Microsystem Technologies 10 (2004) 307–314 © Springer-Verlag 2004
DOI 10.1007/s00542-003-0329-4

Influence of surface stress on frequency of microcantilever-based biosensors

Q. Ren, Y.-P. Zhao

Abstract Microcantilever-based biosensors have been found increasing applications in physical, chemical, and biological fields in recent years. When biosensors are used in those fields, surface stress and mass variations due to bio-molecular binding can cause the microcantilever deform or the shift of frequency. These simple biosensors allow biologists to study surface biochemistry on a micro or nano scale and offer new opportunities in developing microscopic biomedical analysis with unique characteristics. To compare and illustrate the influence of the surface stress on the frequency and avoid unnecessary and complicated numerical solution of the resonance frequency, some dimensionless numbers are derived in this paper by making governing equations dimensionless. Meanwhile, in order to analyze the influence of the general surface stress on the frequency, a new model is put forward, and the frequency of the microcantilever is calculated by using the subspace iteration method and the Rayleigh method. The sensitivity of microcantilever is also discussed.

1 Introduction

In cantilever-based sensing, micromechanical cantilever-based sensors are devices based on the measured changes of physical quantities such as resonance frequency, amplitude, deflection and quality factor (Q), whose applications have been found wildly pervading into physical, chemical, and biological field in recent years [1–5]. Peculiarly, biological applications, due to advances in high sensitivity, increased reliability, greatly reduced size and low costs, make increasing progress. Several groups have been investigating cantilever-based biological sensors [6–10]. When the micromechanical cantilever is used as the detection platform for the chemical or biological properties, one of the microscale effects, surface stress, plays an important role in sensing and must be considered correspondingly. Some experimental results have also demonstrated its significant influence on the physical properties of the cantilever [11, 12]. Therefore, theoretical understanding and analysis on the origin and behavior of the surface stress will be mainly critical in determining resonance frequency, amplitude, deflection as well as Q factor, and will also be useful in optimizing and improving the cantilever structure. Of all the above-mentioned parameters, the resonance frequency is most important.

In biological sensing, the available beam is V-shaped or rectangular micromechanical silicon nitride cantilever generally coated with other metal materials, such as a piece of gold film for the adsorption of the biomolecular on the microcantilever surfaces or a piece of chrome film for good adhesion between the first film and the substrate. To illustrate the problem generally, the rectangle cantilever is considered in this paper and Fig. 1 shows the schematic view of microcantilever as analyzed in [11]. In experiment, when the target moleculars are immobilized on the substrate or react with special film plated on the cantilever, surface stress is caused, defined as force per unit length, which affects the physical properties, mainly the frequency of the microcantilever.

2 Existing models

2.1 Model of the constant axial force acting at the free end of mirocantilever

Assuming the symmetry for the $x-y$ plane of the prismatic beam, the general equation for transverse free vibrations of a beam is

$$EI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}=0 \quad , \tag{1}$$

where the EI is the bending rigidity of beam, w is the deflection, x is the abscissa, t is time, ρ and A are the mass density and the area of beam cross-section, respectively. Here, the bending rigidity, EI, is assumed constant along the beam. For the micromechanical cantilever-based biosensors, the surface stress is caused by biomolecular binding on the substrate shown in Fig. 2. The surface stress can be expressed by the equivalent axial force N and equivalent moment M acting at the free end of the cantilever which are given by [13]

Received: 22 January 2003 / Accepted: 19 May 2003

Q. Ren, Y.-P. Zhao (✉)
State Key Laboratory of Nonlinear Mechanics (LNM),
Institute of Mechanics, Chinese Academy of Sciences,
Beijing 100080, China
e-mail: yzhao@lnm.imech.ac.cn

The supports from the Key Project from the Chinese Academy of Sciences (No. KJCX2-SW-L2), projects from the National Natural Science Foundation of China (No. 10225209, No. 19928205, No. 50131160739 and No. 10072068), and the National "973" project (No. G1999033103) are gratefully acknowledged.

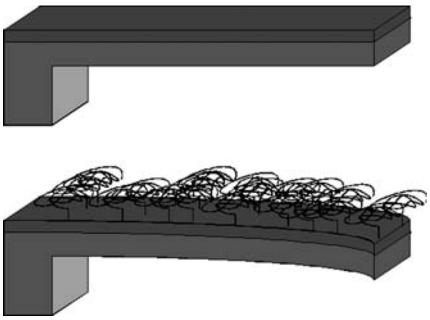

Fig. 1. Schematic of a microcantilever with biomolecule adsorption as analyzed in [11]

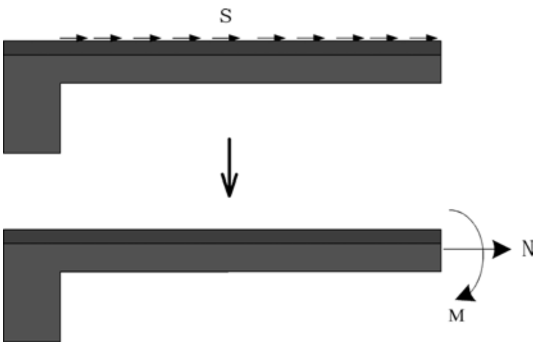

Fig. 2. Schematic of a microcantilever with surface stress action and its equivalent axial force N and equivalent moment M

$$\begin{aligned} N &= sl \\ M &= slh/2 \ , \end{aligned} \tag{2}$$

where s is the surface stress defined as force per unit length, and l, h are the length, the thickness of the cantilever, respectively. Considering the axial force the governing equation for free transverse vibration can be written as [13]

$$EI\frac{\partial^4 w}{\partial x^4} - N\frac{\partial^2 w}{\partial x^2} + \rho A\frac{\partial^2 w}{\partial t^2} = 0 \ . \tag{3}$$

And the resonant frequency of this model is complex and given by Eq. (19) in [13]. This model adopts the assumption that the axial force due to the surface stress should be constant and act at the free end of the cantilever.

2.2 Model of a taut string

The model of a taut string approximates the structure by substituting the spring constant caused by the surface stress for the effective rigidity of the cantilever as shown in Fig. 3. This model is simple to calculate the frequency because it neglects the bending rigidity of the cantilever. For this model, the governing equation for a taut string can be written as [14]

$$N\frac{\partial^2 w}{\partial x^2} + \rho A\frac{\partial^2 w}{\partial t^2} = 0 \ . \tag{4}$$

In [14] and [15], the resonant frequency of the cantilever is given by

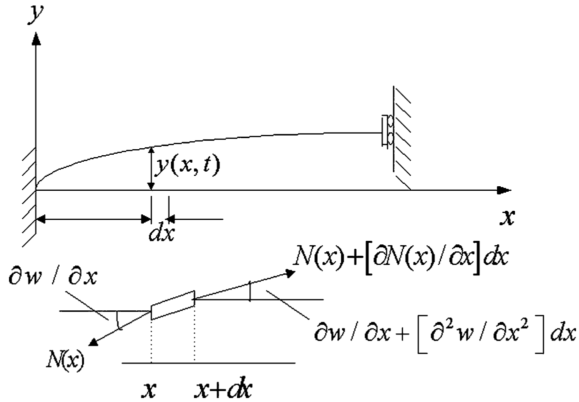

Fig. 3. Schematic of the vibration of a taut string

$$f = \frac{1}{4l}\sqrt{\frac{N}{nm}} = \frac{1}{4}\sqrt{\frac{s}{nm_b}} \ , \tag{5}$$

where N is the axial force equivalent to the surface stress

$$N = \int_0^L s\,\mathrm{d}l = sl \ , \tag{6}$$

m, m_b and n are the mass of unit length, mass of cantilever and the effective coefficient for different shapes of cantilevers, respectively. This model adopts the following assumptions to simplify the calculation of the frequency:

(i) the surface stress is sufficiently great and the bending rigidity is negligible; and
(ii) the surface stress must be positive to satisfy the tensile force.

In general, the whole system can also be treated as an effective mass connected in parallel to two springs with K contributed by bulk property and K_s contributed by the surface stress. This case is same to the model that the constant force acting at the free end of the mirocantilever, while its frequency is expressed by the simple form combined with the taut string model. Yet, it cannot deal with the general surface stress because the aforementioned models adopt the assumption that the axial force due to the surface stress should be constant along the cantilever.

3 Governing dimensionless numbers

Because the influence of the surface stress on the frequency can not be revealed directly from the expression of frequency of the aforementioned models, the dimensional analysis on the frequency of the microcantilever subjected to surface stress is necessary. By making the governing equation dimensionless, some dimensionless numbers with evident physical significance can be obtained. Introducing the dimensionless transformation as follows

$$X = \frac{x}{l}, \quad W = \frac{w}{h}, \quad T = \omega t \ , \tag{7}$$

then $\partial^n w/\partial x^n$ and $\partial^n w/\partial t^n$ are given as follows

$$\begin{aligned} \frac{\partial^n w}{\partial x^n} &= \frac{h}{l^n}\frac{\partial^n W}{\partial X^n} \\ \frac{\partial^n w}{\partial t^n} &= h\omega^n \frac{\partial^n W}{\partial T^n} \end{aligned}, \quad (n = 1, 2, 3 \ldots\ldots) \ . \tag{8}$$

Substituting Eqs. (7) and (8) into Eq. (3), the governing equation can be expressed in the dimensionless form

$$\frac{\partial^4 W}{\partial X^4} - \Pi_1 \frac{\partial^2 W}{\partial X^2} + \Pi_2 \frac{\partial^2 W}{\partial T^2} = 0 \ , \tag{9}$$

where two new dimensionless numbers are defined as follows

$$\Pi_1 = \frac{sl^3}{EI}, \quad \Pi_2 = \frac{\rho A l^4 \omega^2}{EI} \ . \tag{10}$$

The first dimensionless number, Π_1, denotes the ratio of the surface force s to the elastic bending force EI/l^3 per unit length, and the second one, Π_2, denotes the ratio of inertial force $\rho A l \omega^2$ to the elastic bending force EI/l^3 per unit length. The governing equation of microcantilever may be greatly affected by the dimensionless number, Π_1, because the microcantilever generally has the large slenderness ratio or is subjected to the great surface stress.

If the magnitude of the surface stress is approximately the same to that of the bending force, i.e., $\Pi_1 \approx 1$, the equation is given by Eq. (9). For harmonic vibration, one has

$$W = Y \sin(T + \varphi) \ , \tag{11}$$

where φ phase angle after dimensionless transformation. Substituting Eq. (11) into Eq. (9), it yields

$$\frac{d^4 Y}{dX^4} - \Pi_1 \frac{d^2 Y}{dX^2} - \Pi_2 Y = 0 \ . \tag{12}$$

The solution of Eq. (12) can be expressed as

$$Y = A \cos h PX + B \sinh PX + C \cos QX + D \sin QX \ , \tag{13}$$

where

$$\begin{aligned} P &= \sqrt{\frac{\sqrt{\Pi_1^2 + 4\Pi_2} + \Pi_1}{2}} \\ Q &= \sqrt{\frac{\sqrt{\Pi_1^2 + 4\Pi_2} - \Pi_1}{2}} \end{aligned} \ . \tag{14}$$

Taking the following boundary conditions into account

$$\begin{aligned} &Y|_{X=0} = 0 \\ &\left.\frac{dY}{dX}\right|_{X=0} = 0 \\ &\left.\frac{d^3 Y}{dX^3} - \Pi_1 \frac{dY}{dX}\right|_{X=1} = 0 \\ &\left.\frac{d^2 Y}{dX^2}\right|_{X=1} = 0 \end{aligned} \ , \tag{15}$$

the first order frequency equation can be obtained as follows

$$\begin{aligned} 2P^2Q^2 &+ (P^4 + Q^4)\cosh P \cos Q \\ &+ (P^3 Q - PQ^3)\sinh P \sin Q = 0 \end{aligned} \ . \tag{16}$$

This solution of frequency is given in [13], which is expressed by the dimensionless numbers in this paper.

If the magnitude of the surface stress is sufficiently small, i.e., $\Pi_1 << 1$, the second term, $\Pi_1 \frac{\partial^2 W}{\partial X^2}$, in Eq. (9) can be negligible and the equation is expressed as follows

$$\frac{\partial^4 W}{\partial X^4} + \Pi_2 \frac{\partial^2 W}{\partial T^2} = 0 \ . \tag{17}$$

This equation denotes transverse vibration of the beam. The first order frequency is given by

$$f = \frac{\omega_1}{2\pi} = \frac{3.516}{2\pi}\sqrt{\frac{EI}{\rho A l^4}} \ . \tag{18}$$

In this case, the dimensionless number, Π_2, can approximately be calculated as 3.516^2 when substituting the Eq. (18) into the expression of the dimensionless number, Π_2, in Eq. (10) and considering the first order frequency.

If the magnitude of the surface stress is sufficiently great, i.e., $\Pi_1 >> 1$, the first term, $\partial^4 W/\partial X^4$, in Eq. (9) can be negligible and the equation will be expressed as

$$\Pi_1 \frac{\partial^2 W}{\partial X^2} + \Pi_2 \frac{\partial^2 W}{\partial T^2} = 0 \ . \tag{19}$$

This is the governing equation of a taut string expressed by the dimensionless form, as referred in [15]. In [15], the equation of a taut string substitutes that of microcantilever subjected to the surface stress because of sufficiently great surface stress. So the fundamental natural frequency can be calculated as

$$f = \frac{\omega_1}{2\pi} = \frac{1}{4}\sqrt{\frac{s}{nm_b}} \ . \tag{20}$$

In this case, the ratio of the dimensionless number Π_2 to Π_1 can be calculated as $\Pi_2/\Pi_1 = \pi^2/4$, when the first order frequency is taken into account. By using Eq. (20), the calculation of the first order frequency of a cantilever is simplified. However, it requires that the general surface stress should be absolutely positive, and that the axial force equivalent to the surface stresses be sufficiently great. If the general surface stress is negative or is not sufficiently great, Eqs. (19) and (20) can not be applied any more.

4
A new model for general surface stress and the analysis compared with different models

As we discussed in the previous section, the two aforementioned modes are expected to abide by some assumptions, such as tensile surface stress or equivalent force acting at the free end of cantilever. Both models deal with the problem of constant axial force distributed along the cantilever. In reality, since the surface stress coverage varies as shown in Fig. 4, which leads that the axial forces due to surface stress vary along the cantilever, and that the solutions of the frequency of the previous models are not

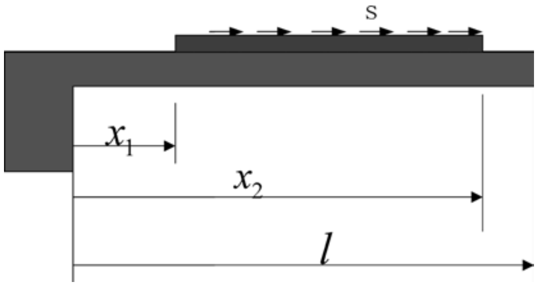

Fig. 4. Schematic of a microcantilever with fractional surface stresses coverage

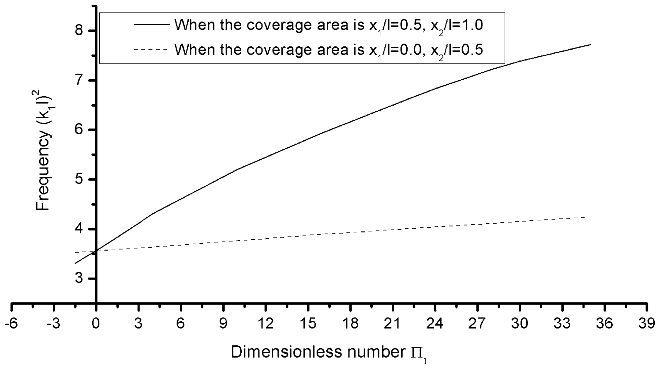

Fig. 5. The frequency when the different coverage area of surface stress

exact. So the aforementioned models cannot deal with the influence of the general surface stress on frequency.

In this case, we put forward a new mode, in which the axial forces due to the surface stress vary along the cantilever, to obtain the analysis of the general surface stress on the frequency.

If the varied axial forces are distributed along the cantilever, the governing equation can be expressed as [16]

$$EI\frac{\partial^4 w}{\partial x^4} - \frac{\partial}{\partial x}\left(N_{(x)}\frac{\partial^2 w}{\partial x^2}\right) + \rho A\frac{\partial^2 w}{\partial t^2} = 0 \ , \tag{21}$$

where the axial force, $N_{(x)}$ varies along the cantilever and is given as follows

$$N_{(x)} = sl \cdot f_{(x)}$$
$$f_{(x)} = \begin{cases} \frac{x_2}{l} - \frac{x_1}{l} & 0 < x < x_1 \\ \frac{x_2}{l} - \frac{x}{l} & x_1 < x < x_2 \end{cases} \ , \tag{22}$$

Substituting Eqs. (7), (8), (11), (22) into Eq. (21), the equation can be expressed in the dimensionless form

$$\frac{d^4 Y}{dX^4} - \Pi_1 \cdot f_{(X)} \cdot \frac{d^2 Y}{dX^2} - \Pi_1 \frac{df_{(X)}}{dX}\frac{dY}{dX} - \Pi_2 Y = 0 \ , \tag{23}$$

whose boundary conditions are taken as revised Eq. (15) in which the term, $\Pi_1 dY/dX$, is deleted. This problem can be solved by modern numerical techniques, such as the subspace iteration technique and the shooting techniques by means of Runge-Kutta integration or Newton method. We choose the subspace iteration technique to solve the first order frequency of this model. Figure 5 shows its result versus dimensionless number, Π_1 when the surface stress covers from the beam root to the beam midpoint ($x_1/l = 0$ and $x_2/l = 0.5$) and from the midpoint to the beam free-end ($x_1/l = 0.5$ and $x_2/l = 1$), respectively.

Although the numerical solution of first order frequency of this model is given, the influence of the surface stress on the frequency can not be obtained directly. In addition, the biologists expect to get a simple and exact formulation about the frequency of cantilever subjected to the surface stress. So the approximate solution of the new model is necessary, and can be given by using the Rayleigh method.

The deflection curve is given by [17]

$$w = \frac{mg}{2EI}\left(\frac{1}{2}l^2x^2 - \frac{1}{3}lx^3 + \frac{1}{12}x^4\right) \ , \tag{24}$$

where m is the density of a microcantilever per unit length, $m = \rho A$. If the axial forces vary along the cantilever, the potential energy of bending in the case can be expressed as

$$V = \frac{EI}{2}\int_0^l \left(\frac{d^2w}{dx^2}\right)^2 dx + \frac{1}{2}\int_0^{x_1}\left(\frac{dw}{dx}\right)^2 (x_2 - x_1)s\,dx$$
$$+ \frac{1}{2}\int_{x_1}^{x_2}\left(\frac{dw}{dx}\right)^2 (x_2 - x)s\,dx$$
$$= \frac{m^2g^2l^5}{40EI}\left\{1 + \frac{5sl^3}{EI}\left[\frac{1}{12}\left(\frac{x}{l}\right)^4 - \frac{1}{10}\left(\frac{x}{l}\right)^5 + \frac{1}{18}\left(\frac{x}{l}\right)^6\right.\right.$$
$$\left.\left. - \frac{1}{63}\left(\frac{x}{l}\right)^7 + \frac{1}{504}\left(\frac{x}{l}\right)^8\right]\Bigg|_{x_1}^{x_2}\right\} \ . \tag{25}$$

If the deflection during vibration is given by $w\cos\omega t$, the maximum kinetic energy of vibration will be

$$T = \frac{m\omega^2}{2}\int_0^l (w)^2 dx = \frac{13}{6480}\frac{m^3g^2\omega^2l^9}{(EI)^2} \ . \tag{26}$$

Putting Eq. (25) equal to Eq. (26), the following expression of the first order frequency is obtained

$$f = \frac{\omega_1}{2\pi} = \frac{9}{2\pi}\sqrt{\frac{2}{13}} \cdot \sqrt{1 + \beta\left(\frac{x_1}{l}, \frac{x_2}{l}\right) \cdot \Pi_1} \cdot \sqrt{\frac{EI}{ml^4}} \ , \tag{27}$$

where

$$\beta = 5\left[\frac{1}{12}\left(\frac{x}{l}\right)^4 - \frac{1}{10}\left(\frac{x}{l}\right)^5 + \frac{1}{18}\left(\frac{x}{l}\right)^6 - \frac{1}{63}\left(\frac{x}{l}\right)^7 + \frac{1}{504}\left(\frac{x}{l}\right)^8\right]\Bigg|_{x_1}^{x_2} .$$

The changes of the first order frequency due to the different positions and the length of the surface stress coverage can be investigated through the parameters $\beta(x_1/l, x_2/l)$. In general, the frequency of cantilever can be expressed as [17]

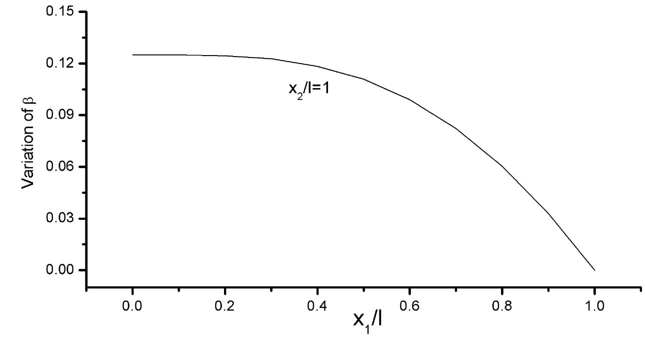

Fig. 6. Variation of β with x_1/l when $x_2/l = 1$

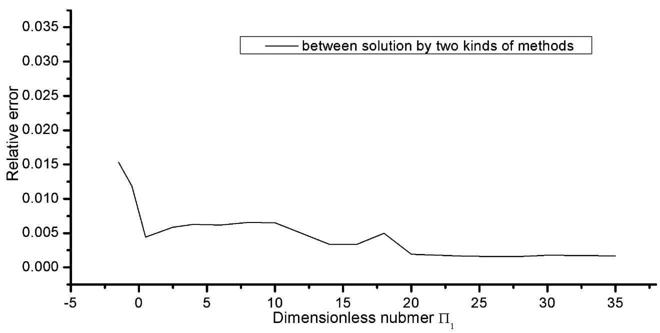

Fig. 7. Relative errors of frequencies calculated by the subspace iteration method and by the Rayleigh method

$$f = \frac{\omega_n}{2\pi} = \frac{(k_n l)^2}{2\pi}\sqrt{\frac{EI}{\rho A l^4}}\ , \tag{28}$$

where the dimensionless number, $(k_n l)^2$, determines the frequency of cantilever and is dependent on the boundary conditions and the applied forces such as the surface stress. When the first order frequency taken into considering, substituting Eq. (27) into Eq. (28), the dimensionless number $(k_1 l)^2$ can be expressed as

$$(k_1 l)^2 = 9\sqrt{\frac{2}{13}} \cdot \sqrt{1 + \beta\left(\frac{x_1}{l}, \frac{x_2}{l}\right) \cdot \Pi_1}\ . \tag{29}$$

Figure 6 shows the variation of β with the changes of x_1/l when $x_2/l = 1$. From this figure, it is clear that the position and the area of the surface stress coverage greatly influence on the frequency. If the rectangle cantilever is considered and the surface stress is linearly distributed along the cantilever ($x_1/l = 0$ and $x_2/l = 1$), the first order frequency can be given by

$$(k_1 l)^2 = 9\sqrt{\frac{2}{13}} \cdot \sqrt{1 + \frac{1}{8} \cdot \Pi_1}\ . \tag{30}$$

This is the simplified first order frequency of the microcantilever subjected to the axial forces linearly distributed along the cantilever using the dimensionless number.

It must be noted that an elastic beam represents a system with an infinitely large number of degrees of freedom. It can perform vibrations of various types. The choosing of a definite shape for the deflection curve in using Rayleigh's method is equivalent to introducing some additional constraints which reduce the system to one having one degree of freedom. Such additional constraints can only increase the rigidity of the system, i.e., increase the frequency of vibration and the frequency has the stable result near the shape of vibration, whose result differs by less than 1.5% from the exact numerical solution [17]. Figure 7 shows the relative error of frequency obtained by the numerical solution and Rayleigh's method. It is seen that the error of the approximate solution for this case is about 0.5 percent with increasing dimensionless number, Π_1. So the approximate solution by using Rayleigh's method can be used to determine the frequency.

In order to compare the frequency of this model with those of the existent models, we should rewrite the formulations of aforementioned models as dimensionless number, $(k_1 l)^2$ versus dimensionless number, Π_1. For the model of the constant force acting at the free end of mirocantilever, substituting Eq. (28) into Eq. (16), Eq. (16) can be expressed as the function of the dimensionless number, Π_1, and $(k_1 l)^2$

$$\begin{aligned} &2P^2Q^2 + (P^4 + Q^4)\cosh P \cos Q \\ &\quad + (P^3Q - PQ^3)\sinh P \sin Q = 0 \end{aligned}\ , \tag{31}$$

where P and Q can be expressed as follow

$$P = \sqrt{\frac{\sqrt{\Pi_1^2 + 4\left[(k_1 l)^2\right]^2} + \Pi_1}{2}}$$
$$Q = \sqrt{\frac{\sqrt{\Pi_1^2 + 4\left[(k_1 l)^2\right]^2} - \Pi_1}{2}}\ . \tag{32}$$

We can obtain the variation of $(k_1 l)^2$ with changes of magnitude of the dimensionless number, Π_1. For the model of taut string, substituting the expression $s = \Pi_1 \cdot EI/l^3$ into Eq. (20) and considering Eq. (28), the first order frequency of a taut string can be expressed as

$$(k_1 l)^2 = \frac{\pi}{2}\sqrt{\Pi_1}\ . \tag{33}$$

In Fig. 8, the continuous line denotes the model of constant force acting at the free end of the cantilever. The dotted line denotes model of a taut string, whose first order frequency is $(k_1 l)^2 = \frac{\pi}{2}\sqrt{\Pi_1}$. The dashed line denotes the model with axial forces linearly distributed along the cantilever, whose first order frequency is $(k_1 l)^2 = 9\sqrt{\frac{2}{13}} \cdot \sqrt{1 + \frac{1}{8} \cdot \Pi_1}$. The horizontal dash-dot line denotes microcantilever under no surface stress, whose first order frequency is $(k_1 l)^2 = 3.516$. The vertical dash-dot line denotes the lower limit of dimensionless number, $\Pi_1 = -\pi^2/4$, which is calculated by substituting the critical load $P_{cr} = -sl = \pi^2 EI/4l^2$ into dimensionless number, Π_1, of Eq. (10). From Fig. 8, the continuous line and the horizontal dash-dot line denote the upper limit and lower limit of the frequencies of the cantilever subjected to the surface stress, respectively. The first order frequency of the microcantilever subjected to the axial force linearly distributed along the cantilever, is between that of a taut string and a cantilever subjected to the constant axial force

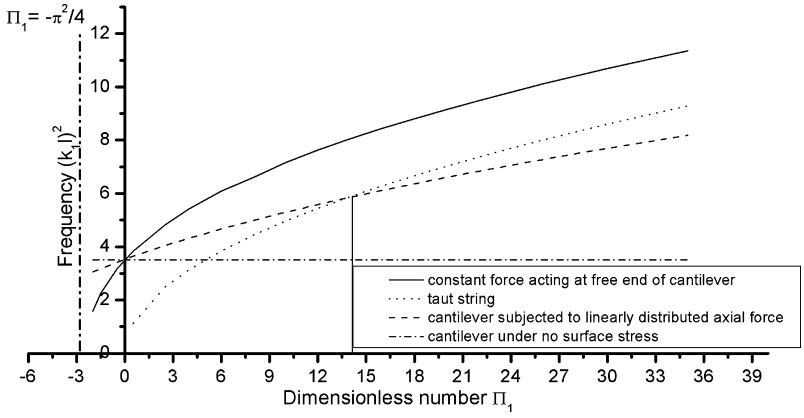

Fig. 8. Frequency $(k_1 l)^2$ versus the dimensionless number, Π_1

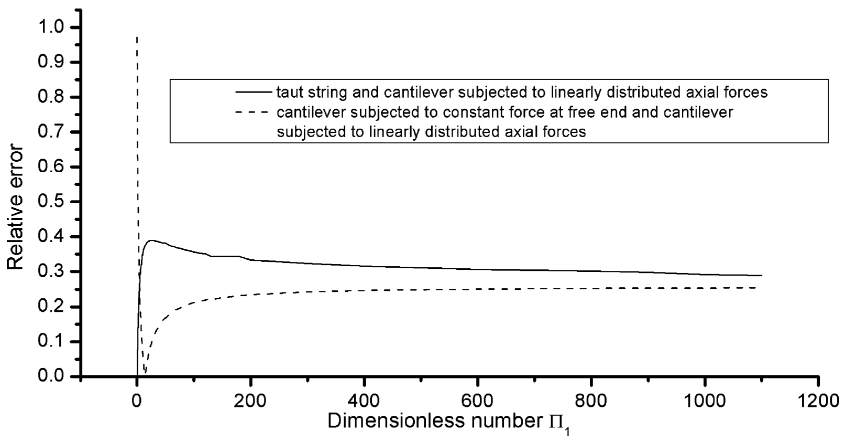

Fig. 9. Relative errors of frequencies of the different models versus the dimensionless number, Π_1

at the free end when the magnitude of dimensionless number, Π_1, is less than 14. The frequencies of those two models, the taut string and the microcantilever subjected to the axial forces linearly distributed along the cantilever, are equal when the magnitude of dimensionless number, Π_1, is approximately 14. When $\Pi_1 > 14$, the frequency of the microcantilever subjected to the axial forces linearly distributed along the cantilever increases more slowly than that of a taut string with increasing Π_1. The reason is that the influence of the bending rigidity on the frequency is more than that of the surface stress when magnitude of dimensionless number, $\Pi_1 < 14$. And with the increasing dimensionless number, Π_1, when $\Pi_1 > 14$, the surface stress plays more important role in the frequency than the bending rigidity, so the first order frequency of the new model is lower than that of a taut string.

In Fig. 9, the continuous line denotes relative error of the frequency of a taut string and the microcantilever subjected to axial force linearly distributed along the cantilever, and the dashed line denotes the relative error of the frequencies of two models, i.e., the microcantilever subjected to the constant force at the free end and the microcantilever subjected to the axial force linearly distributed along the cantilever. In this figure, when the magnitude of dimensionless number, Π_1, is less than 14, the relative error of the frequencies between the models, the taut string and the microcantilever with the axial force linearly distributed along the cantilever, decreases sharply, and the relative error of the frequencies between the models, the microcantilever subjected to the constant force at the free end and the microcantilever with the equivalent axial force linearly distributed along the cantilever, increases dramatically. From Fig. 9, the relative error of two curves approaches 25% when Π_1 approaching infinite. By analyzing Figs. 8 and 9, it can be compared that the frequencies of those two models, i.e., the taut string and the cantilever subjected to the constant axial force at the free end, increase more dramatically than that with equivalent axial force linearly distributed along the cantilever with increasing Π_1. So the influence of the surface stress on the frequency is different in different models and we cannot simply model the microcantilever applied by the general surface stress as the aforementioned models, i.e., the taut string and the microcantilever subjected to the constant axial forces at the free end of the cantilever.

5
Sensitivity of cantilever on surface stress of the new mode

In general, the resonant frequency, f, of an oscillating cantilever can be expressed as

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m^*}} \, , \tag{34}$$

where K is the effective spring constant and m^* is the effective mass of cantilever. The effective spring constant can be expressed as $K = 3EI/l^3$, and the effective mass is related to the mass of the cantilever, m_b, through the relation $m^* = nm_b$, where n is a geometric parameter, and it is 0.24 for a rectangular cross-section. It is clear that biomolecule adsorption can cause the mass to change as

Table 1. The equation of different models classified in accordance with the dimensionless number and the meaning of different equations

Π_1	Equation	Meaning of the equation	Magnitude of Π_2 when the first order frequency taken into account
$\Pi_1 \approx 1$	$\frac{\partial^4 W}{\partial X^4} - \Pi_1 \frac{\partial^2 W}{\partial X^2} + \Pi_2 \frac{\partial^2 W}{\partial T^2} = 0$	Equation of cantilever subjected to a constant axial force at free end	
$\Pi_1 << 1$	$\frac{\partial^4 W}{\partial X^4} + \Pi_2 \frac{\partial^2 W}{\partial T^2} = 0$	Transverse vibration equation of cantilever	$\Pi_2 = 3.516^2$
$\Pi_1 >> 1$	$\Pi_1 \frac{\partial^2 W}{\partial X^2} + \Pi_2 \frac{\partial^2 W}{\partial T^2} = 0$	Governing equation of taut string	$\frac{\Pi_2}{\Pi_1} = \frac{\pi^2}{4}$

well as the variation in spring constant, but contribution of from the mass due to adsorption is negligible [15]. When the microcantilever is subjected to the surface stress due to the biomolecule adsorption, rewriting the Eq. (27) in the form of Eq. (34), the frequency of the microcantilever calculated by the new model can be given as

$$f = \frac{1}{2\pi}\sqrt{\frac{K + K_s}{m^*}} \ , \tag{35}$$

where K_s is the function of $\beta(x_1/l, x_2/l)$ and the dimensionless number, Π_1. If the surface stress coverage is full of the length of the microcantilever, the K_s can be given by

$$K_s \approx \tfrac{1}{8}\Pi_1 K \ . \tag{36}$$

So the change of the resultant surface stress, δs, can be calculated from

$$\delta s = \tfrac{32}{3}\pi^2(f_2^2 - f_1^2)nm_b \ , \tag{37}$$

where f_1 and f_2 are the resonant frequencies before and after adsorption, respectively. This interpretation of frequency shift can be used to calculate the general surface stress due to adsorption. If the adsorption characteristic of the biomolecule is considered, the adsorption amount of the biomolecule can also be calculated in accordance with the change of the surface stress.

To illustrate the degree of adsorption, it is necessary to define the surface-stress sensitivity of a microcantilever as

$$\alpha_s = \lim_{\Delta s \to 0} \frac{1}{f}\frac{\Delta f}{\Delta s} = \frac{1}{f}\frac{\mathrm{d}f}{\mathrm{d}s} \ , \tag{38}$$

in accordance with the mass sensitivity [18, 19], where Δs and $\mathrm{d}s$ are normalized to the active area of the cantilever. Substituting Eq. (35) into Eq. (38), the sensitivity of the surface stress, α_s, is given by

$$\alpha_s = \frac{3}{16\left(K + \frac{3}{8}s\right)} = \frac{1}{16\left(\frac{EI}{l^3} + \frac{1}{8}s\right)} \ . \tag{39}$$

From the above result, we can conclude that the sensitivity of the cantilever is inversely proportional to the effective spring constant when the surface stress is slight. And the surface-stress sensitivity for this case is determined when the microcantilever is fabricated. We can design its geometrical shape and adopt different materials to control the detection sensitivity in various conditions.

6
Conclusions

In conclusion, two issues about the influence of the surface stress on the first order frequency of cantilever-based biosensor are considered in this paper. One is to make dimensionless of the governing equations of the microcantilever subjected to the surface stresses to obtain two models. Thereby, the influence of the surface stress on the first order frequency of the different models is analyzed and compared.

Another issue is to put forward the new model in which the axial forces are not constant but vary along the cantilever. Its first order frequency is obtained by using the subspace iteration method and Rayleigh's method. Adopting this model, the influence of the general surface stress on the frequency is dealt with. And this model is more suitable for the actual situation than other models.

Although the first order frequency of the microcantilever subjected to the general surface stress is obtained by the subspace iteration method and Rayleigh's method, the detection using the frequency in experiment is limited because the microcantilever is generally immerged into the liquid and the scale of cantilever is so little that the viscosity of the liquid and the thermal effects should be considered in real experiment. All those questions will be discussed in later paper.

References

1. **Binnig G; Quate CF** (1986) Atomic force microscope. Phys Rev Lett 56: 930–932
2. **Martin Y; Wickramasinghe HK** (1987) Magnetic imaging by "force microscopy" with Å resolution. Appl Phys Lett 50(20): 1455–1457
3. **Rugar D; Mamin HJ; Guetner P** (1989) Improved fiber-optic interferometer for atomic force microscopy. Appl Phys Lett 55(25): 2588–2930
4. **Oden PI; Thundat T; Wachter EA; Warmack RJ** (1996) Remote infrared radiation detection using piezoresistive mirocantilevers. Appl Phys Lett 69(26): 1–3
5. **Boisen A; Thaysen J; Jensenius H; Hansen O** (2000) Environmental sensors based on micromachined cantilevers with integrated read-out. Ultramicroscopy 82: 11–16
6. **Gotszalk T; Grabiec P; Shi F; Dumania P; Hudek P; Rangelow IW** (1998) Fabrication of multipurpose AFM/SCM/SEP Microprobe with integrated piezoresistive deflection sensor isolated conductive tip. Microelectronic Eng 41/42: 477–480
7. **Fritz J; Baller MK; Lang HP; Rothuizen H; Vettiger P; Meyer E; Guntherodt H; Gerber Ch; Gimzewski K** (2000) Translating biomolecular recognition into nanomechanics. Science 288: 316–318
8. **Moulin AM; O'Shea SJ; Welland ME** (2000) Microcantilever-based biosensors. Ultramicroscopy 82: 23–31
9. **Thaysen J; Yalçinkaya AD; Vettiger P; Menon A** (2002) Polymer-based stress sensor with integrated readout. J Phys D: Appl Phys 35: 2698–2703
10. **Raiteri R; Grattarola M; Berger R** (2002) Micromechanics senses biomolecules. Materials Today, January issue, 22–29

11. **Wu G; Ji H; Hansen K; Thundat T; Datar R; Cote R; Hagan MF; Chakraborty AK; Majumdar A** (2001) Origin of nanomechanical cantilever motion generated from biomolecular interactions. Proc Natl Acad Sci USA 98(4): 1560–1564
12. **Raiteri R; Nelles G; Butt HJ; Knoll W; Skladal P** (1999) Sensing of biological substances based on the bending of microfabricated cantilevers. Sensors Actuators B 61: 213–217
13. **Lu P; Shen F; O'Shea SJ; Lee KH; Ng TY** (2001) Analysis of surface effects on mechanical properties of microcantilevers. Material Phys Mech 4: 51–55
14. **Bishop RED; Johnson DC** (1960) The mechanics of vibration. Cambridge University Press, New York
15. **Chen GY; Thundat T; Wachter EA; Warmack RJ** (1995) Adsorption-induced surface stress and its effects on resonance frequency of microcantilevers. J Appl Phys 77(8): 3618–3622
16. **Landau LD; Lifshitz EM** (1986) Theory of Elasticity. Pergamon Press. Third Edition
17. **Timoshenko S** (1937) Vibration problems in engineering. D.Van Nostrand Company, Inc. Second Edition
18. **Ward MD; Buttry DA** (1996) In situ interfacial mass detection with piezoelectric transducers. Science 249: 1000–1007
19. **Thundat T; Oden PI; Warmack RJ** (1997) Microcantilever sensors. Microscale Thermophys Eng 1: 185–199