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General paper

An Elasto-Plastic Constitutive Relation of Whisker-Reinforced Metal-Matrix Composite

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Abstract: A material model for whisker-reinforced metal-matrix composites is constructed that consists of three kinds of essential elements: elastic medium, equivalent slip system, and fiber-bundle. The heterogeneity of material constituents in position is averaged, while the orientation distribution of whiskers and slip systems is considered in the structure of the material model. Crystal and interface sliding criteria are addressed. Based on the stress-strain response of the model material, an elasto-plastic constitutive relation is derived to discuss the initial and deformation induced anisotropy as well as other fundamental features. Predictions of the present theory for unidirectional-fiber-reinforced aluminum matrix composites are favorably compared with FEM results.

Key words: *Whisker-reinforced metal-matrix composites, Elasto-plasticity, Constitutive relation, Initial and deformation induced anisotropy*

1. INTRODUCTION

The overall elasto-plastic behavior of metal-matrix composites (MMCs) has been a topic of intense research for several decades [1-3]. Many efforts have been made to develop relationships between the microstructure and macro performance of composites. On the basis of Eshelby's method, self-consistent scheme and equivalent inclusion method were used as a basis for the mechanical analysis of fiber or particle reinforced composites. Fibers were also considered as long cylinders [4] or prolate ellipsoids and plates in the form of oblate ellipsoids [5]. As an extension of Eshelby's equivalent inclusion method, Mori-Tanaka method [6] has been widely adopted for composites.

The need for numerical analyses arises because analytical formulations become intractable for composites including the heterogeneity, anisotropy and plasticity behavior under non-proportional loading. The relative experiments are much more difficult and time-consuming. Therefore, the finite element method (FEM) made a great contribution for understanding of the effect of distributed fibers on the elasto-plastic behavior of composites [7-8]. However, it is unable to form an explicit constitutive relation for engineering structural analysis. On the other hand, the constituents of composites vary in size, shape, orientation, and volume-fraction, an opportunity to optimize their properties and performances for specific application under various circumstances [9] is provided. It is a challenge to model whisker-reinforced MMCs for predicting the effect of the heterogeneity, anisotropy and evolution of properties on the overall performance.

Based on the microscopic description of whisker reinforcing and crystal sliding mechanisms [10], mesoscopic material models and constitutive relations for polycstalline metals and whisker-reinforced composites have been established [11-13]. The evolution of the yield surface and deformation-induced anisotropy were predicted [12-15]. Recently, the damage mechanism has been taken into account, so the meso-damage constitutive relation was developed to predict the damage-induced anisotropy, damage-rate effect and failure surface of brittle fiber-reinforced composite material [16-17].

Constituents, microstructure and interface between the matrix and whiskers play a significant role in improving the toughness of metal-matrix and ceramic-matrix composites [18]. The study of various types of the fiber-matrix bonding [19,20] shows that the interface sliding between the matrix and fibers may take place [21]. It can be taken as a pseudo-plastic behavior of fibers [22,23]. Therefore, consideration of the coupling of multiple mechanisms becomes inevitable in a material model for composites.

The present work is concerned with the effect of the anisotropy and multiple mechanisms on the overall elasto-plastic performance of whisker-reinforced MMCs. A material model is constructed, and crystal and interface sliding criteria are addressed. Based on the stress-strain response of the model material, an elasto-plastic constitutive relation is derived followed by discussion of the initial and deformation induced anisotropy as well as other fundamental features. Predictions of the present theory for unidirectional-fiber-reinforced aluminum matrix composites will be compared with FEM results.

2. MATERIAL MODEL

Whisker-reinforced metal-matrix composites (MMCs) are composed of metallic matrix and whiskers. The overall elasto-plastic behavior depends on multiple mechanisms as well as the microstructure of composites. To develop a material model that has characteristics consistent with the composite material, it is assumed that:

- Deformation of the matrix is decomposed into an elastic part $\dot{E}_{\Box e}$ in the crystal grains and a plastic part $\dot{E}_{\Box s}$ caused by crystal sliding. The total strain rate of the matrix is compatible with macro strain rate $\dot{E}$, i.e.

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$$\dot{E} = \dot{E}_{me} + \dot{E}_{ms} \quad (1)$$

• Local stresses $\tilde{S}_g$ in a crystal grain and $\tilde{\tau}_s$ in a slip system are proportional to the average stress S_m and resolved shear stress τ_s of the matrix, i.e.

$$\tilde{S}_g = \tilde{c}_g : S_m \quad (2)$$

$$\tilde{\tau}_s = \tilde{c}_s \tau_s \quad (3)$$

$$\tau_s = P_s : S_m \quad (4)$$

where $\tilde{c}_g$ and $\tilde{c}_s$ are stress heterogeneity factors of the crystal grain and slip system. Note that

$$P_s = \frac{1}{2}(m \otimes n + n \otimes m) \quad (5)$$

is an orientation tensor of the slip system. In Eq.(5), m is a unit vector in sliding direction and n, a unit normal vector of the sliding plane.

• Local strain rate $\dot{\tilde{\varepsilon}}_f$ of a whisker along direction l is proportional to the overall strain rate $\dot{\varepsilon}_f$ in the same direction, i.e.

$$\dot{\tilde{\varepsilon}}_f = \tilde{c}_f \dot{\varepsilon}_f \quad (6)$$

$$\dot{\varepsilon}_f = P_f : \dot{E} \quad (7)$$

where $\tilde{c}_f$ is a heterogeneity factor of the strain rate of the whisker. Here,

$$P_f = l \otimes l \quad (8)$$

is an orientation tensor of the whisker and l, a unit vector.

The symbols with tilde refer to location. $\tilde{c}_g$ and $\tilde{c}_s$ may vary with the stress of the matrix, and $\tilde{c}_f$ may vary with overall strain. The crystal grains, slip systems, and whiskers are incrementally linear, such that

$$\dot{\tilde{E}}_g = \tilde{C}_g : \dot{\tilde{S}}_g \quad (9)$$

$$\dot{\tilde{\gamma}}_s = \frac{1}{\tilde{h}_s} \dot{\tilde{\tau}}_s \quad (10)$$

$$\dot{\tilde{\varepsilon}}_f = \frac{1}{\tilde{E}_f} \dot{\tilde{\sigma}}_f \quad (11)$$

where $\tilde{E}_g$ and $\tilde{C}_g$ are local strain rate and compliance tensors of crystal grains, respectively. The local sliding rate and hardening modulus of slip systems are given by $\tilde{\gamma}_s$ and $\tilde{h}_s$, while $\tilde{\sigma}_f$ and $\tilde{E}_f$ are the local stress rate and Young's modulus of whiskers, respectively.

2.1. The Matrix

Polycrystalline metal matrix consists of many irregular crystal grains. Crystal sliding is the plastic deformation mechanism of the matrix.

2.1.1. Elasticity of the matrix

According to the assumptions, the power stored in crystal grains can be expressed as

$$\dot{W} = \frac{1}{V_m}\int_{V_m} \tilde{S}_g : \dot{\tilde{E}}_g dV = S_m : \frac{1}{V_m}\int_{V_m} \tilde{c}_g : \dot{\tilde{E}}_g dV \quad (12)$$

where V_m is volume fraction of the matrix. Being a power conjugate with S_m, the elastic strain rate tensor in crystal grains can be written as

$$\dot{E}_{me} = \frac{1}{V_m}\int_{V_m} \tilde{c}_g : \dot{\tilde{E}}_g \, dV . \quad (13)$$

The substitution of Eqs.(9) and (2) into Eq.(13) yields

$$\begin{aligned} \dot{E}_{me} &= \overline{C}_{me} : \dot{S}_m \\ \overline{C}_{me} &= \frac{1}{V_m}\int_{V_m} [\tilde{c}_g : \tilde{C}_g : \tilde{c}_g \\ &+ \frac{1}{2}\tilde{c}_g : \tilde{C}_g : (\tilde{S}_m : \frac{d\tilde{c}_g}{dS_m} + \frac{d\tilde{c}_g}{dS_m} : \tilde{S}_m)] dV. \end{aligned} \quad (14)$$

Thus, the elastic behavior of the heterogeneous matrix described by Eq.(14) can be represented by one of a uniform elastic medium with the average compliance tensor $\overline{C}_{me}$. S_m and $\dot{E}_{oe}$ represent the average stress and elastic strain rate tensors of the elastic medium or matrix.

2.1.2. Plasticity of crystal sliding

Consider slip in the direction m on slip planes with the unit normal vector n. The power dissipated by the local slip systems can be expressed as

$$\dot{W} = \frac{1}{V_m}\int_{V_m} \tilde{\tau}_s \dot{\tilde{\gamma}}_s dV = \left\{\frac{1}{V_m}\int_{V_m} \tilde{c}_s \dot{\tilde{\gamma}}_s dV\right\}\tau_s . \quad (15)$$

An average sliding strain rate conjugate with τ_s is

$$\dot{\gamma} = \frac{1}{V_m}\int_{V_m} \tilde{c}_s \dot{\tilde{\gamma}}_s dV . \quad (16)$$

By use of Eqs.(3) and (10), the local sliding strain rate can be derived as

$$\dot{\tilde{\gamma}}_s = \frac{1}{\tilde{h}_s}(\tilde{c}_s \dot{\tau}_s + \dot{\tilde{c}}_s \tau_s) = \frac{1}{\tilde{h}_s}(\tilde{c}_s + \frac{d\tilde{c}_s}{d\tau_s}\tau_s)\dot{\tau}_s . \quad (17)$$

Substituting Eq.(17) into Eq.(16), the local slip systems can be represented by an equivalent slip system with an average hardening modulus , h_s ,and

$$\begin{aligned} \dot{\gamma}_s &= \frac{1}{\overline{h}_s}\dot{\tau}_s, \\ \frac{1}{\overline{h}_s} &= \frac{1}{V_m}\int_{V_m} \frac{\tilde{c}_s}{\tilde{h}_s}(\tilde{c}_s + \frac{d\tilde{c}_s}{d\tau_s}\tau_s) dV \end{aligned} \quad (18)$$

where $\dot{\tau}_s$ and $\dot{\gamma}_s$ denote resolved shear stress and sliding rates of the equivalent slip system.

An orientation distribution of equivalent slip systems can be described with an orientation density ρ_s . If there is no preferred orientation, equivalent slip systems will be

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homogeneously distributed in the 3-D space, and $\rho_s = 1/4\pi^2$.

A slip system may be active when its resolved shear stress reaches a critical value τ_{+cr} in $\boldsymbol{m}$ direction or τ_{-cr} in the opposite direction. An activation criterion of the slip system can be stated as [13]

$$\begin{cases} \text{If } \tau_s = \tau_{+cr} & \text{then } \dot{\gamma}_s > 0, \dot{\tau}_{+cr} = \dot{\tau}_s = \bar{h}_s \dot{\gamma}_s \\ & \text{and } \quad \dot{\tau}_{-cr} = \dot{\tau}_{+cr} - 2\tau_{cr0} \\ \text{If } \tau_s = \tau_{-cr} & \text{then } \dot{\gamma}_s < 0, \dot{\tau}_{-cr} = \dot{\tau}_s = \bar{h}_s \dot{\gamma}_s \\ & \text{and } \quad \dot{\tau}_{+cr} = \dot{\tau}_{-cr} + 2\tau_{cr0} \\ \text{Otherwise} & \quad \dot{\gamma}_s = 0, \ \dot{\tau}_{\pm cr} = 0 \end{cases} \tag{19}$$

In Eq.(19), Prager's kinematics hardening rule is applied, and τ_{cr0} is an initially critical resolved shear stress.

2.2. Whiskers

Experimental results show that the interface siding between the matrix and whiskers may take place when the fiber strain reaches its critical value [19]. The interface debonding is mainly dependent on strain in the fiber [24]. Therefore, the interface sliding may be regarded as pseudo-plasticity of whisker [22]. Hence, the whiskers are regarded as elastic and pseudo-plastic components.

2.2.1. Elasticity of whiskers

In the case of whiskers distributed unidirectionally in the matrix, the power required by the whiskers can be expressed as

$$\dot{W}_f = \frac{1}{V_f}\int_{V_f} \tilde{\sigma}_f \dot{\tilde{\varepsilon}}_f dV = \left\{\frac{1}{V_f}\int_{V_f} \tilde{c}_f \tilde{\sigma}_f dV\right\} \dot{\varepsilon}_f \tag{20}$$

where V_f is the volume fraction of the whiskers. The stress σ_f conjugating with $\dot{\varepsilon}_f$ should be

$$\sigma_f = \frac{1}{V_f}\int_{V_f} \tilde{c}_f \tilde{\sigma}_f dV \ . \tag{21}$$

Thus, the stress rate can be derived as

$$\dot{\sigma}_f = \frac{1}{V_f}\int_{V_f} (\tilde{c}_f \dot{\tilde{\sigma}}_f + \dot{\tilde{c}}_f \tilde{\sigma}_f) dV \ . \tag{22}$$

Substitution of Eqs.(11) and (6) into Eq.(22) yields

$$\begin{aligned} \dot{\sigma}_f &= \bar{E}_f \dot{\varepsilon}_f , \\ \bar{E}_f &= \frac{1}{V_f}\int_{V_f} (\tilde{c}_f^2 \tilde{E}_f + \tilde{\sigma}_f \frac{d\tilde{c}_f}{d\varepsilon}) dV \ \cdot \end{aligned} \tag{23}$$

So the whiskers can be represented by a fiber-bundle with the average stiffness modulus $\bar{E}_f$. Note that $\dot{\sigma}_f$ and $\dot{\varepsilon}_f$ denote the stress and strain rates of the fiber-bundle, respectively.

Statistically, a probability density ρ_f of whiskers can be introduced to describe the orientation distribution of the fiber-bundles in multi-oriented composites. The homogeneous orientation distribution density, ρ_f ,is $1/2\pi$.

2.2.2. Pseudo-plasticity of interface sliding

Interface sliding may take place when the fiber strain reaches its critical value ε_{+cr} in tension or ε_{-cr} in compression. Then, the behavior of the fiber-bundle becomes pseudo-plastic that can be described as

$$\dot{\sigma}_f = \bar{E}_{fs} \dot{\varepsilon}_f \tag{24}$$

where $\bar{E}_{fs}$ is an average interface sliding modulus. An interface-sliding criterion can be stated as

$$\begin{cases} \text{If } \varepsilon_f = \varepsilon_{+cr} & \text{then } \quad \dot{\varepsilon}_{+cr} = \dot{\varepsilon}_f = \dot{\sigma}_f / \bar{E}_{fs} \\ & \text{and } \quad \dot{\varepsilon}_{-cr} = \dot{\varepsilon}_{+cr} - 2\varepsilon_{cr0} \\ \text{If } \tau_s = \tau_{-cr} & \text{then } \quad \dot{\varepsilon}_{-cr} = \dot{\varepsilon}_f = \dot{\sigma}_f / \bar{E}_{fs} \\ & \text{and } \quad \dot{\varepsilon}_{+cr} = \dot{\varepsilon}_{-cr} + 2\varepsilon_{cr0} \\ \text{Otherwise} & \quad \dot{\varepsilon}_{\pm cr} = 0 \end{cases} \tag{25}$$

where $\varepsilon_{\pm cr0}$ are initial critical sliding strains of the fiber-bundle in $\pm \boldsymbol{l}$ directions, and $\varepsilon_{cr0} = (\varepsilon_{+cr0} - \varepsilon_{-cr0})/2$ [12].

3. CONSTITUTIVE RELATION

The whisker-reinforced metal-matrix composite can be modeled with three types of essential elements: matrix, equivalent slip systems and fiber-bundles, which are distributed according to their orientation density ρ_s and ρ_f. The heterogeneity of constituents in position is averaged, while the heterogeneity in orientation is kept in the material model.The elasto-plasticity constitutive equations can be derived as follows.

3.1. Elasticity

Consider a representative elementary volume (REV) of composites with orientation distribution density ρ_s and ρ_f . Before sliding takes place, two kinds of essential elements: elastic medium and fiber-bundles influence on the elastic behavior. Therefore, the total power in REV equals to the summation required by the matrix and fiber-bundles with the orientation distribution density ρ_f , i.e.

$$\boldsymbol{S} : \dot{\boldsymbol{E}} = V_m \boldsymbol{S}_m : \dot{\boldsymbol{E}}_{me} + V_f \int_\Omega \rho_f \sigma_f \dot{\varepsilon}_f d\Omega \tag{26}$$

where $d\Omega$ is a solid angle in direction $\boldsymbol{l}$, V_m and V_f are volume fractions of the matrix and fiber-bundles such that satisfy $V_m + V_f = 1$. Since deformation of elastic medium and fiber-bundles is compatible with overall deformation, the overall stress of the composite should be

$$\boldsymbol{S} = V_m \boldsymbol{S}_m + V_f \boldsymbol{S}_f , \tag{27}$$

that is to say, the total stress is shared by the matrix and fiber-bundles together, where the stress tensor carried by fiber-bundles is written as

$$\boldsymbol{S}_f = \int_\Omega \rho_f \sigma_f \boldsymbol{P}_f d\Omega \ . \tag{28}$$

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By use of Eqs.(7)and(23),the stress rate tensor carried by the fiber-bundles satisfies

$$\dot{S}_f = \overline{K}_f : \dot{E}, \qquad \overline{K}_f = \int_\Omega \rho_f \overline{E}_f P_f \otimes P_f d\Omega. \tag{29}$$

Equation (29) is the stress-strain relation of whisker-network without consideration of the matrix.

The overall stress rate tensor shared by the matrix and fiber-bundles is expressed as

$$\dot{S} = V_m \dot{S}_m + V_f \dot{S}_f. \tag{30}$$

Substituting Eqs.(14) and (29) into Eq.(30) yields

$$\dot{S} = \{V_m \overline{K}_{me} + V_f \overline{K}_f\} : \dot{E} \tag{31}$$

where $\overline{K}_{me} = \overline{C}_{me}^{-1}$ and $\overline{K}_f$ are fourth rank elastic stiffness tensors of the matrix and fiber-bundles.

3.2. Elasto-plasticity

After crystal sliding takes place, besides the power stored in the elastic medium and fiber-bundles in Eq.(26), all active slip systems will dissipate power which can be obtained by superposition, i.e.

$$\dot{W} = \int_\Psi \int_\Phi \rho_s \tau_s \dot{\gamma}_s d\Phi d\Psi = \int_\Psi \int_\Phi \rho_s \dot{\gamma}_s P_s d\Phi d\Psi : S_m. \tag{32}$$

According to the power conjugate principle, the strain rate tensor produced by the crystal sliding can be obtained by use of Eqs.(4), (18) and (32) as:

$$\dot{E}_{ms} = \int_\Psi \int_\Phi \rho_s \dot{\gamma}_s P_s d\Phi d\Psi = \overline{C}_{ms} : \dot{S}_m, \qquad \overline{C}_{ms} = \int_\Psi \int_\Phi \frac{\rho_s}{h_s} P_s \otimes P_s d\Phi d\Psi \tag{33}$$

where $\overline{C}_{ms}$ is an average compliance tensor of the slip systems with ρ_s. Substitution of Eqs.(14) and (33) into Eq.(1) yields

$$\dot{E} = (\overline{C}_{me} + \overline{C}_{ms}) : \dot{S}_m. \tag{34}$$

Equation (34) describes the stress-strain relation of pure matrix.

Thus, the total power of REV equals a sum of power dissipated by the three types of elements, i.e.

$$S : \dot{E} = V_m S_m : \dot{E}_{me} + V_m S_m : \int_\Psi \int_\Phi \rho_s \dot{\gamma}_s P_s d\Phi d\Psi + V_f \left\{ \int_\Omega \rho_f \sigma_f P_f d\Omega \right\} : \dot{E}. \tag{35}$$

The first term on the right-hand side of Eq.(35) is the power stored in the elastic medium; the second is that dissipated by the active slip systems; and the third is the one of fiber-bundles which include the powers stored in fiber-bundles and dissipated by the interface sliding. The total power in Eq.(35) can be rewritten as

$$S : \dot{E} = V_m S_m : \{\dot{E}_{me} + \dot{E}_{ms}\} + V_f S_f : \dot{E}. \tag{36}$$

By substituting Eq.(1) into Eq.(36), the overall stress in REV can be obtained:

$$S = V_m S_m + V_f S_f. \tag{37}$$

The overall stress rate tensor is

$$\dot{S} = V_m \dot{S}_m + V_f \dot{S}_f. \tag{38}$$

Equations (34) and (29) can be substituted into Eq.(38) to yield

$$\dot{S} = \{V_m(\overline{C}_{me} + \overline{C}_{ms})^{-1} + V_f \overline{K}_f\} : \dot{E} \tag{39}$$

where $\overline{C}_{ms}$ is an average compliance tensor of the slip systems and

$$\overline{C}_{ms} = \int_\Psi \int_\Phi \frac{\rho_s}{h_s} P_s \otimes P_s d\Phi d\Psi \tag{40}$$

where

$$h_s = \begin{cases} \infty & \text{when} \quad \tau_{-cr} < \tau_s < \tau_{+cr} \\ \overline{h}_s & \text{when} \quad \tau_s \le \tau_{-cr} \text{ or } \tau_s \ge \tau_{+cr}. \end{cases}$$

$\overline{K}_f$ is an average stiffness tensor related with fiber-bundles

$$\overline{K}_f = \int_\Omega \rho_f E_f P_f \otimes P_f d\Omega \tag{41}$$

in which

$$E_f = \begin{cases} \overline{E}_f & \text{when} \quad \varepsilon_{-cr} < \varepsilon_f < \varepsilon_{+cr} \\ \overline{E}_{fs} & \text{when} \quad \varepsilon_f \le \varepsilon_{-cr} \text{ or } \varepsilon_f \ge \varepsilon_{+cr}. \end{cases}$$

Equations (39), (40) and (41) describe the elasto-plastic constitutive relation of whisker-reinforced metal-matrix composite. The overall stiffness tensor of composites depends not only on basic constituent parameters C_{me}, h_s, and E_f but also on the parameters of microstructure and sliding state of slip systems and fiber-bundles, i.e. ρ_s / h_s and $\rho_f E_f$, which can be determined in combination with the crystal and interface sliding criteria in Eqs.(19) and (25).

4. COMPARISON WITH FEM RESULTS

According to the crystal and interface sliding criteria, the macro yielding of composites means that at least a slip system in matrix is activated or an interface sliding takes place. That is to say, the yield surface of the composite will be the internal envelope hypersurface of hyperplanes determined by $P_s : S_m = \tau_{\pm cr}$ and concomitant hyperplanes in the stress space with those determined by $P_f : E_m =$

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$\varepsilon_{\pm cr}$ .The hypersurface includes the coupling of two sliding mechanisms [12].

The following takes the unidirectional fiber-reinforced B-Al composite with a perfect interface [7-8] as an example. Thus, the macro yielding is dominated by crystal sliding.Therefore, the yield surface is the internal envelope hypersurface of hyperplanes determined only by $\boldsymbol{P}_s : \boldsymbol{S}_m = \tau_{\pm cr}$. In numerical analysis, the REV of the present model material is divided into 876 slip systems homogeneously distributed in the 3-D elastic matrix with a unidirectional fiber-bundle. The elastic properties of constituents are: for the aluminum matrix, E_m=70GPa, ν_m =0.33, and for the boron fiber, E_f=420GPa. The tensile yield stress of the aluminum matrix, $\sigma_s = 2\tau_{cr0}$, that are taken from ref.[8].

The global yield hypersurface of composites in the 6-D stress space can provide an insight into the global elasto-plastic properties of composites. Generally, it can be illustrated by several yield surfaces in the 2-D stress subspaces. For transverse isotropic composites, such as unidirectional fiber-reinforced composites, the yield surface in 6-D stress space can degenerate into one in the 4-D space. The following four groups of yield surfaces can illustrate the global yielding of transverse isotropic composites in the stress space.

Figure 1 shows the predicted yield surfaces of unidirectional fiber-reinforced composites with V_f =0.3 in 4 subspaces: (a) in the $\{S_{11}, S_{12}\}$ subspace with a parameter S_{23}, (b) in the $\{S_{11}, S_{22}\}$ subspace with a parameter S_{12}, (c) in the $\{S_{22}, S_{23}\}$ subspace with a parameter, S_{12}, and (d) in the $\{S_{22}, S_{33}\}$ subspace with a parameter S_{23}. Figures 1 (a) and (b) reflect the yielding characteristics related to the preferred orientation, while Figures 1 (c) and (d) describe these on the transversely isotropic plane.

The predicted results show that the yielding of unidirectional fiber-reinforced composites becomes more difficult in the fiber direction than in the transverse isotropic plane. The predicted results are in qualitative agreement with numerical results obtained by FEM in refs.[7-8]. The present result is obtained by much less calculations than that by FEM because we only analyzed a REV but not a structure. The present constitutive relation can be applied to FEM. The effect of constituents, whisker orientation, and coupling of crystal and interface sliding mechanisms on the yield surface was discussed in ref.[12]. The further theoretical investigation and experimental verification will be the subject of separate publication.

5. FUNDAMENTAL FEATURES OF MODEL

5.1. Assembling

The material model consists of three types of essential elements. Each assembling of the elements may represent a composite that has its respective constitutive equation. The following are three typical examples:

(a) The assembling of the elastic medium, equivalent slip systems and fiber-bundles can model whisker-reinforced metal-matrix composites with meso-structures described by ρ_s and ρ_f . The corresponding constitutive equation is shown in Eqs.(39) to (41), in which the fiber reinforcing, interface sliding and crystal sliding mechanisms were taken into account [12].

(b) Fiber-reinforced polymer composites can be modeled with the elastic matrix and fiber-bundles included the effect of orientation distribution of fiber-bundles ρ_f [11][16]. The corresponding constitutive equation is described in Eqs.(31) and (41), which is based on the fiber

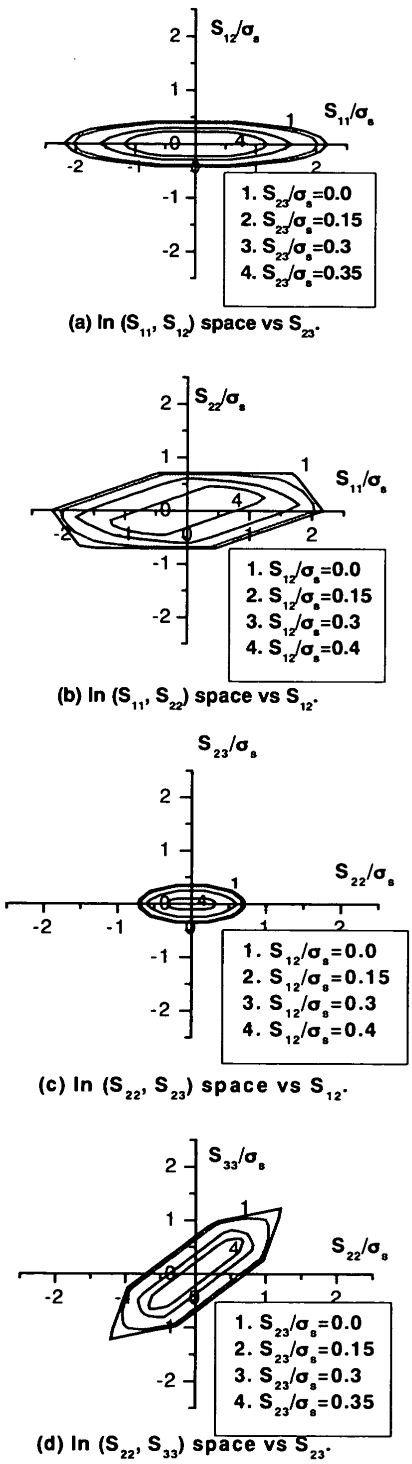

Fig.1. The variations of yield surfaces in 4 subspaces.

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reinforcing and interface sliding mechanisms.

(c) A simple material model is composed of the assembling of the elastic medium and slip systems that could represent polycrystalline metals. Crystal sliding becomes the only physical mechanism in the model, and its constitutive equation is written in Eqs.(34) and (40) including the effect of distribution of slip systems ρ_s [13-15].

5.2. Coupling of multiple mechanisms

The overall elasto-plastic behavior of composites is controlled by multiple mechanisms, which depend closely on the properties of the constituents and their interface. The crystal sliding mechanism could control the plasticity of composites with a strong-bond interface, while that of composites with a weak-bond interface could be dominated by the interface sliding mechanism. Otherwise, the couple of the crystal and interface sliding mechanisms will control the overall elasto-plastic performance of composites.

5.3. Anisotropy

The initial anisotropy can be described by the orientation density of slip systems and fiber-bundles ρ_s and ρ_f. However, plastic deformation can induce anisotropy of composites. The induced anisotropy can be varied with the distribution of activation slip systems and fiber-bundle with interface sliding, which is embodied in the variation of $\rho_s / \bar{h}_s$ and $\rho_f \bar{E}_{fs}$. The induced anisotropy is dependent on a specific deformation or loading path.

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