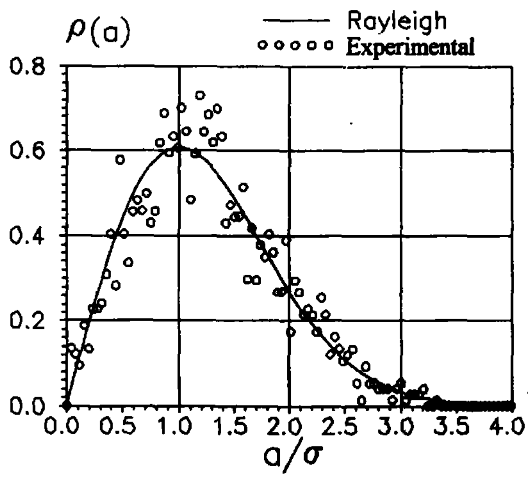

Fig. 3. Probability density of wave height.

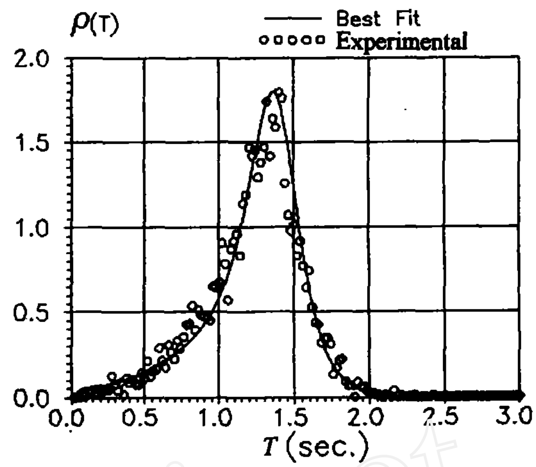

Fig. 4. Probability density of wave period.

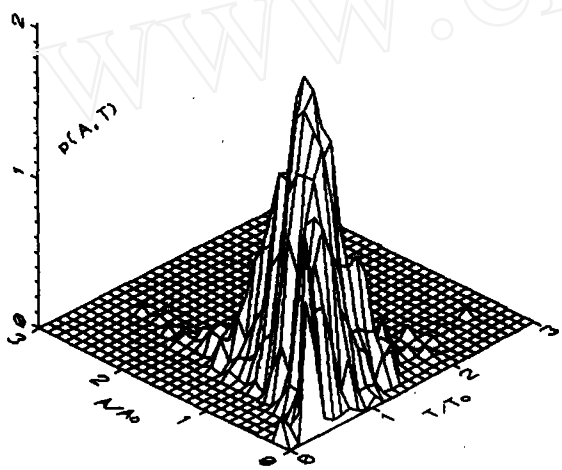

Fig. 5. Joint probability density of wave height and wave period.

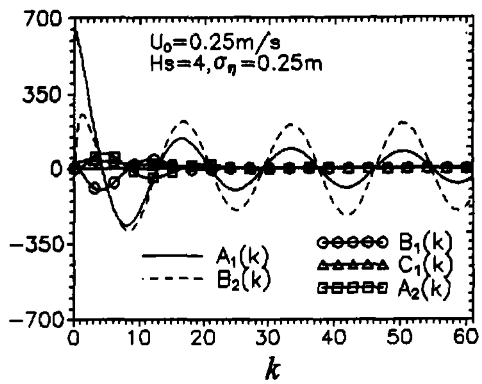

Fig. 6. Expanded force coefficients v.s. wave number.

4.3 Comparison of Calculated and Experimental Forces

Fig. 7 and Fig. 8 show the comparison between the calculated and experimental mean forces and between the calculated and experimental root-mean-square total force respectively.

Fig. 9 and Fig. 10 show the calculated and experimental force spectra of pure wave and wave plus current respectively. They are in good agreement.

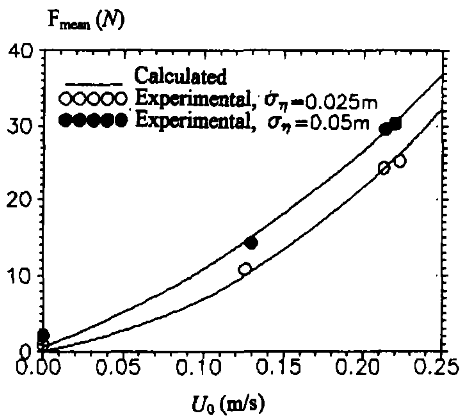

Fig. 7. Comparison between calculated and experimental mean force.

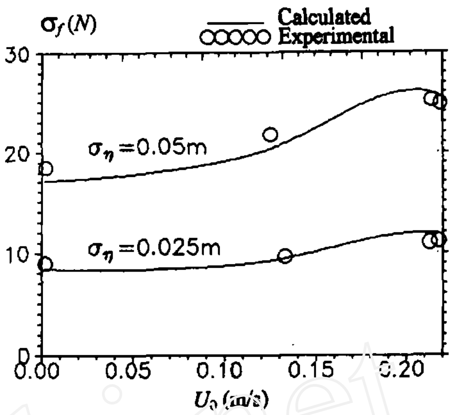

Fig. 8. Comparison between calculated and experimental root-mean-square force.

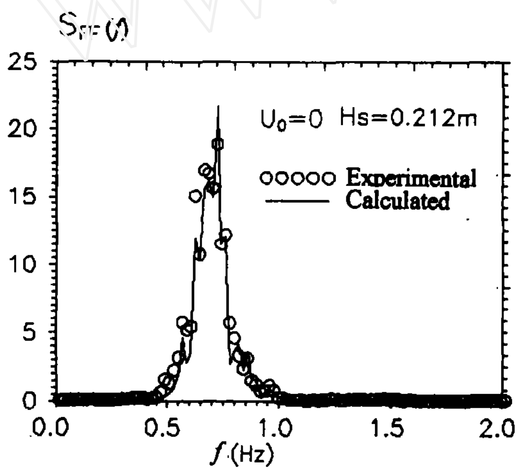

Fig. 9. Comparison between calculated and experimental force spectra. (for pure random wave)

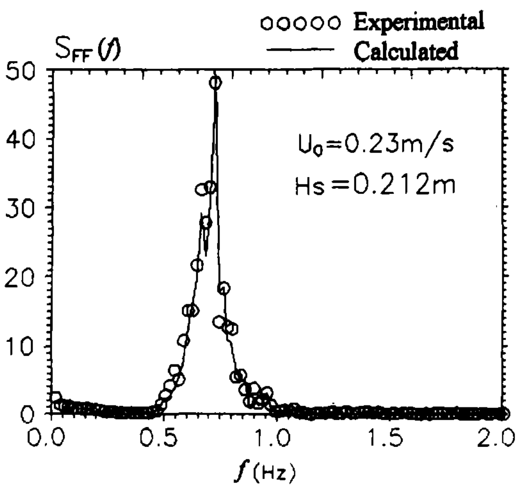

Fig. 10. Comparison between calculated and experimental force spectra. (for current-random wave)

4.4 Force Variation with Wave and Current Parameters

Fig. 11 shows the variation of numerical results of root-mean-square force F_{rms} with significant wave height $H_s = H_{1/3}$ and current velocity U_0. Fig. 12 gives the calculated results of mean force F_{mean} versus H_s and U_0.

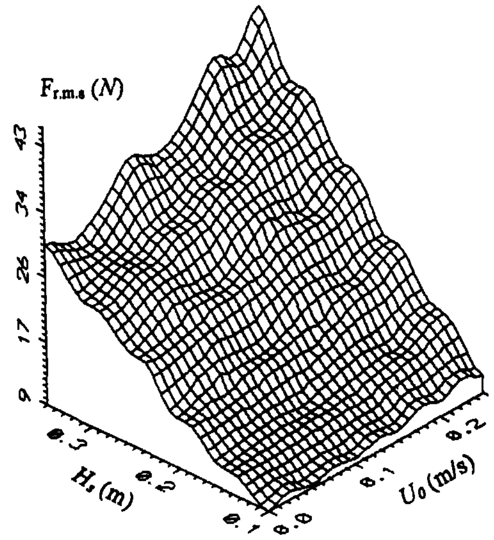

Fig. 11. Root-mean-square force v.s. wave height and current velocity.

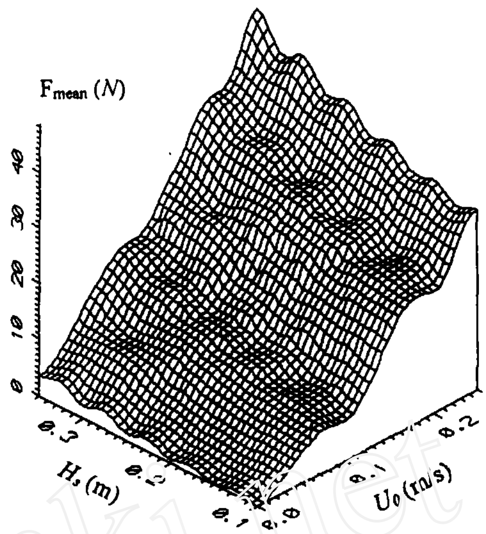

Fig. 12. Mean force v.s. wave height and current velocity.

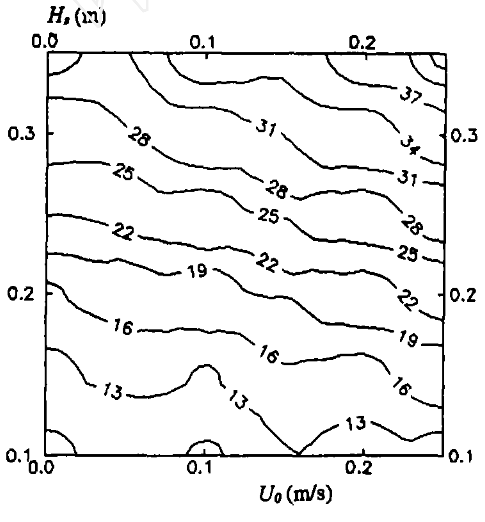

Fig. 13. Contour lines of root-mean-square force v.s. wave height and current velocity.

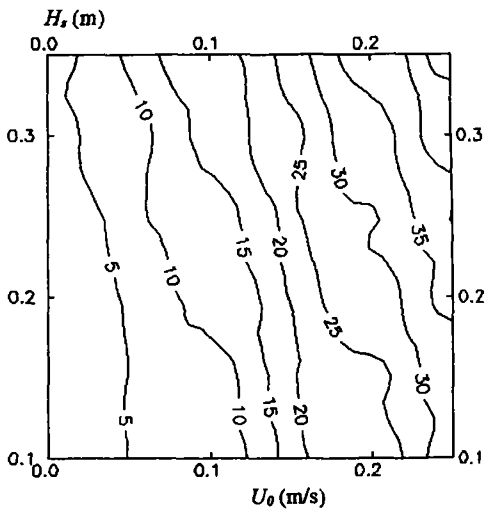

Fig. 14. Contour lines of mean force v.s. wave height and current velocity.

As H_s and U_0 increase, both F_{rms} and F_{mean} increase. The effect of U_0 on F_{mean} or H_s on F_{rms} is larger, but the effect of U_0 on F_{rms} or H_s on F_{mean} is comparatively small.

4.5 Functions of Single Parameter to Describe Mean and RMS Force

From Fig. 13 and Fig. 14 it is known that the contour lines of F_{rms} and F_{mean} are nearly parallel and straight. So an equivalent significant wave height parameter can be introduced:

$$H^* = H_s + \lambda_1 U_0$$

where λ_1 is a constant and in this case, $\lambda_1 = 0.26$.

The root-mean-square force F_{rms} is a function of parameter H^* as shown in Fig. 15.

Similarly, an equivalent velocity can be introduced:

$$U^* = U_0 + \lambda_2 (H_s - H_{s0})$$

where λ_2 is a constant, and in this case $\lambda_2 = 0.3636$. The mean force F_{mean} is a function of parameter U^* as shown in Fig. 16.

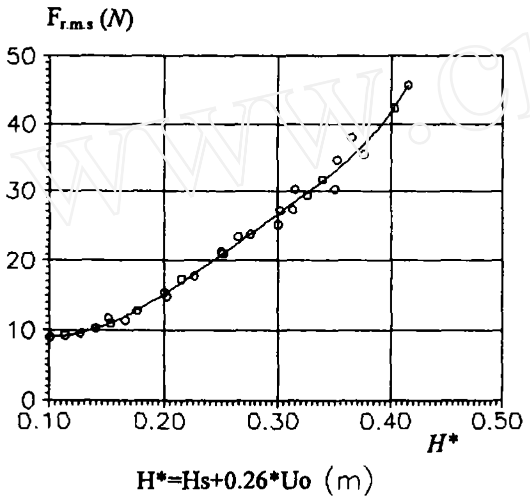

Fig. 15. Root-mean-square force v.s. equivalent significant wave height.

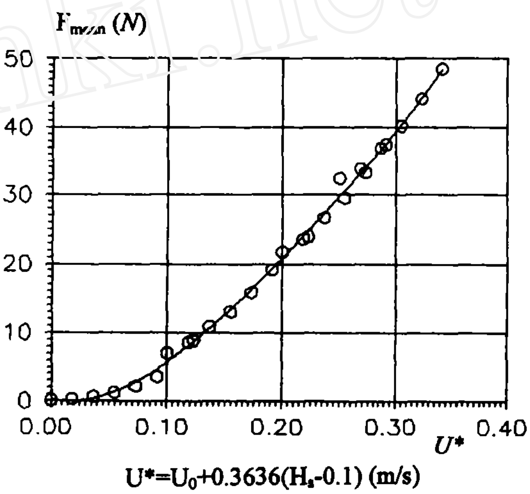

Fig. 16. Mean force v.s. equivalent current velocity.

5. Conclusions

— The results of the linearized random force calculation method mentioned above are in good agreement with experiments.

— The mean or root-mean-square force of random waves on a framework depends only on a single parameter (equivalent current velocity or equivalent wave height).

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