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Numerical analysis on intermittent flow in a longitudinal section of Bingham fluid along a slope

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Abstract **The transition process of intermittent flow in a longitudinal section of Bingham fluid from initial distribution to fully developed state was numerically investigated in this paper. The influences of slope ϑ, dimensionless runoff Q^* and viscosity μ_0^* on the dimensionless surge speed U^* were also presented in a wide range of parameters. By one typical example, the intermittent flow possessed wave characteristics and showed a supercritical flow conformation for a fully developed flow. The distributions of gravity and bed drag along the flow path and the velocity distribution of flow field were also analyzed.**

Keywords: Bingham fluid, debris flow, intermittent flow, surge speed, bed drag.

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Debris flow is frequently happening natural catastrophe. The composition of debris is very complicated and its composition changes from place to place in China. For example, the granule magnitude varies from micrometers to meters except water. Thus, the great difference in mechanical properties and motion characteristic is distinguished among granular and water compositions. For mud consisting of water and fine granule, the motion of each composition is almost the same. But the motion of coarse granules[1)] is quite different from pasty medium as well as for different granule size.

In studying debris flow, methodology is often referred to studying of sediment flow in hydraulics. But remarkable differences are distinguished between sediment and debris flows. (1) The mud in debris is non-Newtonian and possesses remarkable yield stress while water is Newtonian. (2) The sand granule in sediment flow nearly shares the same velocity with water and the concentration gradient in the flow direction can be neglected although the sand granule settles with a low velocity much less than water flow speed and there is a certain concentration gradient in the vertical direction. However in debris flow, especially for the intermittent flow, there are remarkable differences between the velocity of coarse granule and that of pasty mud and a considerable concentration gradient lies in both horizontal and vertical directions[1—3]. (3) Sediment flow, except the hyper-concentrated sediment flow, is almost continuous while viscous debris flow is mostly intermittent. Flow stoppage often happens between two surges[3,4]. In the turbulent head of a surge, the flow shows obviously two-dimensionality (the vertical velocity component has the same order of magnitude as the horizontal one).

It is impossible at the initial stage of debris flow research that all of the complicated factors are included since the debris phenomenon is too complicated. At that time, only the first difference mentioned above was taken into account.

Single-fluid model has been extensively used for a long time[3—8] in which only one fluid (the mixture consisting of mud and coarse granule) is included and the motion of the mixture is studied. The diffusion (i.e. the relative motions) between various components is neglected.

Because of collisions among coarse granules, each of them possesses random velocity in addition to the average velocity. The random velocity can induce additional stresses, i.e. normal and shear stresses[9,10]. These stresses have been included in the rheological relation in Bagnold's, Takahashi's and Chen's models[5,6,9]. It seems that these models belonged to two-fluid model because they had taken the motion difference between mud and coarse granules into account. Actually, that is not the case. These models really belong to single-fluid model because there is only one set of conservation equations in these models and the average velocity of coarse granules is the same as the velocity of mud.

If the single-fluid model is used, the composition of the mixture element cannot be changed during the movement. If the distribution of composition is homogeneous in initial, the homogeneous state remains unchanged no matter how accurate rheological relation has been used. Therefore, the single-fluid model cannot interpret the phenomenon in viscous debris flow that the coarse granules tend to concentrate to the turbulent head and the coarse granule concentration there is higher than that elsewhere.

The diffusion model and two-fluid model were used to study debris flow since the 1990s[1,2,11]. But the progress is not satisfactory because the problem is rather difficult.

For some special topics in debris flow, such as

1) In this paper, the motion of granules is always referred to the average motion of a great quantity of granules, not the motion of individual granule.

non-homogeneous distribution of granule concentration, the diffusion or two-fluid model must be used even for qualitative investigation. For other topics, such as intermittent flow impacting walls, only the qualitative results can be obtained using the single-fluid model because of the remarkable effects induced by non-homogeneous concentration distribution. But for the surge speed research, some quantitative results can be offered using the single-fluid model. They can be regarded as a first approximation if a further investigation in which the influences of velocity difference between various components and non-homogeneous distribution will be taken into account is needed. In comparison with the relative velocity of the components and the non-homogeneous distribution, the slope J and the bulk properties, such as mixture density ρ_m, viscosity μ_0 and yield stress τ_0, are the main factors affecting the surge speed U.

In analogy with hyper-concentrated sediment flow, the continuous flow of viscous debris flow is unstable[12,13], any small distribution on the free surface can be enlarged, so that viscous debris flow in most case moves intermittently and flow stoppage appears between one surge and another. Such flow differs greatly from continuous flows.

For most continuous flow, the free surface fluctuates weakly so that it works everywhere in the flow field that the vertical velocity v is much smaller than the horizontal u ($v \ll u$). Therefore, the averaged equations in transverse cross-section can be reasonable to describe the flow motion. But for intermittent flows, velocity components u and v at the turbulent head have the same order of magnitude and one-dimensional equations are not reasonable for approximation.

Material velocity $u(t, x, z)$ and its average $\overline{u}(t,x)$ are studied in continuous flow. However, what we are interested in for an intermittent flow is the surge speed $U(t)$ with which the flowing medium moves as a whole. It is noted that $U(t)$ is a wave speed and not material velocity in any sense of average. In fact, the average velocity $\overline{u}(t,x)$ at any cross section is less than $U(t)$.

The relation between average velocity $\overline{u}$ and water depth h (or between average velocity $\overline{u}$ and flow rate per unit width $\dot{Q}$ where $\dot{Q} = h\overline{u}$) is studied in continuous flows. But in the intermittent flow, U is not dependent on flow rate per unit width $\dot{Q}$ (its dimension is L^2T^{-1}) but on the runoff per unit width Q (its dimension is L^2). Thus, the function $\overline{u} = f_c(\dot{Q})$ is quite different from the function $U=f_i(Q)$ and we can not expect to get $U=f_i(Q)$ through correcting $\overline{u} = f_c(\dot{Q})$. Till now, there are few works on the function $U=f_i(Q)$ carried out although the function $\overline{u} = f_c(\dot{Q})$ has been studied extensively. Savage and Hutter[14] studied the motion process and the asymptotic state of granular flow on a rough incline. Huang and Garcia[7] studied the process of generating bed layer of Bingham fluid on a steep slope. The head height of a surge is decreased with reducing of runoff.

Because of the yield stress, a residual bed layer with thickness h_{cr} is generated along its path when an amount of fluid moves forward. Meanwhile, the fluid quantity is reduced with the marching fluid. If more fluid is imposed on the generated bed-layer, it moves continuously and the fluid quantity remains constant until it comes to a dry bed. The focus in this paper is to study the two-dimensional unsteady motion of fluid on an infinite generated bed-layer. It is noted that the total amount of fluid, Q, remains unchanged during its process. Its configuration and moving speed will be automatically regulated and a final fully developed state can be arrived if the slope is long enough.

For a developing intermittent flow, its configuration and moving speed change with time and they are also related to the initial distribution of Q. But for a fully developed intermittent flow, the configuration and moving speed will be steady and independent of time and initial conditions, so that the research possesses more scientific importance. In this paper, two-dimensional governing equations in a longitudinal section are used to describe the unsteady flow of Bingham fluid with a certain amount of fluid imposed on the generated bed-layer. The configuration was captured and the relation between U and Q was studied for a fully developed intermittent flow.

Just as the water flow is studied before we study sediment flow, the mudflow should be studied before we study debris flow with two-fluid model because mud is mainly component of debris. Generally, the rheology of mud is very close to Bingham fluid. Therefore, the investigation of this paper reflects the main character and rules of mudflow and offers a preparation for studying intermittent viscous debris flows. The results can also be considered as the first approximation for them when the equivalent viscosity and yield stress of the debris are set for Bingham fluid.

Three independent variables, such as dimensionless viscosity μ_0^* $(=\mu_0\rho^{1/2}g/\tau_0^{3/2})$, runoff per unit width Q^* $Q^*(=Q\rho^2g^2/\tau_0^2)$ and slope ϑ $(J=\sin\vartheta)$, are used to describe the fully developed intermittent flow of Bingham fluid. Where g is the gravity acceleration and ρ, μ_0 and τ_0 are the density, viscosity and yield stress of Bingham fluid respectively. The influence of these three parameters to dimensionless surge speed $U^*\left(=U\sqrt{\rho/\tau_0}\right)$ was discussed in this paper.

1 Governing equations, constitutive relations and auxiliary conditions

(i) Constitutive relations. If $u(=V_1)$ and $v(=V_2)$ are assumed as the two components of velocity $V_i(i=1, 2)$, the constitutive relation of a Bingham fluid can be expressed as[16]

$$\begin{cases} \tau_{ij} = 2\mu_0\left(1+\dfrac{q_0}{q}\right)e_{ij}, & \tau_a^2 > \tau_0^2, \\ e_{ij} = 0, & \tau_a^2 \geqslant \tau_0^2, \end{cases} \tag{1}$$

where τ_{ij} and e_{ij} are viscous stress and strain rate tensors respectively,

$$e_{ij} = \frac{1}{2}\left(\frac{\partial V_i}{\partial x_j}+\frac{\partial V_j}{\partial x_i}\right). \tag{2}$$

The definitions of τ_a, q and q_0 and their expansions in two-dimensional flow are as follows:

$$\tau_a = \left[\frac{1}{2}\tau_{ij}\tau_{ji}\right]^{1/2} = \left[\frac{1}{2}\tau_{xx}^2+\tau_{xy}^2+\frac{1}{2}\tau_{yy}^2\right]^{1/2}, \tag{3}$$

$$q = \left[2e_{ij}e_{ji}\right]^{1/2} = \left[2\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2\right]^{1/2}, \tag{4}$$

$$q_0 = \frac{\tau_0}{\mu_0}. \tag{5}$$

For a pure shear flow (where v=0 and u=$u(y)$), $e_{xx} = e_{yy}$ =0, $e_{xy} = e_{yx} = \frac{1}{2}\partial u/\partial y$, $q = \partial u/\partial y$, $\tau_{xx} = \tau_{yy} = 0$ and $\tau_a = \tau_{xy} = \tau_{yx}$ because $\partial u/\partial x = \partial v/\partial x = \partial v/\partial y = 0$. Then, eq. (1) can be simplified as

$$\begin{cases} \dfrac{\tau_{xy}}{\tau_0} = \dfrac{\tau_{yx}}{\tau_0} = \dfrac{q}{q_0}\left(1+\dfrac{q_0}{q}\right) = \dfrac{q}{q_0}+1, & \tau_{xy}^2 > \tau_0^2, \\ \dfrac{1}{q_0}\dfrac{\partial u}{\partial y} = \dfrac{q}{q_0} = 0, & \tau_{xy}^2 \leqslant \tau_0^2. \end{cases} \tag{6}$$

Introducing a function $f(q/q_0)$ as the constitutive relation of a Bingham fluid, eqs. (1) and (6) can be approximated as

$$\tau_{ij} = 2\mu_0 f(q/q_0)e_{ij} \approx 2\mu_0\left\{1+\frac{q_0}{q}\left[1-\exp\left(-K\frac{q}{q_0}\right)\right]\right\}e_{ij}, \tag{1$'$}$$

$$\tau_{xy}/\tau_0 = (q/q_0)f(q/q_0) \approx \frac{q}{q_0}\left\{1+\frac{q_0}{q}\left[1-\exp\left(-K\frac{q}{q_0}\right)\right]\right\}, \tag{6$'$}$$

where K is a specified positive constant. Fig. 1 presents the constitutive relation in a pure shear flow. It is noted that the exponential terms on the right hand side of eqs. (1)′ and (6)′ are introduced to avoid the multi-value of function $(q/q_0)f(q/q)$ at $q = 0$. Obviously, the larger the value of K, the better approximation to the Bingham fluid can be made. In our computation, K is set as 1000.

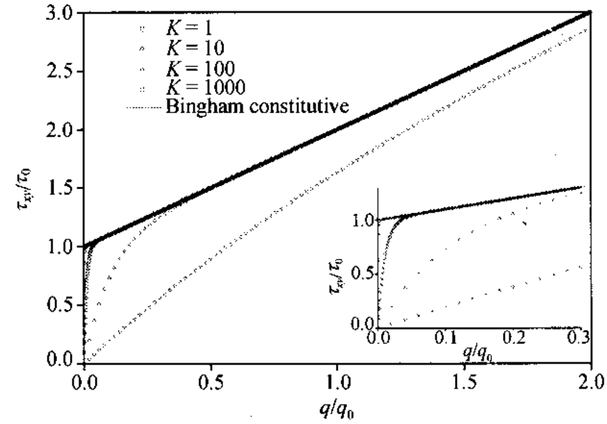

Fig. 1. Constitutive relation for Bingham fluid at different K.

(ii) Governing equations and its dimensionless parameters. In Cartesian coordinates, the two-dimensional incompressible equations for Bingham fluid can be expressed as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0, \tag{7}$$

$$\frac{\partial u}{\partial t}+\frac{\partial(uu)}{\partial x}+\frac{\partial(vu)}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu_0}{\rho}\frac{\partial}{\partial x}\left(2f\frac{\partial u}{\partial x}\right) + \frac{\mu_0}{\rho}\frac{\partial}{\partial y}\left[f\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+g\sin\vartheta, \tag{8}$$

$$\frac{\partial v}{\partial t}+\frac{\partial(uv)}{\partial x}+\frac{\partial(vv)}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu_0}{\rho}\frac{\partial}{\partial x}\left[f\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right] + \frac{\mu_0}{\rho}\frac{\partial}{\partial y}\left(2f\frac{\partial v}{\partial y}\right)-g\cos\vartheta, \tag{9}$$

where p is pressure. Taking ρ, g, τ_0, $\tau_0^{1/2}\rho^{-1/2}$, $g^{-1}\tau_0\rho^{-1}$, $g^{-1}\tau_0^{1/2}\rho^{-1/2}$ and $g^{-1}\tau_0^{3/2}\rho^{-1/2}$ as reference density, acceleration, pressure, velocity, length, time and viscosity respectively, the following dimensionless parameters with superscript asterisk are introduced:

$$\tau_{ij}^* = \tau_{ij}/\tau_0, \quad p^* = (p-p_0)/\tau_0,$$

$$u^* = u\sqrt{\rho/\tau_0}, \quad v^* = v\sqrt{\rho/\tau_0}, \quad U^* = U\sqrt{\rho/\tau_0},$$

$$x^* = x\rho g/\tau_0, \quad y^* = y\rho g/\tau_0,$$

$$h^* = h\rho g / \tau_0, \quad h_{cr}^* = h_{cr}\rho g / \tau_0 = 1/\sin\vartheta, \qquad (10)$$

$$Q^* = Q\rho^2 g^2 / \tau_0^2, \qquad (11)$$

$$t^* = t\, g\sqrt{\rho / \tau_0},$$

$$\mu_0^* = \mu_0 \rho^{1/2} g / \tau_0^{3/2} = (g\sqrt{\rho/\tau_0})/q_0 = (q_0^*)^{-1}, \quad (12)$$

where $h = h(t, x)$ represents the free surface of the intermittent flow. The reference length is related to the critical thickness of the residual layer $h_{\text{cr}} = (= \tau_0 / \rho g \sin\vartheta)$ (equal to the critical thickness at a vertical wall). Eqs. (7)—(9) and (1)′ can also be used as the dimensionless equations and constitutive relation when ρ and g are set as unit and q_0 is replaced by $(\mu_0^*)^{-1}$. From eqs. (8) and (9) the reciprocal of μ_0^* corresponds to the Reynolds number defined by the reference velocity and length mentioned above. If the surge speed U and square of the runoff are used as reference velocity and length, then Reynolds number is defined as

$$\text{Re} = \frac{\rho U Q^{1/2}}{\mu_0} = \frac{U^*(Q^*)^{1/2}}{\mu_0^*}. \qquad (13)$$

For a fully developed intermittent flow, the influence of initial conditions and boundary conditions at inlet and exit can be neglected. However, what we studied numerically was the process from initial time to the fully developed state and a limited computational domain is used, so a certain initial condition and the boundary conditions on all sides are needed.

(iii) Initial conditions. The fluid initial position is shown in Fig. 2 and the computational domain is from x^*=0 to $x^* = x_M^*$, which may be extended if needed in computation. The thickness of the residual layer on the bed is $h^* = h_{\text{cr}}^* = 1/\sin\vartheta$ everywhere. The total quantity of fluid Q^* is imposed on the residual layer from x^*=0 to $x^* = S_1^* + S_2^*$.

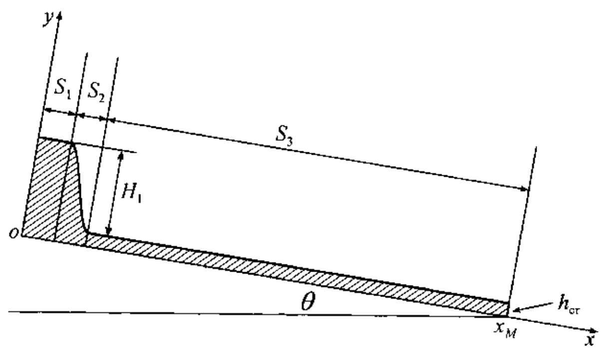

Fig. 2. Computational domain and initial fluid distribution.

(iv) Boundary conditions

(1) u^*=0 and $\partial v^*/\partial x^* = 0$ were specified for a slide wall at x^*=0.

(2) $\partial u^*/\partial x^* = 0$ and $\partial v^*/\partial x^* = 0$ were specified at $x^* = x_M^*$.

(3) No-slip condition, i.e. $u^* = 0$ and $v^* = 0$, was set on the bed at $y^* = 0$.

(4) On the free surface where the pressure is atmospheric p_0 and the tangential stress vanishes, $p^* \approx 0$ and $\partial V_t^*/\partial n = 0$ were set and the surface tension was assumed negligible. $\partial V_t^*/\partial n$ is a normal derivative for tangential velocity.

2 Results and discussion

(i) One typical example. The MAC method was used to track the free surface. Nine marks were set in each grid with the same spatial step in x and y directions $(\Delta x^* = \Delta y^* = 0.1/\sin\vartheta)$. Ten grids were included in a critical residual layer h_{cr}^*. For a fully developed intermittent flow, three independent dimensionless parameters are Q^*, μ_0^* and ϑ. In this example, the parameters were listed as follows: $\mu_0^*=1$, $\vartheta = 0.1745\,(10^\circ)$, which corresponds to $h_{\text{cr}}^* = 5.7588$, and $Q^* = 18(h_{\text{cr}}^*)^2 \approx 597$, which corresponds to $S_1^* = S_2^* = 2h_{\text{cr}}^*$ and $H_1^* = 6h_{\text{cr}}^*$ (see Fig. 2).

In Fig. 3, fully developed flow is arrived at t^* = 24. The head height $h_{\max}^*$ is about 13.6 $(h_{\max}^* - h_{\text{cr}}^* \approx 7.86)$ and the length of the intermittent flow L^* is about 122. The free surface corresponding to the specified runoff remains with time marching and U^* is almost constant (U^*=5.29). The velocity distributions in some cross sections are shown in Fig. 4 and the velocity in the head region is shown in Fig. 4(b).

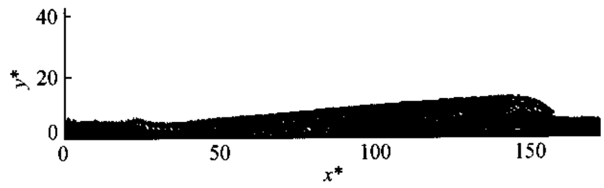

Fig. 3. Typical free surface of a fully developed intermittent flow.

Fig. 5 presents the drag and gravity distribution along the slope. Although the integrated drag and gravity are equal to each other for a fully developed intermittent flow, they are unequal at each transverse cross-section. The gravity is larger than drag in the head region, but less than drag in the body and trail of a surge. As the difference of pressure is small, it can be concluded that the fluid

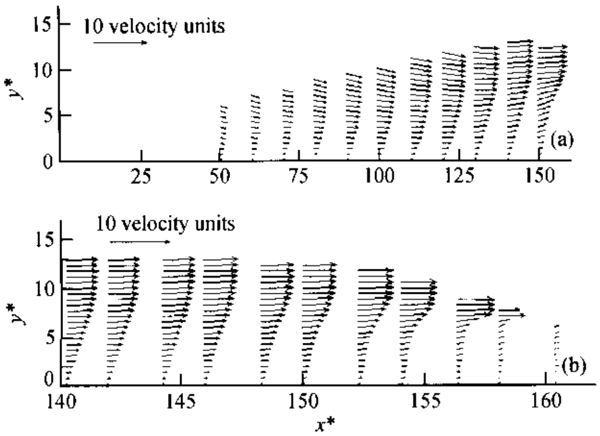

Fig. 4. Velocity distribution in the flow field.

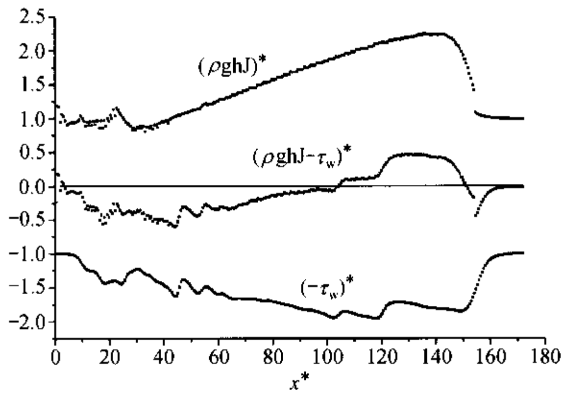

Fig. 5. Distribution of gravity and drag along the slope.

in the head region is accelerated and the fluid elsewhere is decelerated. It can be seen that the inertia force plays a key role in momentum equilibrium in an intermittent flow, so that any approximate method neglecting inertia force (such as that deducing Buckingham equation) is not suitable.

For a static observer, the intermittent flow moves forwards and the material velocity at each point is positive but less than U^* and changes with time. For an observer traveling with the surge, the fully developed intermittent flow is steady. The material velocity at each point is then negative. The incoming flow layer with a height h_{cr}^* and a velocity ($-U^*$) expands upwards and (according to the continuity equation) reduces its velocity promptly while it arrives at the turbulent head. Then h^* is increased and the negative velocity decreases rapidly as the flow moves backwards. After h^* reaches its maximum, it begins to decrease and the negative velocity to increase gradually. The fluid recovers to their initial height $h^*=h_{cr}^*$ and negative velocity ($-U^*$) as it leaves the surge.

As shown in Fig. 4, the transverse distribution of velocity, $u(y)$, in the rear part of the surge is close to that of homogeneous and continuous flow, i.e. in the top part there is a flow core with a thickness h_{cr} where the velocity distribution is nearly uniform but the fluid velocity in lower part is at a parabolic distribution. However, the velocity is much non-homogeneous at the turbulent head. For example, at the section of $x^*=155$, the fluid layer on the bed just enters the turbulent head, has not been drove and moves slowly. But the upper fluid is almost at maximum velocity (nearly equal to the speed U). The slip velocity between two layers becomes small when they move backwards.

It can be seen from Fig. 3 that the fluid layer on the bed is not disturbed until the arrival of the surge. It is the characteristic of a supercritical flow. The supercritical flow requires that the flow velocity is greater than the speed of shallow water wave. In this example, the dimensionless speed of shallow water wave is 2.38 while the dimensionless surge speed is 5.29. In fact, the fully developed intermittent flow must be a supercritical flow. In other words, an intermittent flow with lower speed will always attenuate and can not reach a fully developed state.

(ii) The surge speed of a fully developed intermittent flow. In this paper, twenty-nine cases of intermittent

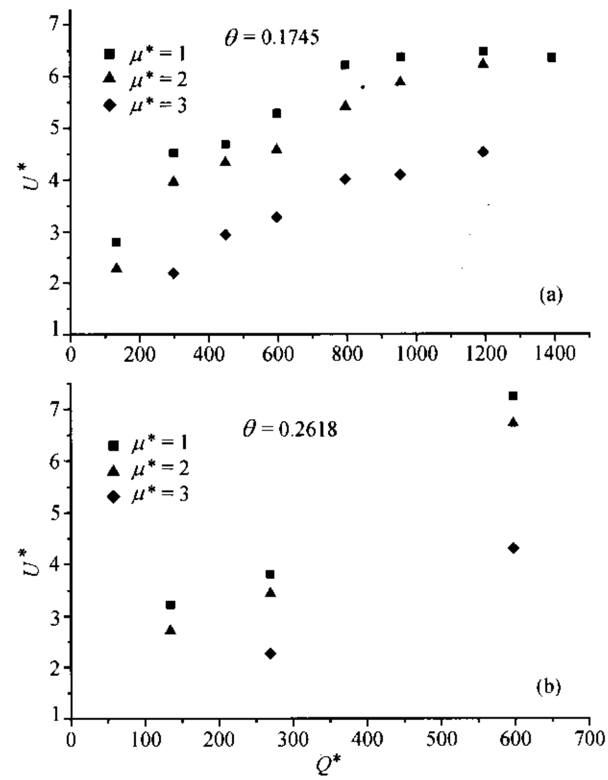

Fig. 6. Surge speed of fully developed intermittent flows.

flow were carried out. The parameters are listed as follows: ϑ=0.1745(10°) and 0.2618(15°), μ_0^*=1, 2, and 5, and Q^* is from 130 to 1400.

In all these examples, except that mentioned in section 2(i), the steps $\Delta x^* = \Delta y^* = 0.2/\sin\vartheta$ are used. Fig. 6 presents the computed surge speed U^* of fully developed intermittent flows. In logU^* and log($Q^* \sin\vartheta/\mu_0^*$) plane, the relation is nearly linear (Fig. 7), so that

$$U^* = C(Q^* \sin\vartheta/\mu_0^*)^n, \quad (14)$$

$$U = C(Qg \sin\vartheta/\mu_0)^n \rho^{(3n-1)/2} \tau_0^{(1-n)/2}, \quad (15)$$

where $n \approx 0.37$ and $C \approx 1$.

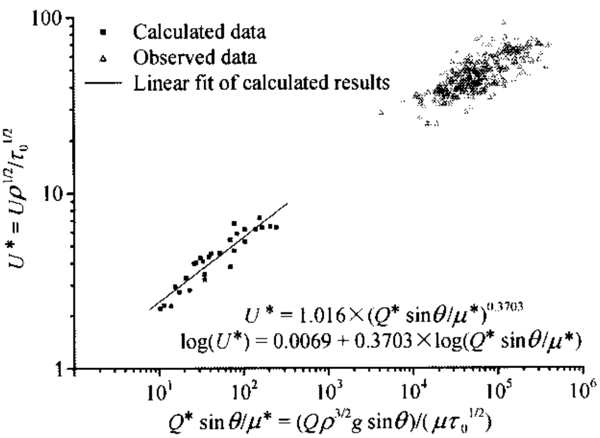

Fig. 7. Comparison of computed and observed surge speed.

Till now, there are almost no test data on fully developed intermittent flow of Bingham fluid because water flume is not long enough. For qualitative comparison, the 334 observation data on the debris flow in Jiangjia Ravine on 11 July, 1991 is also shown in Fig. 7. Because of no data for the equivalent viscosity and yield stress of the debris, the equivalent viscosity (16 Pa · s) and yield stress (30.8 Pa) were evaluated from the measured density of debris samples (ρ_m = 2080 kg/m^3) using eqs. (3)—(12) and figs. (3)—(14) in ref. [18].

It can be seen from Fig. 7 that the computed results agree tendentiously with the observed data. It must be noted that debris is not a perfect Bingham fluid, errors may appear in the data of the equivalent viscosity and yield stress as well as in the observed data on the surge speed and runoff per unit width. The Ravine is really not a two-dimensional straight flume and it is difficult to say the intermittent flow be fully developed. So, we say such agreement is perhaps fortunate. But it is more or less proved that Bingham fluid model and two-dimensional analysis in a longitudinal section is capable of describing intermittent flows. The results can be regarded as the first approximation.

It can be seen from eq. (15) that the surge speed U is increased with increasing Q and ϑ and decreasing μ_0. It is surprised that U is also increased with increasing τ_0 when Q, ϑ and μ_0 are given.

Generally speaking, with increasing τ_0 the drag on bed will be increased and flow speed will be decreased. However, what we study is such a situation that Bingham fluid moves on a generated bed-layer. In the condition that Q, ϑ and μ_0 remain constant, the critical thickness of residual layer, i.e. the thickness of the generated bed-layer, will increase with increasing τ_0.

Let's suppose a certain amount Q of fluid 1 and fluid 2 possessing different yield stresses τ_{01} and τ_{02} ($\tau_{02} > \tau_{01}$) respectively. Fluid 1 moves on a thinner bed-layer possessing a thickness $h_{cr,1}$ than that (with a thickness $h_{cr,2}$,, $h_{cr,2} > h_{cr,1}$) on which fluid 2 moves. Fluid 2 may move faster than fluid 1 because fluid 2 possesses a greater gravity component along a slope and $(\partial u/\partial y)|_{y=0}$ is smaller on a thicker bed. U increases with increasing τ_0 only if the two effects mentioned above exceed the effect that drag increase with τ_0.

In fact, such a phenomenon also occurs in homogeneously continuous flow. From the Buckingham equation the average velocity $\bar{u}$ can be expressed as

$$\bar{u} = \frac{\rho g \sin\vartheta}{3h\mu_0}(h-h_{cr})^2\left(h+\frac{1}{2}h_{cr}\right)$$
$$= \frac{\rho g \sin\vartheta(h-h_{cr})^2}{3\mu_0}\left[1+\frac{1}{2}\frac{h_{cr}}{(h-h_{cr})+h_{cr}}\right].$$

It can be seen from the equation above that with increasing h_{cr} (namely τ_0) the average velocity $\bar{u}$ is decreased when the total height h remains constant and increased when (h–h_{cr}) remains constant. The flow core with a thickness h_{cr} in continuous flow is corresponding to the residual layer with the same thickness in an intermittent flow. The remaining thickness outside the core or residual layer, (h–h_{cr}), is constant in continuous flow, but variable in an intermittent flow. The runoff Q is an integral of (h–h_{cr}) over x. The condition "(h–h_{cr}) remains constant" corresponds to the condition "runoff remains constant".

3 Conclusions

(1) The surge speed U able to describe the bulk motion of an intermittent flow is a wave speed, not any kind of material velocity. The material velocities $u(t, x, y)$ or their average values $\bar{u}(t,x)$ in the flow field are all smaller than U.

(2) Intermittent flow is unsteady one. When it reaches a fully developed state, it can be treated as steady if the coordinate system is attached to the flow.

(3) The fully developed intermittent flow belongs to a supercritical flow and the speed U must be larger than the speed of shallow water wave in the residual layer.

(4) For a fully developed intermittent flow, at the turbulent head the gravity component along a slope is larger than the drag and the fluid is accelerated, and the very reverse occurs in the rear part. Inertia plays a key role in the momentum equilibrium.

(5) The transverse distribution of velocity, $u(y)$, is much non-homogeneous. With marching of time, the turbulent head travels forwards and the fluid in this region draws back gradually from head to the rear part because the surge speed is greater than the fluid velocity. At the same time, the average material velocity and the non-homogeneity of velocity distribution is reduced.

(6) Three independent dimensionless parameters Q^*, μ_0^* (see eqs. (11) and (12)) and ϑ are capable of describing the fully developed intermittent flow of Bingham fluid.

(7) The results indicate that, for a fully developed intermittent flow, the speed of bulk motion can be expressed as $U = C(Qg\sin\vartheta/\mu_0)^n \rho^{(3n-1)/2}\tau_0^{(1-n)/2}$. It is increased with increasing ϑ, Q and τ_0/ρ and decreasing μ_0/ρ.

It is reasonable to regard viscous debris as a homogeneous Bingham fluid for the research of the surge speed of an intermittent flow in the sense of first order approximation.

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