ELSEVIER Applied Mathematics and Computation 144 (2003) 1–2

APPLIED MATHEMATICS AND COMPUTATION

www.elsevier.com/locate/amc

A variational approach to the Burridge–Knopoff equation

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Abstract

A variational principle is obtained for the Burridge–Knopoff model for earthquake faults, and this paper considers an analytic approach that does not require linearization or perturbation.

Keywords: Burridge–Knopoff model; Earthquake; Variational principle

Investigations of the dynamics of slowly driven threshold systems have triggered renewed interest in the nonlinear dynamics of earthquake faults. The governing equation describing dynamics of earthquake faults is the well-known Burridge–Knopoff equation [1–3], which reads

$$\frac{\partial^2 u}{\partial t^2} = \xi^2 \frac{\partial^2 u}{\partial x^2} - u - F(u_t) + vt, \tag{1}$$

where $u_t = \partial u / \partial t$, u is the normalized displacement, ξ is the normalized sound velocity, v is the dimensionless driving velocity, F is the dimensionless friction force.

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doi:10.1016/S0096-3003(02)00379-X

Adomian [3] applied the decomposition method to the solution of the Burridge–Knopoff equation, in this paper, by the semi-inverse method [4,5], we obtain the following variational principle:

$$J(u) = \iint \left\{ \frac{1}{2}\left(\frac{\partial u}{\partial t}\right)^2 - \frac{1}{2}\xi^2\left(\frac{\partial u}{\partial x}\right)^2 - \frac{1}{2}u^2 + \Phi(u_t) + vut \right\} \mathrm{d}x\,\mathrm{d}t, \tag{2}$$

where Φ is calculated by the relation

$$\frac{\partial}{\partial t}\left(\frac{\partial \Phi}{\partial u_t}\right) = F(u_t). \tag{3}$$

In view of finite element method [6], the displacement u is approximated as

$$u(t,x) = \sum_{i=1}^{N} N_i u_i, \tag{4}$$

and

$$\frac{\partial u}{\partial t} = \sum_{i=1}^{N} \left(\frac{\partial u_i}{\partial t} N_i\right). \tag{5}$$

The variational equation (2) provides a more complete theoretical basis not only for the finite element applications [6], but also for many direct variational methods such as Ritz's, Trefftz's and Kantorovitch's methods. We can easily obtain an oscillatory mode by the Ritz's method.

References

[1] P.G. Akishin, M.V. Altaisky, I. Antoniou, A.D. Budnik, V.V. Ivanov, Burridge-Knopoff model and self-similarity, Chaos, Solitions and Fractals 11 (2000) 207–222.

[2] P. Hahner, Y. Drossinos, Dynamics of a creep-slip model of earthquake faults, Physica A 260 (1998) 391–417.

[3] G. Adomian, The Burridge-Knopoff model, Applied Mathematical and Computation 77 (1996) 131–132.

[4] J.H. He, Semi-inverse method of establishing generalized variational principles for fluid mechanics with emphasis on the turbomachinery aerodynamics, International Journal of Turbo & Jet-Engines 14 (1) (1997) 23–28.

[5] J.H. He, Variational theory for linear magneto-electro-elasticity, International Journal of Non-Linear Science and Numerical Simulation 2 (4) (2001) 309–316.

[6] G.L. Liu, A new finite element with self-adapting built-in discontinuity for shock-capturing in transonicflow, International Journal of Non-Linear Science and Numerical Simulation 1 (1) (2000) 25–30.