

## **The Effect of Nucleation Time on the Growth of a Microvoid in a Viscoelastic Material**

J.K. Chen<sup>1</sup>, Z.P. Huang<sup>1</sup> and S.L. Bai<sup>2</sup>

<sup>1</sup>LNM, Institute of Mechanics, Chinese Academy of Sciences,  
Beijing 100080, China P.R.

<sup>2</sup>Department of Mechanics and Engineering Sciences,  
Peking University, Beijing 100871, China P.R.

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### **ABSTRACT**

In this paper, discussions are focused on the growth of a nucleated void in a viscoelastic material. The in situ tensile tests of specimens made of high-density polyethylene, filled with spherical glass beads (HDPE/GB) are carried out under SEM. The experimental result indicates that the microvoid nucleation is induced by the partially interfacial debonding of particles. By means of the Laplace transform and the Eshelby's equivalent inclusion method, a new analytical expression of the void strain at different nucleation times is derived. It can be seen that the strain of the nucleated void depends not only on the remote strain history, but also on the nucleation time. This expression is also illustrated by numerical examples, and is found to be of great usefulness in the study of damage evolution in viscoelastic materials.

### **1 INTRODUCTION**

It has been found that one of the controlling failure mechanisms in a particulate-reinforced composite material is the nucleation, growth and eventual coalescence of microvoids [1,2]. If the strength of particles is sufficiently high, the microvoids' nucleation may take place at the interface between particles and the matrix [3,4]. In the condition of low stress triaxiality, the microvoids' nucleation occurs due to the partially interfacial debonding of particles [5]. In this paper, discussions are focused on the growth of a nucleated void in a viscoelastic material. Firstly, in situ tensile tests of the specimen made of high-density polyethylene filled with glass beads are performed under SEM. The experiment shows that the microvoid nucleation takes place due to the partially debonding of the interface between particles and the matrix. Then, the growth of a nucleated void embedded in an infinite viscoelastic material is theoretically studied. The expression of the void strain is derived, and it can be seen that the strain of this nucleated void

depends not only on the remote strain history, but also on the nucleation time. Finally, the influences of the remote strain history and the nucleation time on the void strain are discussed by the numerical examples.

## 2 THE IN SITU TEST OF HDPE/GB UNDER SEM

The test specimen is made of the high-density polyethylene (HDPE) filled with glass beads. In the manufacturing process, the glass beads are treated with coupling agents and then mixed with HDPE powder in a chamber of a high speed mixer and a twin-screw extruder. The pelleted extruder is injection molded into the tensile specimen. The geometry of the specimen is shown in Fig.1.

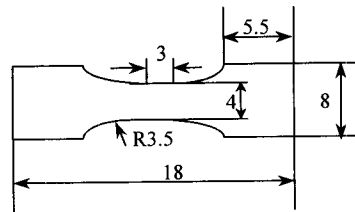


Fig.1. Dimension of in situ test specimens with thickness of 3 mm.

The in situ tensile tests are performed under SEM with the loading rate of about 0.2mm/min. The SEM photographs of the microvoid nucleation and growth are shown in Fig.2.

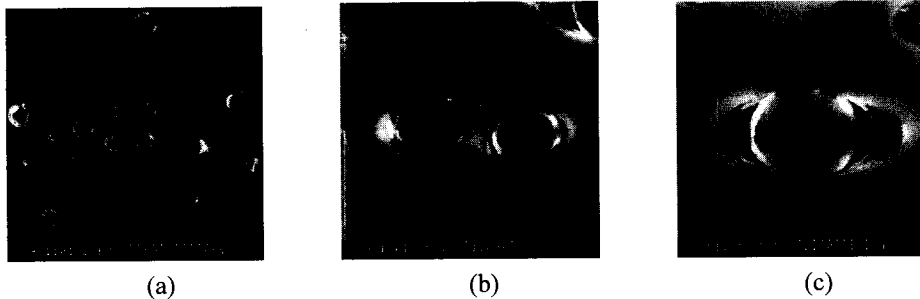


Fig.2. SEM photographs of microvoids' nucleation and growth  
 (a): microvoids' nucleation under the remote stress  $\sigma = 5\text{MPa}$  ;  
 (b),(c): microvoids' growth under the remote stress  $\sigma = 15.8\text{MPa}$

It can be seen that the microvoids' nucleation takes place due to the partially interfacial debonding. Because the glass beads can hardly deform, they will be considered as rigid particles. Hence, it may be assumed from the above experimental observation that under the axial symmetric loading condition, the component of the void stress along the tensile direction and the components of the void strain perpendicular to the tensile direction are equal to zero, i.e.,

$$\sigma_{v11} = 0, \quad \varepsilon_{v22} = \varepsilon_{v33} = 0 \quad (1)$$

### 3 THE DEPENDENCE OF THE VOID STRAIN ON THE NUCLEATION TIME

Now consider a spherical rigid inclusion embedded in an infinite viscoelastic matrix. The constitutive relation of the matrix material is given in the form of Stieltjes' convolution as follows:

$$\sigma_o = \mathbf{L} * d \epsilon_o, \text{ (or) } \epsilon_o = \mathbf{J} * d \sigma_o \quad (2a,b)$$

where  $\mathbf{L}(t)$  and  $\mathbf{J}(t)$  are the fourth order relaxation modulus and creep compliance, respectively.

The Lapalce transform of Eq.(2a) may be written as

$$\overline{\sigma}_o = s \overline{\mathbf{L}} : \overline{\epsilon}_o \quad (3)$$

where the symbols “—” and “s” denote the Laplace transform and the transform variable respectively.

Suppose a void is nucleated by the partial debonding at the interface between the rigid inclusion and the matrix at time  $t'$ . We are interested in calculating the strain of this nucleated void at time  $t$  ( $\geq t'$ ). For simplicity, we assume that the Poisson's ratio of the matrix material can be taken as a constant. Since the rigid inclusion (GB) is spherical, we further assume that the shape of the void is spheroidal.

The void strain is graphically sketched in Fig.3, and may be regarded as the superposition of two sub-problems shown in Fig.4 and Fig.5.

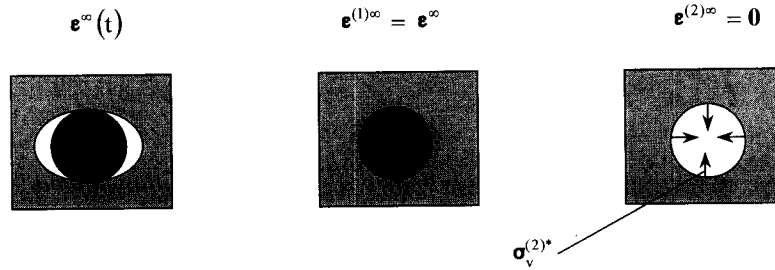


Fig.3. The growth of a nucleated void in an infinite matrix

Fig.4. Sub-problem (1) a rigid inclusion in the same matrix material

Fig.5. Sub-problem (2) a void with eigen-stress in the same matrix material

In sub-problem (1), a spherical rigid inclusion is embedded in a viscoelastic matrix material subjected to a remote strain history, which is identical with  $\epsilon^\infty(t)$  in the original problem. The shape and the size of this rigid inclusion are also the same as those in Fig.3. The interface is supposed to be well bonded during the deformation process, and the stress of the inclusion will be

denoted by  $\sigma_p^{(1)}$ . According to the Eshelby's equivalent inclusion method, the stress in the inclusion in sub-problem (1) may be obtained and expressed by:

$$\sigma_p^{(1)} = \int_{-\infty}^t \mathbf{L}(t-\tau) : \mathbf{S}^{-1} : \frac{d}{d\tau} \boldsymbol{\varepsilon}^{\infty}(\tau) d\tau \quad (4)$$

where  $\mathbf{S}$  is the Eshelby's tensor.

In sub-problem (2), a void is embedded in the same material, but the remote strain is zero. The shape and the size of the void at time  $t < t'$  are the same as those of the rigid inclusion shown in Fig.4. However, the void strain at time  $t$  ( $t \geq t'$ ) in Fig.3 is equal to the one in Fig.5.

The void shown in Fig.5 may also be regarded as an inclusion with the eigenstress  $\sigma_v^{*(2)}$ . The corresponding stress of this inclusion can be transformed to give

$$\bar{\sigma}_v^{(2)} = s\bar{\mathbf{L}}_v : \bar{\boldsymbol{\varepsilon}}_v + \bar{\sigma}_v^{*(2)} \quad (5)$$

where  $\mathbf{L}_v$  denotes the elastic modulus of the void. Now let us introduce  $\bar{\boldsymbol{\varepsilon}}^*$  such that

$s\bar{\mathbf{L}}_v : \bar{\boldsymbol{\varepsilon}}_v = s\bar{\mathbf{L}} : (\bar{\boldsymbol{\varepsilon}}_v - \bar{\boldsymbol{\varepsilon}}^*)$ . Then Eq.(5) may be written in the form:

$$\bar{\sigma}_v^{(2)} = s\bar{\mathbf{L}} : (\bar{\boldsymbol{\varepsilon}}_v - \bar{\boldsymbol{\varepsilon}}^*) + \bar{\sigma}_v^{*(2)} = s\bar{\mathbf{L}} : (\bar{\boldsymbol{\varepsilon}}_v - \bar{\boldsymbol{\varepsilon}}^* - \bar{\boldsymbol{\varepsilon}}_v^{*(2)})$$

where

$$\bar{\boldsymbol{\varepsilon}}_v^{*(2)} = -s\bar{\mathbf{J}} : \bar{\sigma}_v^{*(2)} \quad (6)$$

Noting that the above inclusion in Fig.3 is actually a void, we have

$$\bar{\mathbf{L}}_v = \mathbf{0}, \quad \bar{\boldsymbol{\varepsilon}}_v = \bar{\boldsymbol{\varepsilon}}^* \quad (7)$$

Hence, by virtue of the Eshelby's equivalent inclusion method, the transform of the void strain may be written as

$$\bar{\boldsymbol{\varepsilon}}_v = \mathbf{S} : (\bar{\boldsymbol{\varepsilon}}^* + \bar{\boldsymbol{\varepsilon}}_v^{*(2)}) \quad (8)$$

Substitution of Eq.(6) and Eq.(7) into the above equation yields:

$$\bar{\boldsymbol{\varepsilon}}_v = -(\mathbf{I} - \mathbf{S})^{-1} : \mathbf{S} : s\bar{\mathbf{J}} : \bar{\sigma}_v^{*(2)} \quad (9)$$

Thus, the void strain, as the inverse transform of Eq.(9), may be expressed by

$$\mathbf{e}_v = -(\mathbf{I} - \mathbf{S})^{-1} : \mathbf{S} : \int_{-\infty}^t \mathbf{J}(t - \tau) : \frac{d}{d\tau} [\boldsymbol{\sigma}_v^{(2)*}(\tau)] d\tau \quad (10)$$

In the above equation,  $\boldsymbol{\sigma}_v^{(2)*}$  should be determined from the condition of Eq.(1):

$$\begin{aligned} \sigma_{v11}^{(2)*} &= -\sigma_{p11}^{(1)} u(t - t') \\ \sigma_{v22}^{(2)*} &= \sigma_{v33}^{(2)*} = -A_2 \sigma_{p11}^{(1)} u(t - t') \end{aligned} \quad (11)$$

where  $u(t)$  is a step function, and  $A_2$  is obtained from Eqs.(1) and (10). The analytical expression of  $A_2$  will not be given here owing to the limitation of space. However, it can be seen that the void strain is a function of the remote strain history and the nucleation time  $t'$ .

It should be pointed out that Eq.(10) can also be used to calculate the void strain in the matrix material, which contains a large number of rigid inclusions and nucleated microvoids, if the interaction between these inclusions and microvoids can not be neglected. Several averaging schemes, such as the Mori-Tanaka Scheme and generalized self-consistent Mori-Tanaka scheme (e.g. cf. [7]), may be employed to take into account this interaction if these inclusions and microvoids are randomly distributed. For example, in the Mori-Tanaka Scheme, the remote strain  $\mathbf{e}^\infty$  in Eq.(4) should be replaced by the average strain of the matrix material  $\langle \mathbf{e}_0 \rangle$ .

#### 4 NUMERICAL EXAMPLES AND CONCLUSIONS

Now consider an infinitely extended viscoelastic material that contains a rigid spherical inclusion, and is subjected to the applied strain history at infinity. The constitutive relation of the matrix material is assumed to be the Maxwell model with the relaxation modulus:

$$E(t) = E_0 \exp(-t/t_m) \quad (12)$$

where  $t_m$  is the relaxation time of the matrix material.

Based on Eq.(10), the void strain under a constant remote strain rate is calculated numerically, and is plotted in Fig.6 and Fig.7, in which  $(\epsilon_{v11})^{\text{elastic}}$  denote the components of the void strains in the tensile direction when the relaxation time tends to infinite. It can be seen from these figures that the dependence of the void strain on the nucleation time is quite clear.

The influence of the remote strain history on the void growth is also studied. Two kinds of remote strain histories will be considered: 1) the constant strain rate,  $\epsilon_{11}^\infty = a_1 t$  ( $a_1 = 1/40(1/s)$ ), all other components  $\epsilon_{ij}^\infty = 0$ ; 2) the accelerated strain rate,  $\epsilon_{11}^\infty = a_2 t^2 / 2$  ( $a_2 = 1/20(1/s^2)$ ), all other components  $\epsilon_{ij}^\infty = 0$ . The void strains corresponding to the above two remote strain histories,

(but with the same remote strain at time  $t=1$  s), are compared and shown in Fig.7.

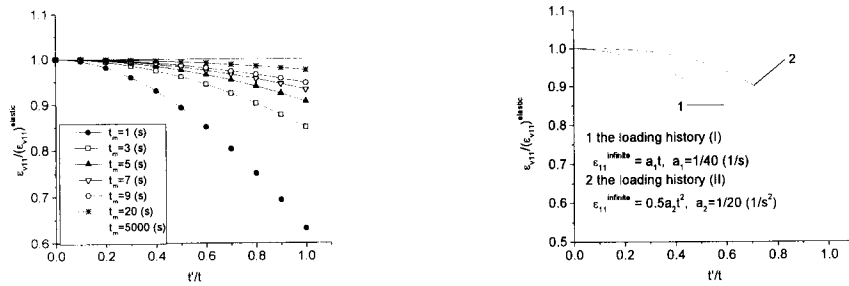


Fig.6. The relations between the component of the void strain  $\varepsilon_{v11}$  and the nucleation time  $t'$  for different relaxation times

Fig.7. The influence of loading history on the component of void strain  $\varepsilon_{v11}$

It may be concluded from the above calculations that:

(1) The void strain, shown in Fig.6, is a function of the nucleation time  $t'$ . In the case that  $t'=0$ , the void strain calculated from Eq.(10) will be the same as  $(\varepsilon_{v11})^{\text{elastic}}$ , namely,  $\varepsilon_{v11}/(\varepsilon_{v11})^{\text{elastic}} = 1$ .

However, for a monotonically increasing remote strain history, the void strain will be a monotonically decreasing function of the nucleation time.

(2) The void strain depends also on the remote strain history. The influence of the remote strain history on the void growth is shown in Fig.7. For the same remote strain at time  $t$ , the void strains at time  $t$  for different remote strain histories may be different.

(3) The dependence of void strain on the relaxation time is shown in Fig.6. It can be seen that the void strain is a monotonically increasing function of the relaxation time. As the relaxation time tends to infinity, the matrix material becomes an elastic one, and the void strain depends on the remote strain only.

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+ to whom all correspondences should be addressed

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