Transition to Chaos in the Floating Half Zone Convection *

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The transition process from steady convection to chaos is experimentally studied in thermocapillary convections of floating half zone. The onset of temperature oscillations in the liquid bridge of floating half zone and further transitions of the temporal convective behaviour are detected by measuring the temperature in the liquid bridge. The fast Fourier transform reveals the frequency and amplitude characteristics of the flow transition. The experimental results indicate the existence of a sequence of period-doubling bifurcations that culminate in chaos. The measured Feigenbaum numbers are $\delta_2 = 4.69$ and $\delta_4 = 4.6$, which are comparable with the theoretical asymptotic value $\delta = 4.669$.

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Transition to chaotic convection in dissipative dynamical systems is a subject of great interest because of its theoretical and practical applications in the fluid mechanics. The route to chaotic convection in natural thermal convection has been the subject of fundamental research.[1−5] It has been shown that there are many routes to the chaotic convection, depending on the typical parameters such as the Rayleigh number, Prandtl number and geometric aspect.

There is also another interesting convection mechanism besides buoyant convection. Thermocapillary convection is a typical subject of microgravity fluid mechanics and also supports new dissipative systems similar to Rayleigh–Benard convection. The flow driven by the gradient of surface tension is called the thermocapillary convection. Thermocapillary convection has received increasing attention because of its relevance in the containerless method of floating-zone crystal growth and material science. Thermocapillary convection has been studied numerically and experimentally under normal gravity and microgravity. There have been many works of floating-zone convection focused on onset of oscillation in which the steady convection transits to oscillatory convection. The onset of oscillation is only the initial period of transition to chaos. A complete route to chaos of the new sort of dissipative system is attractive in the theoretical aspect and in the application aspect of space materials science. The transition process of thermocapillary convection will be great interesting. A route involving a series of successive period doubling bifurcations to chaotic convection has been given by the method of numerical simulation for two-dimensional and unsteady mode of the floating half zone in Ref. [6].

The half-zone liquid bridge model is a typical model to study thermocapillary convection. In some ground-based studies, small scales of the liquid bridge usually are adopted to reduce the buoyancy effect as compared to the thermocapillary convection. In our present experiment, the onset of temperature oscillations in the liquid bridge of diameter $d_0 = 4\,\mathrm{mm}$ and further transitions of the temporal convective behaviour are detected by measuring temperatures. A route to chaotic convection is presented and focused on the bifurcation feature during the transition process. Period doubling bifurcation is obtained by the real time analysis of the spectra, which is accomplished by Fourier transforms. The Feigenbaum constants are experimentally obtained to be close to the theoretical value.

A liquid bridge of 10cst silicone oil was floated in the gap between two coaxial rods with the same diameter $d_0 = 4\,\mathrm{mm}$. A temperature difference was applied between the upper and lower rods, and the cases of the upper rod heated are analysed. The temperatures were measured by thermocouples at both sides of the rods. Two PID-controllers (EUROTHERM 904 controller) were used to control the heating rate and the temperature difference between the upper and lower rods. A thermocouple in diameter 0.02 mm was inserted into the liquid medium to measure the temperature. This thermocouple was located at about $0.2l$ from the upper rod and just beneath the free surface (l is the length of liquid bridge). Another two thermocouples were fixed at the both rods. The temperature signals were recorded simultaneously by a PC-supported data acquisition system composed of KEITHLEY 2182 and KEITHLEY 2000. KEITHLEY 2182 was used to acquire the temperature inside the liquid bridge. The temperature resolution of KEITHLEY 2182 is 0.001°C. KEITHLEY 2000, which is a multimeter, was used to acquire the temperatures of the upper and lower rods. The temperature resolution of KEITHLEY 2000 is 0.01°C. The temperature of the heater was controlled by PID-controllers up to an accuracy better than ±0.05°C. The temperature

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difference ΔT between the rods has been set with an accuracy better than $\pm 0.05^\circ$C and has been measured with the thermocouples. A small-scale liquid bridge is an interesting system to study transitions to chaos. The smallness of the fluid volume involves rather higher frequencies of oscillatory flow. The oscillation frequency f is of the order of 1 Hz and therefore allows fast data- sampling and a relatively short experiment duration. Scanning rates of $R = 10$ Hz and the number of sampling points of $n = 512$ or 1024 were employed. Times series of temperatures were transformed in the frequency domain by fast Fourier transformation (FFT) in real time. In the experiment, the thermocouple data versus time are acquired a computer, and the temperature spectra could be observed on the computer monitor online or at any time. The liquid bridge, temperature controller and data acquisition system used in the experiments are schematically shown in Fig. 1

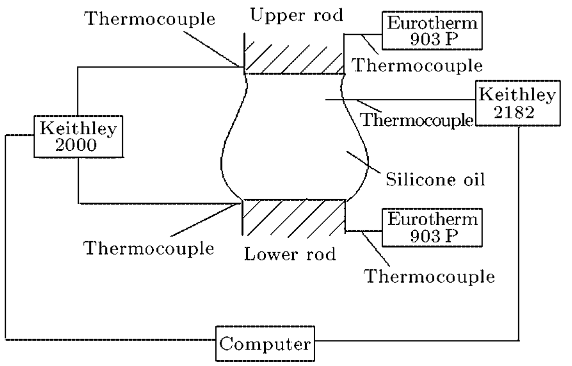

Fig. 1. Schematic diagram of the experimental configuration.

The geometrical parameters, such as the diameter d_0 of two rods, aspect ratio $A = l/d_0$ and volume ratio of liquid bridge, are important for the onset of oscillation and transition process. The volume ratio of liquid bridge can also be described by the relative radius ratio d_m/d_0 with d_m being the minimum diameter of the liquid bridge. The liquid bridge with $d_0 = 4$ mm, $A = l/d_0 = 0.96$ and $d_m/d_0 = 0.9$ was adopted in our experiment. The applied temperature difference ΔT between two ends of the liquid bridge increases slowly with time. It is necessary to find the accurate moment corresponding to the onset of oscillation and the appearance of a subharmonic bifurcation. When the subharmonic bifurcation appears, the temperature ramp profile can be stopped and temperature difference between the rods holds constant. The typical heating rates are 0.1°C/s and 0.01°C/s and the typical holding time between two steps is 5–8 min in the present experiments. At the initial period, the heating rates are 0.1°C/s. The critical applied temperature difference $(\Delta T)_{\rm c}$, which describes the onset from the symmetric and steady convection to periodic and asymmetric oscillatory convection, can be detected in the process of increasing the applied temperature difference by the analysis of time series of temperatures acquired by thermocouples positioned at one point in the liquid bridge. The onset of oscillation is observed. Then the applied temperature difference is further increased, but very slow heating rate is applied, the subharmonic bifurcations successively appear as the applied temperature difference increases. In this process, the applied temperature differences are slowly increased step by step (each step $\delta T = 0.1^\circ$C, heating rate is 0.01°C/s), and each ΔT is maintained for 5–8 min corresponding to acquired 3000–4800 temperature signals at one point inside the liquid bridge and the applied temperature difference data in each time interval in the present experiment. The precise control of applied temperature differences and high temperature resolutions of KEITHLEY 2182 and KEITHLEY 2000 make it possible to find the accurate moment corresponding to the onset of a subharmonic bifurcation. The temperatures at the point inside the liquid bridge were recorded and the Fourier spectrums were analysed in real time at each step. Hence, we can find the accurate instant of the start of the subharmonic bifurcation, such as the frequency locking at $f/2$, $f/4$, $f/8$, $f/16$. Figure 2–6 show a series of temperature signals with oscillations measured at a distance of (0.8 ± 0.1) mm below the upper rod beneath the free surface. Panels (b) in Figs. 2–6 show the corresponding Fourier spectra of (a). Panels (c) in Figs. 2–6 show the applied temperature difference across the liquid bridge.

In the experiment, the temperature fluctuation appears in Fig. 2(a) as the applied temperature difference increases. Figure 2(c) show the applied temperature difference versus time and we can find $\Delta T = 36.55 \pm 0.05^\circ$C. Simultaneously a peak appears in spectrum of Fig. 2(b). A harmonic oscillation with the main-peak frequency $f_0 = 0.58$ Hz is given by the spectrum analysis of Fourier transformation at the applied temperature difference $(\Delta T)_c = 36.55^\circ$C. This means that an oscillatory convection with a single frequency appears in the liquid bridge.

The main-peak frequency increases slightly with the further increase of the applied temperature difference. When $\Delta T = 48.01 \pm 0.05^\circ$C, one more peak of $f_0/2$ ($f_0 = 0.67$ Hz) appears in the spectrum as shown in Fig. 3(b). The fractal frequency is locked at $f_0/2$ at $\Delta T = 48.01^\circ$C. When $\Delta T = 61.65 \pm 0.05^\circ$C, one more peak of $f_0/4$ ($f_0 = 0.80$ Hz) appears in the spectrum shown in Fig. 4(b). The fractal frequencies are locked at $n/4$ ($n = 1, 2, 3$) of the main-peak frequency when $\Delta T = 61.65^\circ$C. For $\Delta T = 64.56 \pm 0.05^\circ$C, one more peak of $f_0/8$ ($f_0 = 0.89$ Hz) appears in the spectrum shown in Fig. 5(b). For $\Delta T = 65.15 \pm 0.05^\circ$C, one more peak of $f_0/16$ ($f_0 = 0.89$ Hz) appears in the spectrum shown in Fig. 6(b). The liquid bridge is broken for higher values of ΔT.

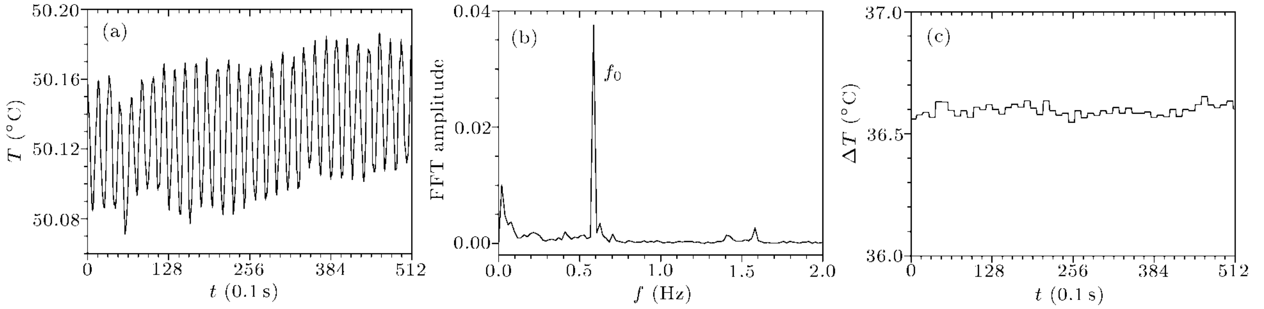

Fig. 2. Evolution of temperature (a) and the corresponding spectrum (b) at one point in the liquid bridge, and applied temperature difference (c) for onset of oscillation.

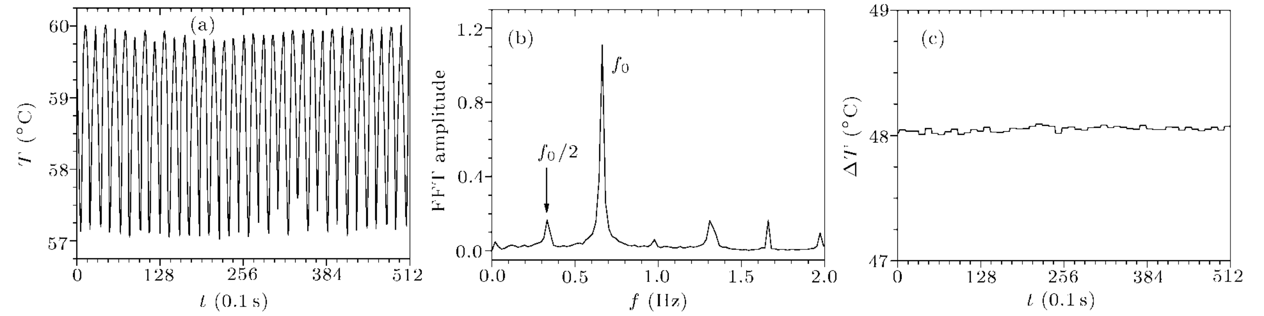

Fig. 3. Evolution of temperature (a) and the corresponding spectrum (b) at one point in the liquid bridge, and applied temperature difference (c). Here $f_0/2$ appears.

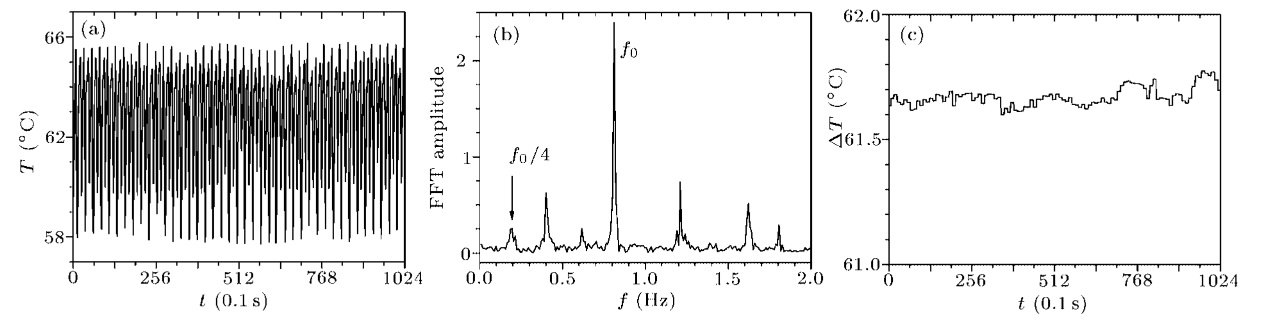

Fig. 4. Evolution of temperature (a) and the corresponding spectrum (b) at one point in the liquid bridge, and applied temperature difference (c). Here $f_0/4$ appears.

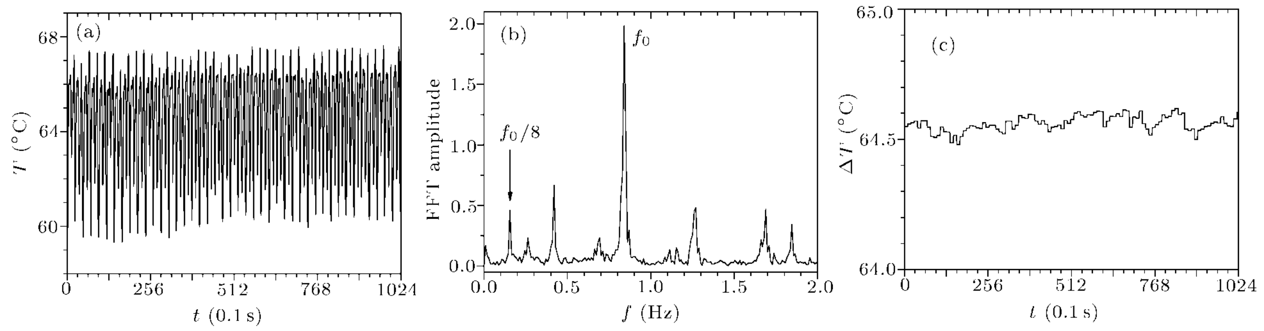

Fig. 5. Evolution of temperature (a) and the corresponding spectrum (b) at one point in the liquid bridge, and applied temperature difference (c). Here $f_0/8$ appear.

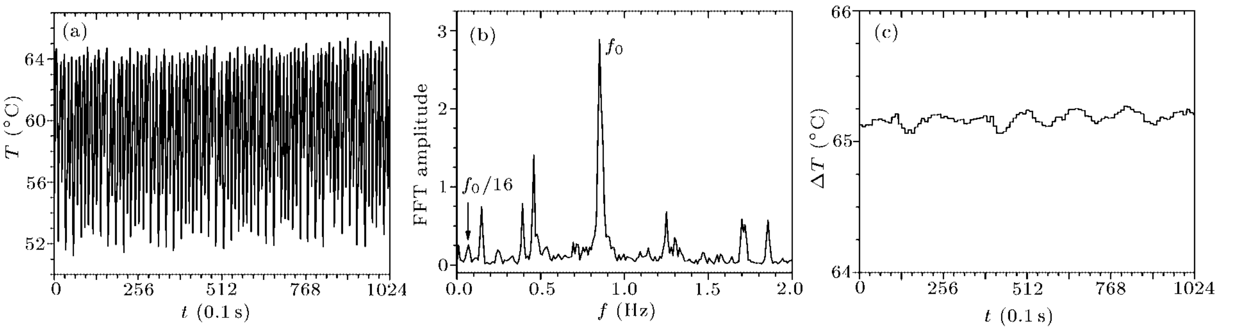

Fig. 6. Evolution of temperature (a) and the corresponding spectrum (b) at one point in the liquid bridge, and applied temperature difference (c). Here $f_0/16$ appears.

From Feigenbaum's general theory for the fluctuation spectrum of the route to chaos, the constraint a_n should asymptotically approach the relation

$$\delta_n = (a_{n+1} - a_n)/(a_{n+2} - a_{n+1}) \rightarrow \delta,$$

where $\delta = 4.6642016$. If we compute the Feigenbaum number δ for the last three bifurcations, we can obtain

$$\begin{aligned}\delta_2 &= (f_4 - f_2)/(f_8 - f_4)\\ &= (61.65 - 48.01)/(64.56 - 61.65) = 4.69 \pm 0.05,\\ \delta_4 &= (f_8 - f_4)/(f_{16} - f_8)\\ &= (64.56 - 61.65)/(65.19 - 64.56) = 4.6 \pm 0.1.\end{aligned}$$

The present results are close to the value of universal constant $\delta = 4.6642016$ given by Feigenbaum's general theory.[1]

The floating half zone on the earth is a special type of mechanical dissipative systems, in which the convection is a mixture of the thermocapillary convection driven by the non-uniformity of surface tension and the Benard convection driven by the buoyancy. Since we have used small typical scale liquid bridge in our experiment, the thermocapillary convection is a dominant one. Early in the liquid bridge experiments, most interests concentrate on the onset of oscillation, which concern only the start of the route to chaos. There are many routes to chaos that depend on the Prandtl number, Rayleigh number and geometric aspect of the floating half zone, and a fixed route will be determined for fixed parameters. In the present study, we have used a 10cst silicone oil with $d_0 = 4\,\text{mm}$, $A = l/d_0 = 0.96$ and $d_m/d_0 = 0.9$ and obtained a route from steady convection to chaos passing through the $f_0/2$, $f_0/4$, $f_0/8$, and $f_0/16$ period doubling bifurcations. The Feigenbaum constant obtained in our experiment is very close to the theoretical value. This means that the route of subharmonic bifurcations is typical in the transition process of thermocapillary convection. The results of the subharmonic bifurcations support the idea that the oscillation in liquid bridges of floating zone is induced by internal instability.

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