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EXPLAINING THE ANALOGY BETWEEN DISLOCATION LINE IN CRYSTAL AND VORTEX FILAMENT IN FLUID

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Introduction

In Sir G.I.Taylor's famous paper [1], he introduced a dislocation in crystals by the analogy of a vortex in a fluid. Nevertheless, Sir G.I.Taylor did not elaborate on the analogy explicitly. The objective of this letter is to explain why this analogy exists.

Main points of the analogy

Dislocation is the most important two-dimensional, or line, defect in solid, it is responsible for nearly all aspects of the plastic deformation of metals. The existence of a dislocationlike defect is necessary to explain the low values of yield stress observed in real crystals. Fig 1 illustrates the Burgers vectors of edge and screw dislocations. Peach and Koehler (1950) [2] pointed out that certain analogy exists between the elastic deformation field round a dislocation line and the magnetic field of constant line currents (Cf. [3,4]), the current is replaced by the

Burgers vector, which must be constant along the dislocation line, like the current.

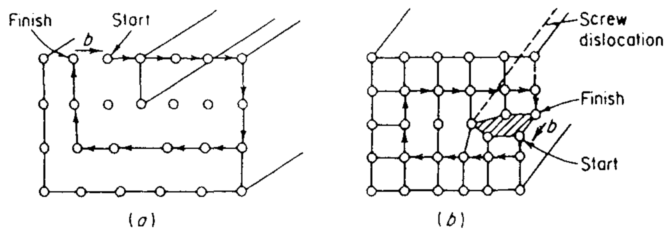


Fig. 1. A illustration of Burgers vector of edge and screw dislocations in crystal.

It is well-known that a dislocation in crystal has the fundamental properties as follows:

- A dislocation line cannot end inside of a crystal. Because a dislocation represents the boundary between the slipped and unslipped region of a crystal, topographic considerations require that it either must be a closed loop or else must end at the free surface of a crystal or at a grain boundary.
- Burgers vector is constant along the dislocation line, this property of dislocation is commonly called "law of conservation of the Burgers vector" in the medium.

In some conceptual aspect, Barenblatt suggested an analogy between failure of solid and fully developed turbulence [5,6], and this analogy was found to be fruitful. Actually, in the turbulence phenomenon we have the fluid instead of the deformable solid, the vortices instead of cracks and defects (such as pores, vacancies, dislocations, etc.). The turbulent flow contains a cascade of interacting vortices of various length scales, quite in the same manner, there exists in deformable solid a cascade of crack-like defects, pores, vacancies, dislocations, etc. which are interacting.

Recent results indeed suggest that the fine scales of turbulent flow include a tangle of very intense and slender vortex filaments [7]. The filaments are actually tubes with an approximately circular cross-section, their diameter is of the order of the Kolmogorov dissipation scale

$$\eta = \left(\upsilon^3 / \varepsilon\right)^{1/4}, \tag{1}$$

where υ and ε are the kinetic viscosity and the mean energy dissipation per unit mass, respectively. She et al [8] pointed out that in isotropic incompressible turbulence at moderately high Reynolds number, highly intermittent vortex structures, illustrated by Fig.2, are typically tube-like (Cf. [9]). Moffatt et al. [10] suggested that vortex filaments are the "sinews" of turbulence.

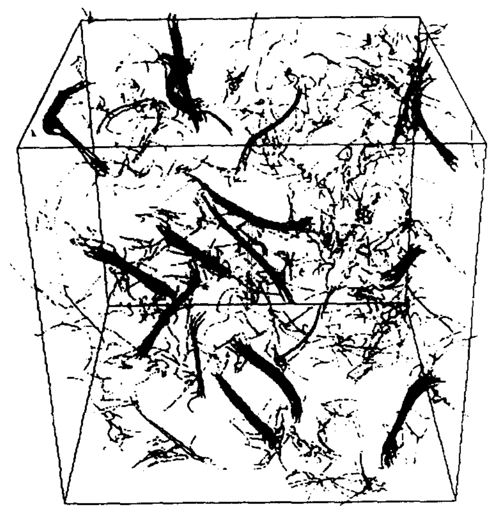


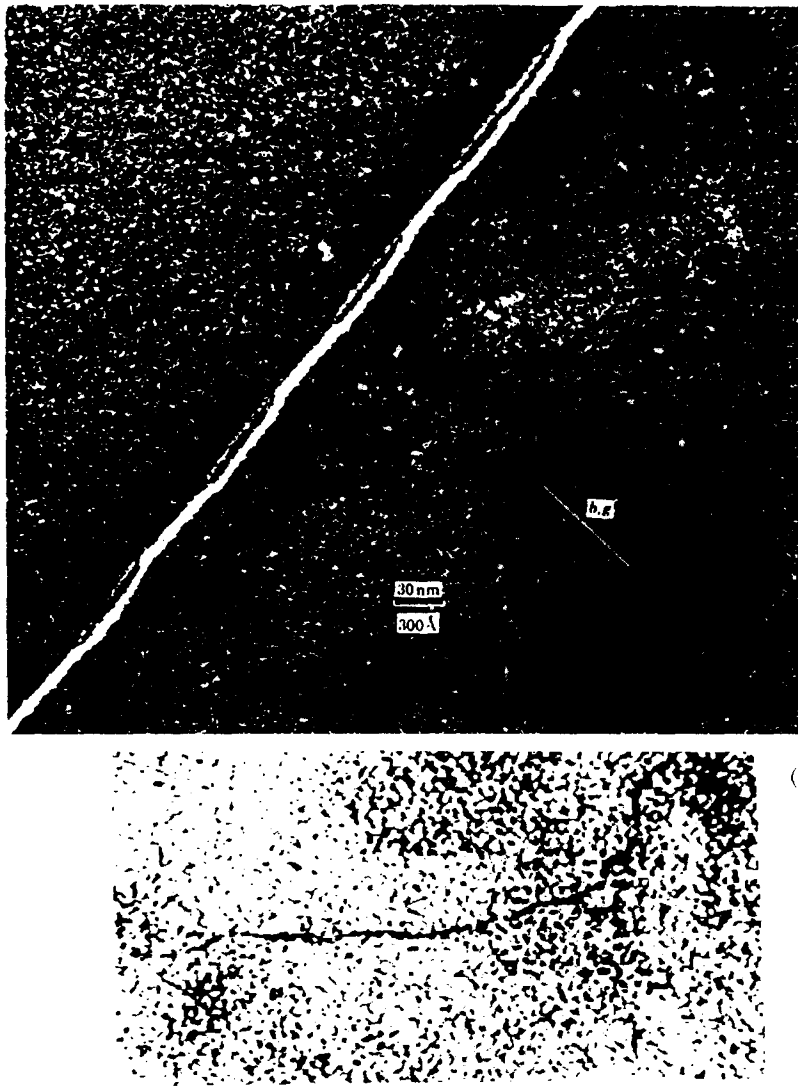
Fig.2. Intermittent vortex filaments in a three-dimensional turbulent fluid simulated on a computer [8]

For comparison, Fig. 3(a) shows a dislocation in an annealed silicon specimen [11], and Fig. 3(b) shows a typical vorticity filament in a turbulent flow [12].

The well-known Helmholtz's theorem about vortex filaments is [13]

- Vortex filaments are either closed, go to infinity or end on solid boundaries. A vortex filament cannot end at any point within the fluid (Cf. [14]).
- The circulation about a vortex filament is everywhere the same.

It is easily seen that the basic properties for both dislocation and vortex filament are the same.



(a)

(b)

Fig. 3. (a) A image of a dislocation in an annealed silicon specimen [11];
(b) A vortex filament in a turbulent flow (exposure time 0.001 s) [12].

It is worthy to point out that a perfect analogy between hydrodynamics and electrodynamics, which, according to a remark of Helmholtz, has helped greatly in the development of either science [15]. As a system of electric currents in linear conductors is surrounded by magnetic field lines, so a system of vortex filaments is surrounded by stream lines. The famous Biot-Savart law applies in both cases. It is obvious that the current intensity I corresponds to the vortex circulation $\Gamma = \oint \bar{u} \cdot d\bar{s}$, the current density $\bar{J}$ to the vortex vector $\bar{\omega} = \mathrm{curl}\bar{u}$, where $\bar{u}$ is the vector field of the motion of the fluid. This analogy shows that the flow $\bar{u}$ corresponds to the magnetic field strength $\bar{H}$ and the rotation $\bar{\omega}$ to the electric field strength. For example, the magnetic field strength round a sufficiently long and straight linear conductor with current intensity I is

$$H = \frac{I}{2\pi r}, \tag{2}$$

where r is the distance of a point from the line.
The velocity field induced by a sufficiently long and straight vortex filament with circulation Γ is of the magnitude [13]

$$u = \frac{\Gamma}{2\pi r}, \tag{3}$$

For a straight screw dislocation with Burgers vector b in isotropic medium [5], if we take cylindrical polar coordinates z, r, ϕ, with the z-axis along the dislocation line. The displacement along the dislocation line is $u_z = b\phi / 2\pi$, the only non-zero induced shear strain in the medium round the dislocation line is

$$\gamma_{\phi z} = \frac{b}{2\pi r}. \tag{4}$$

It is obvious from eqns (2)~(4) that the Burgers vector b for screw dislocation corresponds to the circulation Γ of a vortex filament and the current intensity I of a linear conductor, the shear strain round the screw dislocation to the flow velocity around the vortex filament and the magnetic field strength round a linear conductor.

Acknowledgements

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