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NUMERICAL SOLUTIONS FOR THE TRANSIENT FLOW IN THE HOMOGENOUS CLOSED CIRCLE RESERVOIRS

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ABSTRACT: There are many fault block fields in China. A fault block field consists of fault pools. The small fault pools can be viewed as the closed circle reservoirs in some case. In order to know the pressure change of the developed formation and provide the formation data for developing the fault block fields reasonably, the transient flow should be researched. In this paper, we use the automatic mesh generation technology and the finite element method to solve the transient flow problem for the well located in the closed circle reservoir, especially for the well located in an arbitrary position in the closed circle reservoir. The pressure diffusion process is visualized and the well-location factor concept is first proposed in this paper. The typical curves of pressure vs time for the well with different well-location factors are presented. By comparing numerical results with the analytical solutions of the well located in the center of the closed circle reservoir, the numerical method is verified.

KEY WORDS: fluid flow in porous media, transient flow, closed circle reservoir, numerical solution

1 INTRODUCTION

There are many fault block fields in China, such as the Shengli Oilfield, the Zhongyuan Oilfield, the Jidong Oilfield and so on. The small fault pools can be viewed as the closed circle reservoirs in some case. In order to know the pressure change of the developed formation and provide formation data for developing the reservoirs reasonably, the transient flow should be researched. Scientists at home or aboard have studied the closed circle reservoirs for several decades. In 1949, Van Everdingen and Hurst[1] presented the transient pressure solution in Laplace space for the cylinder water invaded reservoir. Ditzs[2] introduced an approximate processing method by using the shape factor in the circle reservoirs. Raghavan[3] presented the pressure transient solution with the effective well-bore radius in the Laplace space. In China, Kong[4] has studied this problem and obtained the pressure solution in Laplace space and real space for transient flow. Reference [5] has given some new shape factors. Most of these studies have been focused on the analytical or semi-analytical solutions[6~9] for the transient flow problem with the well located in the center of the circle reservoirs. In this paper, we will use the automatic mesh generation technology[10~12] and the finite element method to solve the transient flow problem in which the well is located in an arbitrary position of the closed circle reservoirs. This numerical method will provide a new useful means for interpretation of well test data in the fault pools.

2 DESCRIPTION OF PHYSICAL AND MATHEMATICAL MODEL

The physical model used in our paper can be described as follows:

(1) The reservoir is a horizontal, homogenous and closed circle formation. The homogeneous formation means that the characteristic parameters such as the thickness, permeability and the porosity in the reservoir take the same values.

(2) The well is located in an arbitrary position of a closed circle reservoir. It perforates the developing formation and products with a constant rate.

(3) The fluid in formation is a kind of constant viscosity and slight compressed Newtonian fluid.

(4) The fluid flow through the formation is a laminar flow, which follows Darcy's Law.

(5) The total testing process is at a constant

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temperature, the effects of gravity and the other physicochemical changes are neglected.

From the description of the physical model, the mathematical model can be described as follows:

Flow governing equation

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=\frac{\phi\mu C_t}{3.6k}\frac{\partial p}{\partial t}$$

$$(x,y)\in\left\{\sqrt{x^2+y^2}<R_{\mathrm{e}}\right\}\cap\left\{\sqrt{(x-x_{\mathrm{w}})^2+(y-y_{\mathrm{w}})^2}>r_{\mathrm{w}}\right\} \quad (1)$$

Initial condition

$$p(t=0)=p_{\mathrm{i}}$$

$$(x,y)\in\left\{\sqrt{x^2+y^2}\le R_{\mathrm{e}}\right\}\cap\left\{\sqrt{(x-x_{\mathrm{w}})^2+(y-y_{\mathrm{w}})^2}>r_{\mathrm{w}}\right\} \quad (2)$$

Outer boundary condition

$$\left.\frac{\partial p}{\partial n}\right|_{\Gamma_{\mathrm{e}}}=0 \qquad \Gamma_{\mathrm{e}}=\left\{\sqrt{x^2+y^2}=R_{\mathrm{e}}\right\} \quad (3)$$

Inner boundary condition

$$2\pi h r_{\mathrm{w}}\frac{k}{\mu}\left.\frac{\partial p}{\partial n}\right|_{\Gamma_{\mathrm{i}}}=1.1574\times10^{-2}qB+\frac{C}{3.6}\frac{\partial p_{\mathrm{w}}}{\partial t}$$

$$\Gamma_{\mathrm{i}}=\left\{\sqrt{(x-x_{\mathrm{w}})^2+(y-y_{\mathrm{w}})^2}=r_{\mathrm{w}}\right\} \quad (4)$$

where B is the volume factor, $\mathrm{m}^3/\mathrm{m}^3$; C is the wellbore storage factor, $\mathrm{m}^3/\mathrm{MPa}$; C_t is the total compress coefficient for the test formation, $\mathrm{m}^3/\mathrm{MPa}$; h is the net pay of the test formation, m; k is the permeability of the reservoir, $\mu\mathrm{m}^2$; p is the pressure in the formation, MPa; p_{i} is the initial formation pressure; p_{w} is the pressure in the wellbore, MPa; q is the production rate, m^3/d; r is the distance of the point in the formation to the wellbore center; R_{e} is the radio of the circle reservoir, m; r_{w} is the wellbore radius, m; t is the time, h; (x,y) is the coordinate of the point in the reservoir; $(x_{\mathrm{w}},y_{\mathrm{w}})$ is the coordinate of the wellbore center; ϕ is the porosity of the reservoir, percent number; μ is the viscosity of the fluid in the formation, mPa·s; Γ_{e} is the external boundary of the circle closed reservoir; Γ_{i} is the boundary of the wellbore.

In order to make the equations in a general form, the above equations are normalized as follows:

Flow governing equation

$$\frac{\partial^2 p_{\mathrm{D}}}{\partial X_{\mathrm{D}}^2}+\frac{\partial^2 p_{\mathrm{D}}}{\partial Y_{\mathrm{D}}^2}=\frac{1}{C_{\mathrm{D}}\mathrm{e}^{2S}}\frac{\partial p_{\mathrm{D}}}{\partial T_{\mathrm{D}}}$$

$$(X_{\mathrm{D}},Y_{\mathrm{D}})\in\left\{\sqrt{X_{\mathrm{D}}^2+Y_{\mathrm{D}}^2}<R_{\mathrm{eD}}\right\}\cap\left\{\sqrt{(X_{\mathrm{D}}-X_{\mathrm{wD}})^2+(Y_{\mathrm{D}}-Y_{\mathrm{wD}})^2}>1\right\} \quad (5)$$

Initial condition

$$p_{\mathrm{D}}(T_{\mathrm{D}}=0)=0$$

$$(X_{\mathrm{D}},Y_{\mathrm{D}})\in\left\{\sqrt{X_{\mathrm{D}}^2+Y_{\mathrm{D}}^2}\le R_{\mathrm{eD}}\right\}\cap\left\{\sqrt{(X_{\mathrm{D}}-X_{\mathrm{wD}})^2+(Y_{\mathrm{D}}-Y_{\mathrm{wD}})^2}>1\right\} \quad (6)$$

Outer boundary condition

$$\left.\frac{\partial p_{\mathrm{D}}}{\partial n}\right|_{\Gamma_{\mathrm{eD}}}=0 \qquad \Gamma_{\mathrm{eD}}=\left\{\sqrt{X_{\mathrm{D}}^2+Y_{\mathrm{D}}^2}=R_{\mathrm{eD}}\right\} \quad (7)$$

Inner boundary condition

$$\left.\frac{\partial p_{\mathrm{D}}}{\partial n}\right|_{\Gamma_{\mathrm{iD}}}=-1+\frac{\partial p_{\mathrm{wD}}}{\partial T_{\mathrm{D}}}$$

$$\Gamma_{\mathrm{iD}}=\left\{\sqrt{(X_{\mathrm{D}}-X_{\mathrm{wD}})^2+(Y_{\mathrm{D}}-Y_{\mathrm{wD}})^2}=1\right\} \quad (8)$$

where C_{D} is the dimensionless wellbore storage coefficient, $C_{\mathrm{D}}=\dfrac{0.1592C}{\phi hC_t r_{\mathrm{w}}^2}$; p_{D} is the dimensionless pressure, $p_{\mathrm{D}}=\dfrac{kh(p_{\mathrm{i}}-p)}{1.842\times10^{-3}q\mu B}$; p_{wD} is the dimensionless pressure in the wellbore, $p_{\mathrm{wD}}=\dfrac{kh(p_{\mathrm{i}}-p_{\mathrm{w}})}{1.842\times10^{-3}q\mu B}$; R_{D} is the dimensionless distance, $R_{\mathrm{D}}=\dfrac{r}{r_{\mathrm{we}}}$; R_{eD} is the dimensionless radio of the circle reservoir, $R_{\mathrm{eD}}=\dfrac{R_{\mathrm{e}}}{r_{\mathrm{we}}}$, where r_{we} is the effective wellbore radius, $r_{\mathrm{we}}=r_{\mathrm{w}}\cdot\mathrm{e}^{-S}$, m; S is the skin factor, dimensionless; T_{D} is the dimensionless time, $T_{\mathrm{D}}=\dfrac{t_{\mathrm{D}}}{C_{\mathrm{D}}}$; t_{D} is the dimensionless time, $t_{\mathrm{D}}=\dfrac{3.6kt}{\phi\mu C_t r_{\mathrm{w}}^2}$; X_{D} is the dimensionless horizontal coordinate of the point in the reservoir, $X_{\mathrm{D}}=\dfrac{x}{r_{\mathrm{we}}}$; X_{wD} is the dimensionless horizontal coordinate of the wellbore center, $X_{\mathrm{wD}}=\dfrac{x_{\mathrm{w}}}{r_{\mathrm{we}}}$; Y_{D} is the dimensionless vertical coordinate of the point in the reservoir, $Y_{\mathrm{D}}=\dfrac{y}{r_{\mathrm{we}}}$; Y_{D} is the dimensionless vertical coordinate of the wellbore center, $Y_{\mathrm{wD}}=\dfrac{y_{\mathrm{w}}}{r_{\mathrm{we}}}$; Γ_{eD} is the dimensionless external boundary of the circle closed reservoir; Γ_{iD} is the dimensionless boundary of the wellbore.

3 NUMERICAL SOLUTIONS

In order to solve the transient flow problem for the well located in an arbitrary position of the closed circle reservoir, we generate the finite element meshes by using the mesh generation method given in the literatures[10~12]. The finite element meshes are shown in Fig.1. The meshes of the region near the wellbore are shown in Fig.2. Then, we can derive the finite element equations for every unit in the calculation region, which are shown as follows

$$\iint \varphi_{\rm i}^e \left(\frac{\partial^2 p_{\rm D}^e}{\partial X_{\rm D}^2} + \frac{\partial^2 p_{\rm D}^e}{\partial Y_{\rm D}^2} - \frac{1}{C_{\rm D}{\rm e}^{2S}} \frac{\partial p_{\rm D}^e}{\partial T_{\rm D}} \right) {\rm d}A = 0$$

$$i = 1, 2, 3 \tag{9}$$

where $\varphi_{\rm i}^e$ is the interpolating function; $p_{\rm D}^e$ is the pressure at every node of the calculating element.

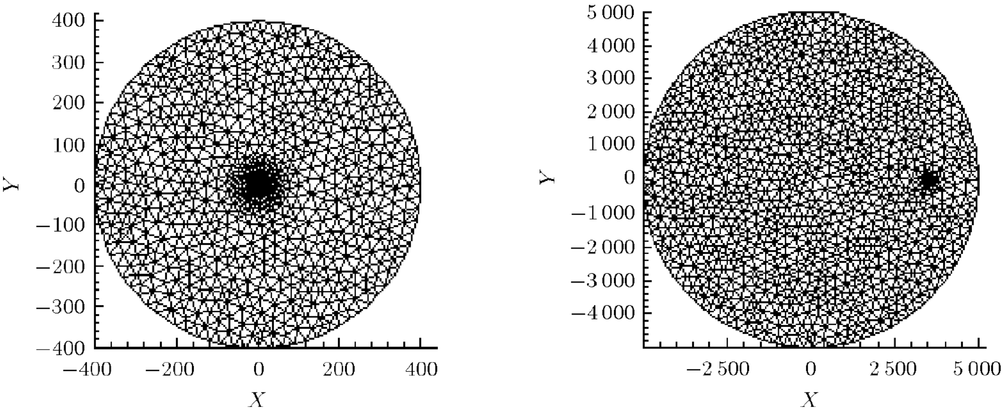


Fig.1 The triangle meshes of the reservoir studied in this paper

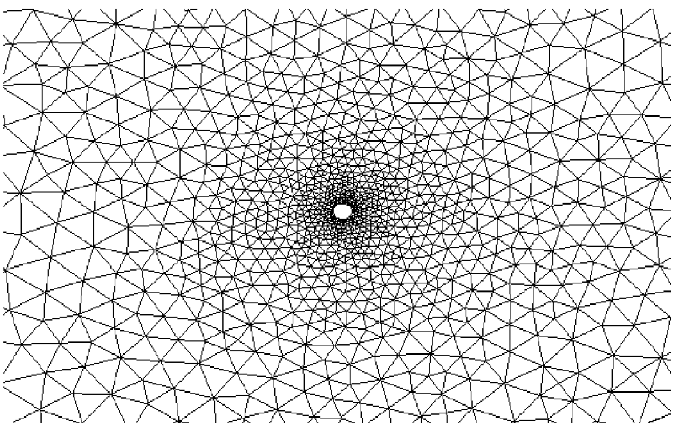


Fig.2 The meshes of the region near the wellbore

Using the finite element method, we solved the transient flow problem for the well located in an arbitrary position of the closed circle reservoirs. For the first case, we solved problem for the well in the center of the closed circle reservoir (see Fig.1). By using the different values of $C_{\rm D}{\rm e}^{2S}$, the typical curves of the wellbore pressure vs time are calculated, as shown in Fig.3.

In order to know the pressure change in the developed formation, we calculate the pressure diffusion process. The pressure contours at different times are shown in Fig.4.

For the second case, we solved problem for the well in an arbitrary position of the closed circle reservoir. In order to describe the well location in the reservoir, we propose a new concept—the well-location fac-

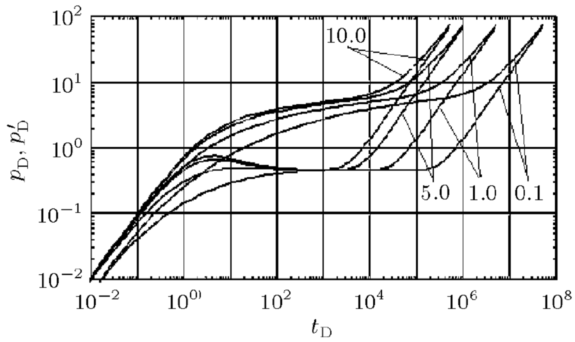


Fig.3 The typical curves with different $C_{\rm D}{\rm e}^{2S}$ values ($C_{\rm D}{\rm e}^{2S}$ = 10, 5, 1, 0.1)

tor which is a geometry character parameter. The well-location factor γ_{wp} is defined as the ratio of the distance between the well location and the center r to the radius of the circle reservoir R_{e}. That is $\gamma_{\mathrm{wp}} = r_{\mathrm{wp}}/R_{\mathrm{e}}$, where r_{wp} is the distance between the wellbore center and the circle reservoir center, $r_{\mathrm{wp}} = \sqrt{x_{\mathrm{w}}^2 + y_{\mathrm{w}}^2}$. Under this concept, we calculate the typical curves of the wellbore pressure with the different well-location factors. The numerical solutions are shown in Fig.5.

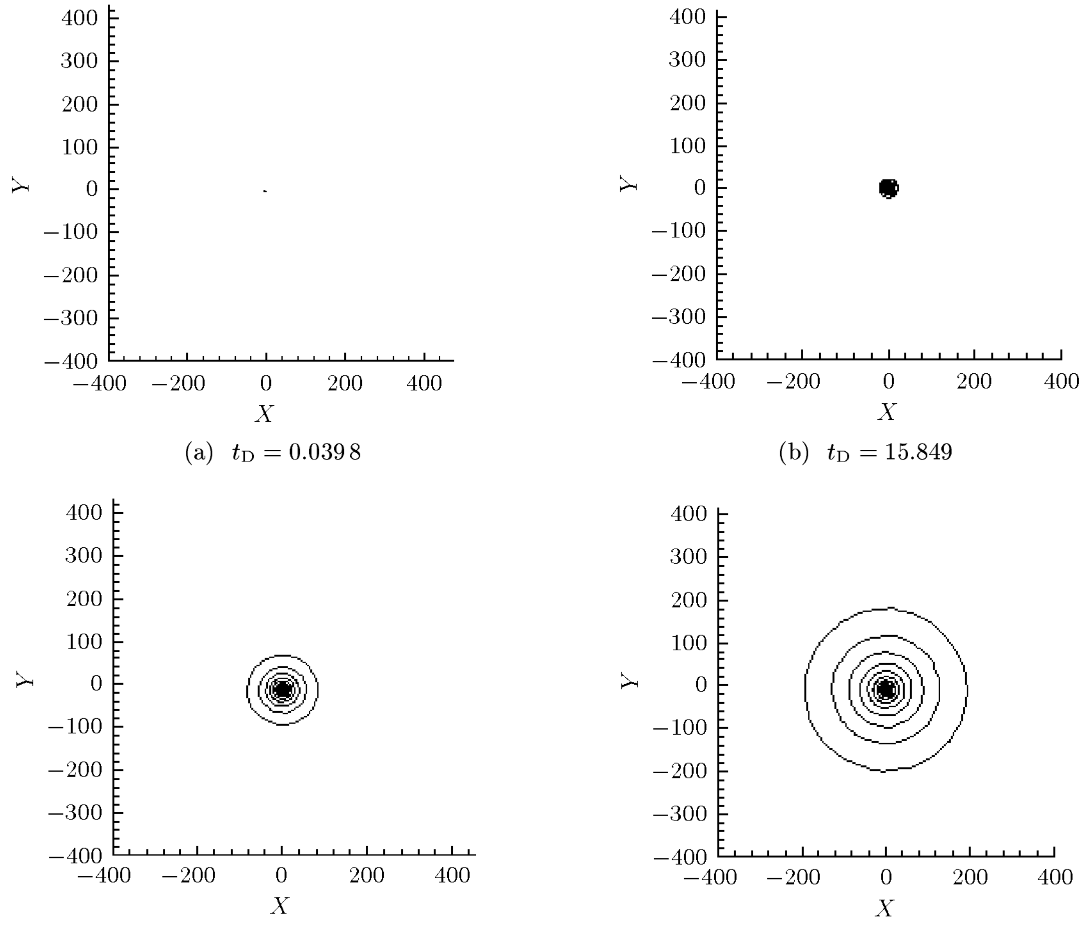


Fig.4 The pressure diffusing process for the well located in the center of the reservoir

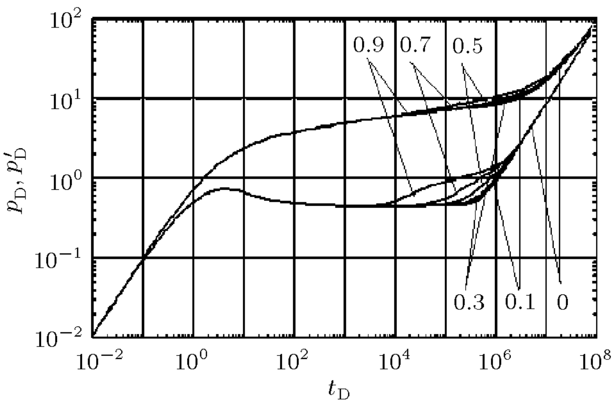
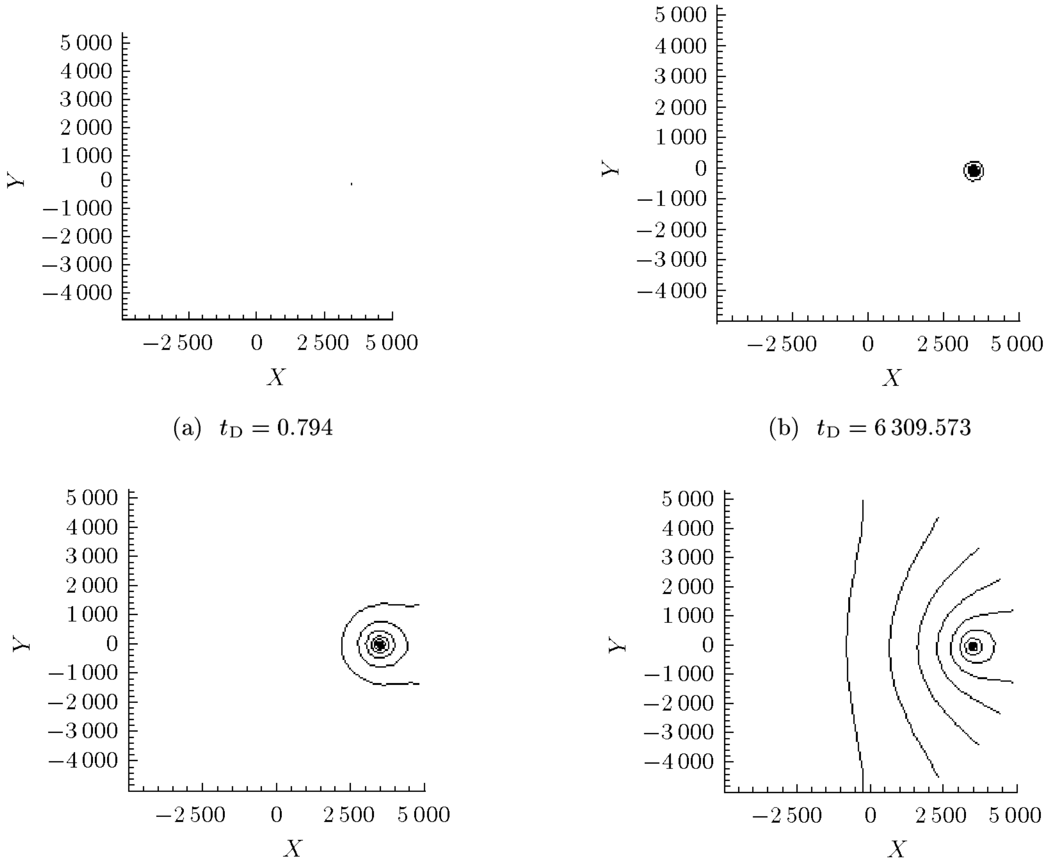


Fig.5 The typical curves of the wellbore pressure with the different well-location factors ($\gamma_{\mathrm{wp}} = 0.9$, 0.7, 0.5, 0.3, 0.1, 0)

When $\gamma_{\mathrm{wp}} = 0.7$, the pressure diffusion process of the off-center well in the closed circle reservoir are shown in Fig.6.

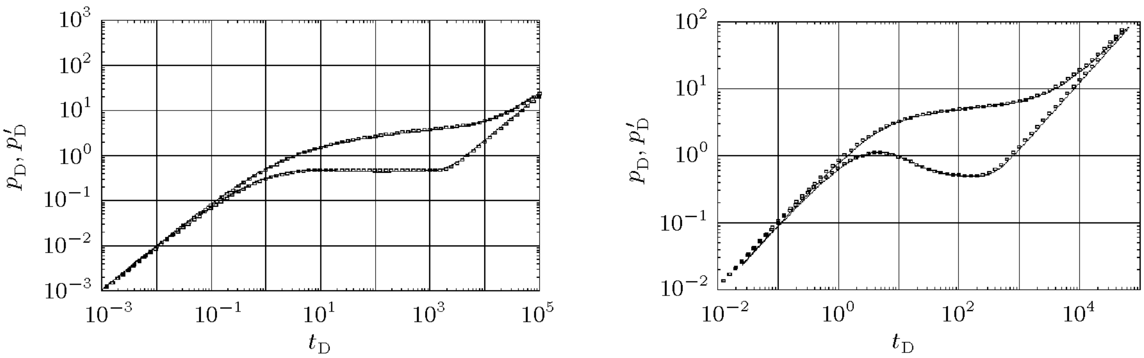
In order to test the validity of our numerical method and the accuracy of the numerical results, we compare our numerical results for the well in the center of the closed circle reservoir with the analytical solutions given in Ref.[3]. The compared results are shown in Fig.7.



(a) $t_\mathrm{D} = 0.794$ (b) $t_\mathrm{D} = 6\,309.573$

(c) $t_\mathrm{D} = 125\,892.541$ (d) $t_\mathrm{D} = 10^9$

Fig.6 The pressure diffusion process of the off-center well in the closed circle reservoir ($\gamma_\mathrm{wp} = 0.7$)



(a) $C_\mathrm{D}\mathrm{e}^{2S} = 1$ (b) $C_\mathrm{D}\mathrm{e}^{2S} = 100$

Fig.7 The Comparison of the numerical results with the analytical solutions (where the lines indicate the analytical solutions and the points indicate the numerical results

4 CONCLUSIONS

In this paper, we solved the transient flow problem in the closed circle reservoirs by using numerical method, especially for the well located in an arbitrary position in the closed circle reservoir. The pressure diffusion processes of the well at the center and at an arbitrary position in the closed circle reservoir are visualized. The well-location factor concept which can describe the geometry characters of the well and the reservoir is first proposed in this paper. The typical curves of the wellbore pressure for the well with different well-location factors are presented. By comparing numerical results of the well located in the center of the closed circle reservoir with the analytical solutions, our numerical method is verified.

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