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Weibull modulus for diverse strength due to sample-specificity

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Abstract

By sample specificity it is meant that specimens with the same nominal material parameters and tested under the same environmental conditions may exhibit different behavior with diversified strength. Such an effect has been widely observed in the testing of material failure and is usually attributed to the heterogeneity of material at the mesoscopic level. The degree with which mesoscopic heterogeneity affects macroscopic failure is still not clear. Recently, the problem has been examined by making use of statistical ensemble evolution of dynamical system and the mesoscopic stress re-distribution model (SRD). Sample specificity was observed for non-global mean stress field models, such as the cluster mean field model, stress concentration at tip of microdamage, etc. Certain heterogeneity of microdamage could be sensitive to particular SRD leading to domino type of coalescence. Such an effect could start from the microdamage heterogeneity and then be magnified to other scale levels. This trans-scale sensitivity is the origin of sample specificity. The sample specificity leads to a failure probability Φ_N with a transitional region $0 < \Phi_N < 1$, so that globally stable and catastrophic modes could co-exist. It is found that the scatter in strength can fit the Weibull distribution very well. Hence, the Weibull modulus is indicative of sample specificity. Numerical results obtained from the SRD for different non-global mean stress fields show that Weibull modulus increases with increasing sample span and influence region of microdamage.

1. Introduction

Diversification of material strength has been observed for a long time. A number of statistical theories have been proposed to explain such a phenomenon. Among them is the Weibull distribution for a chain consisting of links in series [1]. This approach [2,3] makes use of the statistical theories of fiber bundles. It was indicated in [2] that the strength of the fiber bundles tends to follow the normal distribution, provided that the load re-distribution is globally uniform and the fibers are infinite. In material engineering, the Weibull modulus m_f has been used as a measure of strength diversity for years. For most metals, it is over 20 while for ceramics it is usually less than 10. The measurement of m_f is a time consuming task. More precisely, the physical implication of strength diversity remains to be understood.

The transition from damage accumulation to failure of materials has been attributed to mesoscopic heterogeneity. Sample specific behavior for specimens tested under identical macroscopic conditions shows that the failure of the specimens do occur at various critical threshold. It has been

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known that neither percolation nor re-normalization group theories widely used in equilibrium transition could describe the phenomenon properly [4–6]. The underlying mechanism arises from the contingent sensitivity of the non-linear evolution to mesoscopic configuration of samples [6,7]. This implies that there prevails a group of sensitive states: their minor perturbation could lead to diversification via successive evolution. This results in divergent and contingent behavior. In what follows, statistical ensemble evolution of dynamical systems as governed by SRD is used in order to show how mesoscopic heterogeneity and stress re-distribution affect macroscopic failure and hence strength diversification.

2. Stress re-distribution model (SRD)

Damage evolution owing to re-distribution of mesostress contributes to the scatter of material strength. Such a behavior will be examined by making use of statistical ensemble evolution of dynamical system, in conjunction with the mesostress re-distribution model. The model consists of a periodical chain with a period of N parallel sites $X = (x_i, i = 1, 2, \ldots, N)$ [8,9]. Here, N is a dimensionless span between mesoscopic unit and macroscopic sample. There are two options for each site, $x_i = 0$ and $x_i = 1$ that denote unbroken and broken sites, respectively. $n = \sum_{i=1}^{N} x_i$ is the total number of broken sites and $p = n/N$ is the damage fraction. The sum of states in the phase space of the chain Ω_N has to be calculated according to a combined theory. For example, $\Omega_N = 52488$ for $N = 20$ and $\Omega_N \approx 8.03 \times 10^{57}$ for $N = 200$. Clearly, this is a huge ensemble.

The dynamics of damage evolution requires a knowledge of the condition under which a particular site would break. This causes the stress to re-distribute. This corresponds to the core in the SRD models. Suppose that all sites have the same strength σ_c. That is to say, a unit will break, if the local stress σ becomes greater than its strength σ_c. Hence, the stress becomes non-uniform following a particular evolution of damage pattern. In order to cover different types of stress re-distribution, consider the following SRD models.

2.1. Global mean field (GMF) model

This model assumes that the load is always uniformly shared by all unbroken sites, i.e., for a chain with damage fraction p, a uniform stress σ is given by $\sigma = \sigma_0/(1-p)$, where σ_0 is the nominal stress. The macroscopic strength σ_f of a sample with initial damage fraction p_0 can then be obtained as $\sigma_f = (1-p_0)\sigma_c$. Inversely, for a sample under a nominal stress σ_0, the failure threshold with damage fraction p_c is determined by $p_c = 1 - (\sigma_0/\sigma_c)$. This indicates that for the GMF model specificity is absent.

2.2. Stress concentration (SC) model

On the two sides of a broken cluster, there prevails influence regions δ, where unit breaks due to stress concentration. This model assumes stress elevation arising from a hole or a crack tip. Clearly, the largest cluster of broken sites would correspond to the highest stress concentration and the origin of eventual failure. It follows that the largest broken cluster is the most sensitive microstructure. If it could be monitored, it would be possible to predict failure. Failure governed by this model shows sample specificity. For its failure prediction, more information on mesostructure is required than that of the GMF model, which depends only on a knowledge of initial damage fraction p_0 and site strength σ_c. For simplicity, a local mean stress concentration (LMSC) model with influence region δ will be discussed.

2.3. Cluster mean field (CMF) model [8,9]

The nominal stress of broken cluster is shared uniformly by its two neighboring clusters. That is to say, a site in an s-intact cluster separating an l- and r-broken cluster supports a stress, Fig. 1:

$$\sigma = \left(1 + \frac{l+r}{2s}\right)\sigma_0. \tag{1}$$

The site-breaking condition can thus be expressed by

$$L = \frac{2s}{1+r} \leqslant \frac{\sigma_0}{\sigma_c - \sigma_0} = L_c, \tag{2}$$

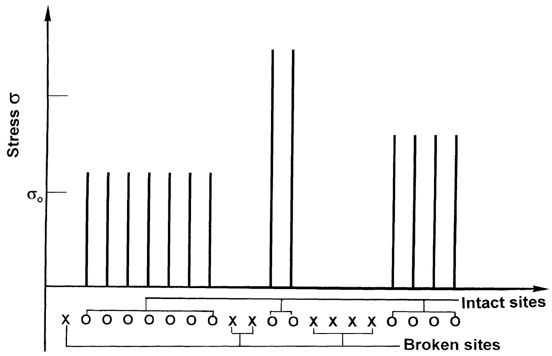

Fig. 1. Sketch of cluster mean field (CMF) model.

where L and L_c are dimensionless and critical ligaments, respectively. This model provides a quantitative assessment of the interaction between the neighboring broken cluster. In this case, the mesostructure that is sensitive to eventual failure is not clearly identified with the largest cluster of broken sites in SC model. The sensitive structure can only be revealed during the course of damage evolution. For example, the series of breaking is a sensitive structure:

$$s_j = \mathrm{Int}[(j + 1 + \Sigma s_j)L_c/2], \tag{3}$$

when $L_c = 1,\ s_j = 1, 2, 3, 5, 8, 13, 20, 30, \ldots,$ is similar to Fibonacci series [9]. Clearly, complexity of specificity is caused by interaction of broken clusters.

3. Sample specificity and failure probability

Loosely speaking, non-linear evolution leads to two different modes of pattern flow in phase space according to their final states. They are the global stable (GS) type, where damage occurred but not fracture and evolution induced catastrophe (EIC), where complete fracture [6] has taken place. According to the models LMSC and CMF, stress fluctuations can always occur owing to random distribution of broken sites as a result of mesoheterogeneity. The stress distribution changes with the evolution of damage pattern. Hence, for the CMF, GS or EIC cannot be evaluated by the macroscopic parameters involving the initial damage fraction p_0 and nominal stress σ_0. In other words, the system shows sample-specific behavior, i.e., macroscopic uncertainty while the formulation of failure should be statistical.

The probability of EIC modes, i.e., the failure probability $\Phi_N(p_0, \sigma_0)$ should be known. An examination of evolution of all points in phase has been carried out for short chains, such as $N \leqslant 30$ and long chains by means of the slice sampling method [8–10].

For the GMF model, there is a deterministic and clear-cut boundary between GS ($\Phi_N = 0$) and EIC ($\Phi_N = 1$) modes as given by

$$\sigma_0 = (1 - p_0)\sigma_c. \tag{4}$$

The curves in Fig. 2 display the variations of the normalized stress σ_0/σ_c with the initial damage fraction p_0 for the transitional region. The curves for $\Phi_N = 0$ and $\Phi_N = 1$ correspond, respectively, to the GS and EIC mode for CMF model with $N = 200$. Here, the failure can be determined uniquely by the macroscopic parameters. However, for all other models with stress fluctuations,

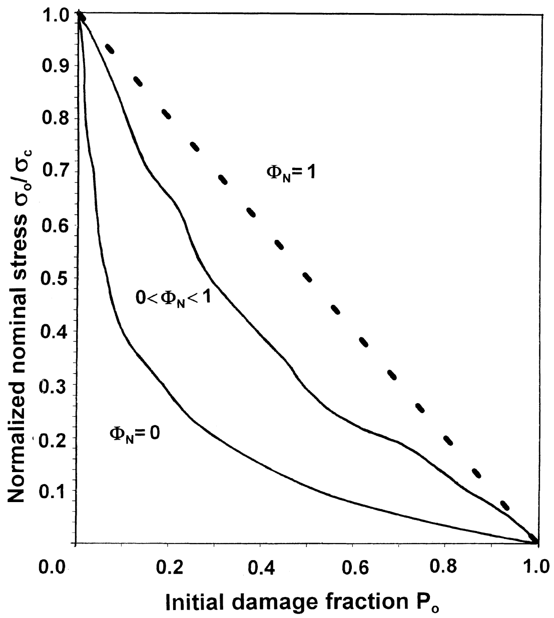

Fig. 2. Failure probability $\Phi_N(p_0, \sigma_0)$ showing the transitional region.

such as LMSC and CMF, there is a transitional region $0 < \Phi_N < 1$ between the GS ($\Phi_N = 0$) and EIC ($\Phi_N = 1$) regions in the macroscopic parameter space (p_0, σ_0), Fig. 2. In the transitional region, GS and EIC modes co-exist. Macroscopic uncertainty of failure shows up.

4. Diversification of strength

Let the nominal stress σ_0 be increased gradually. Observe the transition of the sample from the GS to EIC mode and determine the ultimate strength σ_f. The distribution density of ultimate strength $f(\sigma_f)$ can be calculated. As usual, an effective width of the strength distribution can be defined as

$$\Delta\sigma_f(N, p_0) = \left[\int_0^\infty (\sigma_f - \bar{\sigma}_f)^2 f(\sigma_f; N, p_0)\, d\sigma_f \right]^{1/2}. \tag{5}$$

The average strength of the chain is

$$\bar{\sigma}_f(N, p_0) = \int_0^\infty \sigma_f \cdot f(\sigma_f; N, p_0)\, d\sigma_f. \tag{6}$$

There is also an approximate scaling law

$$\Delta\sigma_f = gN^{-\gamma} \tag{7}$$

and $g = 0.1032$ is in the range of $N \sim (10^1\text{–}10^3)$. $\Delta\sigma_f/\bar{\sigma}_f$ is around 0.1. This demonstrates the importance of strength diversification.

The problem can be formulated in a different way by adopting one-dimensional periodical chain that behaves according to SRD, especially CMF and LMSC. Instead of assuming an initial damage fraction p_0, and constant site strength σ_c, a prescribed distribution density of site strength $c(\sigma_c)$ is used together with Weibull function having expectation of 1 and modulus m_c. Note that this is a distribution of mesostrength σ_c. With the numerical simulation of damage evolution of samples, the macrostrength σ_f is obtained. Finally, the macrostrength of samples can be fitted by a Weilbull distribution:

$$W(\sigma_f; N, \delta) = 1 - \exp\left[-\left(\frac{\sigma_f}{\eta}\right)^{m_f} \right]. \tag{8}$$

Besides the two fitting parameters η and m_f (Weibull modulus), there are three others involving the sample span N, the size of influence region δ and the modulus m_c of distribution $c(\sigma_c)$ that measure the initial mesoheterogeneity. Fig. 3 shows a Weibull distribution fitting for data from 2000 samples using the CMF model with $N = 5000$ and $m_c = 2$. Note that the Weibull distribution fits the data very well for $m_f = 19.0$.

Diversification of macrostrength is due to the amplifying effect of mesoheterogeneity during nonlinear evolution. Fig. 4 shows how the Weibull modulus of macrostrength m_f increases with that of mesostrength m_c for $N = 2000$.

The dependence of Weibull modulus of macrostrength m_f on the sample span N is given in Fig. 5 for the CMF model with $m_c = 2$; it increases rapidly at first and then levels off after $N > 2 \times 10^4$. This indicates that large sample span N can improve the strength diversity. Similar trend is found for the Weibull modulus of macrostrength m_f with increasing size of influence

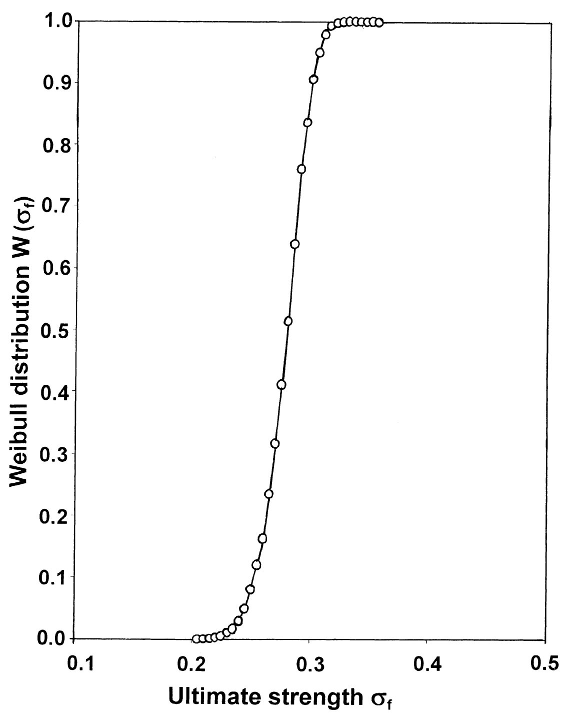

Fig. 3. Weibull distribution fit for macrostrength.

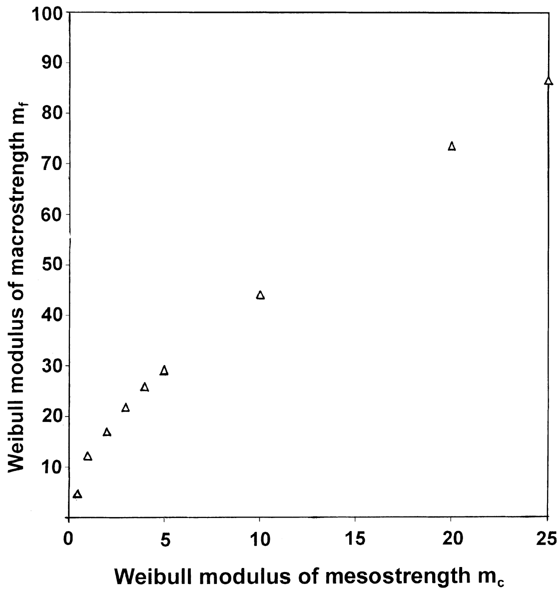

Fig. 4. Weibull modulus of macrostrength versus that of mesostrength for $N = 2000$.

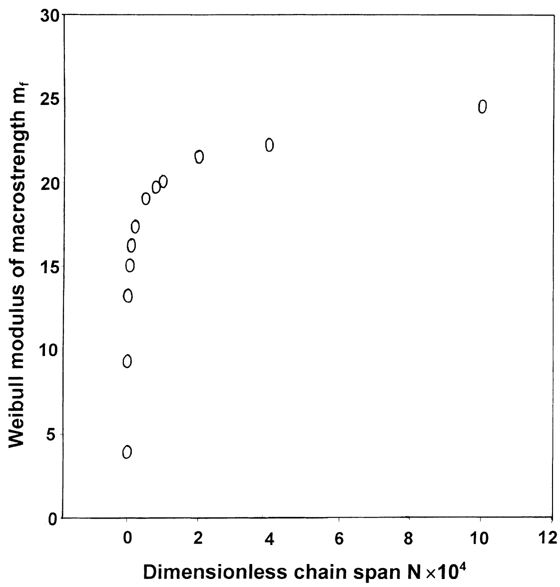

Fig. 5. Weibull modulus of macrostrength versus dimensionless chain span for $m_c = 2$.

region δ, Fig. 6. It is noteworthy that if the size of influence region δ is greater than the spacing between two adjacent broken clusters, the load

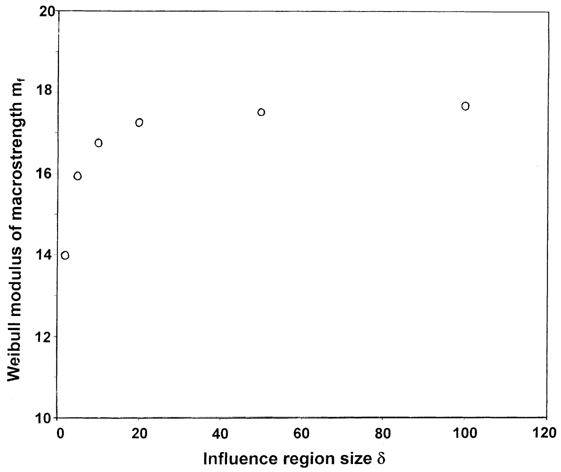

Fig. 6. Weibull modulus of macrostrength versus influence region size for $N = 2000$ and $m_c = 2$.

would be uniformly supported. This corresponds to the stress re-distribution of CMF model. Hence, it is not surprising that LMSC model behaves as that of CMF (Fig. 6), when δ tends to the chain size.

5. Concluding remarks

The following conclusions can be made:

- Sample specificity is observed for various SRD such as those of cluster load sharing, stress concentration, etc., except the global mean field model that sweeps away all mesoheterogeneity and fails to look at the problem of sample specificity.
- Different SRD reflects different sensitive to heterogeneity. Due to the interaction between broken clusters, the cascade of damage evolution magnifies the effect of initial heterogeneity and demonstrates complex specificity, a statistical description of which is the failure probability Φ of macroscopic parameters.
- Due to sample specificity, there prevails a diversity of macrostrength that could be fitted to a Weibull distribution. It is found that Weibull modulus increases with increasing sample span, mesoscopic heterogeneity and influence region.

Acknowledgements

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