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The stability of a vertical single-walled carbon nanotube under its own weight

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Abstract

It has been reported recently that single carbon nanotubes were attached to AFM tips to act as nanotweezers. In order to investigate its stability, a vertical single-walled carbon nanotube (SWCNT) under its own weight is studied in this paper. The lower end of the carbon nanotube is clamped. Firstly the governing dimensionless numbers are derived by dimensional analysis. Then the theoretical analysis based on an elastic column model is carried out. Two ratios, i.e., the ratio of half wall thickness to radius (t/R) and the ratio of gravity to elastic resilience ($\rho gR/E$), and their influences on the ratio of critical length to radius are discussed. It is found that the relationship between the critical ratio of altitude to radius and ratio of half thickness to radius is approximately linear. As the dimensionless number $\rho gR/E$ increases, the compressive force per unit length (weight) becomes larger, thus critical ratio of altitude to radius must become smaller to maintain stability. At last the critical length of SWCNT is calculated. The results of this paper will be helpful for the stability design of nanotweezers-like nanostructures.

Keywords: Carbon nanotubes; Stability; Critical length; Nanotweezer

1. Introduction

Carbon nanotubes are made of tiny sheets of graphite, which are the hexagonal lattices of carbon. In this specific case, carbon nanotubes are thought to be made by wrapping into tight cylinders. Since the discovery of carbon nanotubes [1], numerous experimental and theoretical investigations have been carried out. It is shown that carbon nanotube can potentially be used in various fields because of its superior mechanical, structural and electronic properties to other known materials [2–5]. These nanotubes would be the preferred building block for supporting system for the space elevator [6]. The nanotubes would be woven into long strands and then these ribbons of nanotubes would be entwined into one paper-thin and metre-wide ribbon. The commercial production of carbon nanotubes in the near future will turn the space elevator into reality. From the point of view of materials selection, the resonant performance index of single-walled carbon nanotube (SWNT) resulting in E/ρ is one order of magnitude higher than current materials for MEMS, therefore SWCNT possess extraordinary performance for resonant applications [7]. The large-scale synthesis of pure carbon nanotubes is the fundamental requirement for proposed applications during those investigations. The most prevalent method to synthesize aligned carbon nanotubes is to make them grow perpendicularly to the surface of substrate. An isolated and long carbon nanotube will offer more opportunities for both fundamental research and technological applications. S. Akita et al. [8] attached two carbon nanotubes on the metal electrodes patterned on a conventional Si tip to make up a nano-electro-mechanical systems (NEMS) tweezers (Fig. 1). The application of a dc voltage to the two nanotube arms induces their movement to approach each other. It is clearly that the stability of carbon nanotube under various kinds of loading (electrostatic, weight, van der Waals force, etc.)

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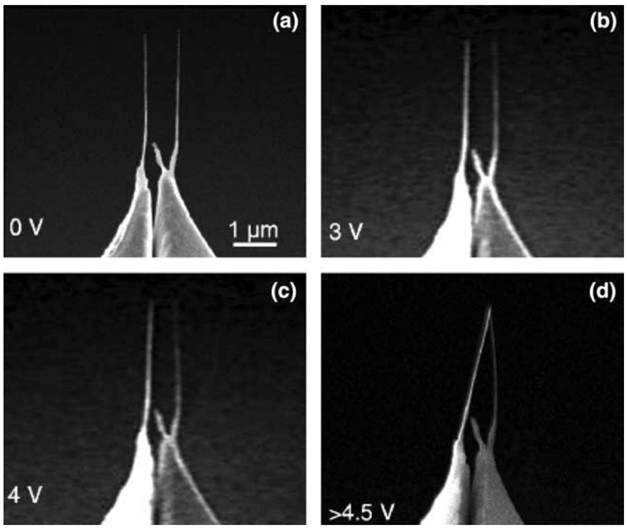

Fig. 1. SEM of the motion process of nanotube arms in a pair of nanotweezers as a function of the applied voltage (from [8]).

plays an important role in the manipulation, and particularly, a vertical SWCNT can lose its stability due to its own weight since its weight is linearly proportional to its length. As a result, the critical length of the arm (SWCNT) is a crucial parameter for the design of the NEMS tweezers.

The aim of the present paper is to find the governing dimensionless parameters and, what is more important, give the formula of the maximum length for a vertical SWCNT with its lower end clamped. The analysis is based on an elastic column model.

2. Dimensional analysis

Consider the stability of a vertical SWCNT under its own weight, with the lower end clamped, as schematically shown in Fig. 2. Without loss of generality, we first take a dimensional analysis for this problem to find the governing dimensionless parameters. The functional relationship for the critical stable length of a vertical SWCNT is

$$l_{\mathrm{cr}} = f(E, \rho g, R, t), \tag{1}$$

where l_{cr} is the critical length of SWCNT, E the Young's modulus, ρ the density of nanotube, g acceleration of gravity, R the radius of nanotube middle plane, and t the half wall thickness of SWCNT.

The corresponding dimensions of the quantities in Eq. (1) are

$$[l_{\mathrm{cr}}] = [R] = [t] = L;$$

$$[E] = ML^{-1}T^{-2};$$

$$[\rho g] = ML^{-2}T^{-2}.$$

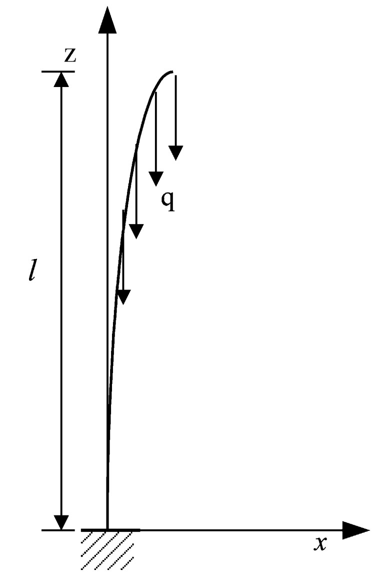

Fig. 2. Schematic of the single-walled carbon nanotube under its weight, the lower end being clamped.

Then, the basic dimensions are M, L and T. By Buckingham's π-theorem, Eq. (1) can be reduced to the dimensionless form as follows

$$\frac{l_{\mathrm{cr}}}{R} = f\left(\frac{\rho g R}{E}, \frac{t}{R}\right). \tag{2}$$

Dimensional analysis gives the general relationships among the independent dimensionless numbers as in Eq. (2). Therefore, the governing dimensionless numbers for l_{cr}/R are $\rho gR/E$ and t/R. The dimensionless number $\rho gR/E$ is the ratio of gravity to elastic resilience. The dimensional analysis assists our understanding of this stability problem. However, the relative importance of each dimensionless number has to be determined by analytical and numerical studies.

3. Elastic column model

To determine the specific function in Eq. (2), elastic column model is used. The governing equation is given by [9] as follows

$$EIx''' + q(l - z)x' = 0, \tag{3}$$

where I is the moment of inertia, q the weight of the nanotube per unit length, l the length of SWCNT and z is measured from the lower end (See Fig. 3).

At the clamped end ($z = 0$) both the displacement and the rotation angle are zero. At the free end ($z = l$), the boundary conditions of zero bending moment and shear force must be satisfied, and the zero shear force boundary condition for the free end $EIx'''(z = l) = 0$ can be automatically satisfied from Eq. (3). Therefore, the boundary conditions are

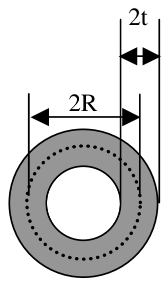

Fig. 3. Schematic of cross-section of SWCNT.

$$x' = 0 \text{ when } z = 0, \tag{4}$$

$$x'' = 0 \text{ when } z = l, \tag{5}$$

The moment of inertia for a nanotube with radius of middle plane, R, and thickness, $2t$, is

$$I = \frac{1}{4}\pi\left[(R+t)^4 - (R-t)^4\right]. \tag{6}$$

The weight of the nanotube per unit length can be described as

$$q = \rho g \pi\left[(R+t)^2 - (R-t)^2\right]. \tag{7}$$

Thus Eq. (3) can be rewritten as

$$2(R^2+t^2)Ex''' + 4\rho g(l-z)x' = 0. \tag{8}$$

The dimensionless numbers are introduced as

$$X = \frac{x}{R}, \quad Z = \frac{z}{R}, \quad x' = X', \quad x''' = \frac{1}{R^2}X'''.$$

Thus Eq. (8) becomes the dimensionless form

$$2\left[1+\left(\frac{t}{R}\right)^2\right]X''' + \frac{4\rho g R}{E}\frac{(l-z)}{R}X' = 0. \tag{9}$$

The dimensionless forms of boundary conditions in (4) and (5) become

$$X' = 0 \text{ when } Z = 0, \tag{10}$$

$$X'' = 0 \text{ when } Z = \frac{l}{R}. \tag{11}$$

Let $X' = U$, Eq. (9) can be rewritten as

$$2\left[1+\left(\frac{t}{R}\right)^2\right]U'' + \frac{4\rho g R}{E}\frac{(l-z)}{R}U = 0. \tag{12}$$

The corresponding boundary conditions are

$$U = 0 \text{ when } Z = 0, \tag{13}$$

$$U' = 0 \text{ when } Z = \frac{l}{R}. \tag{14}$$

Let $t_1 = 1+(t/R)^2$, $P = 4\frac{\rho g R}{E}$, $L = \frac{l-z}{R}$, then

$$2t_1U'' + PLU = 0. \tag{15}$$

Let $\eta = \frac{2}{3}\sqrt{TL^3}$, $V = UT^{-\frac{1}{6}}L^{-\frac{1}{2}}$, Eq. (15) can be rewritten as Bessel equation

$$\frac{d^2V}{d\eta^2} + \frac{1}{\eta}\frac{dV}{d\eta} + \left(1 - \frac{1/3^2}{\eta^2}\right)V = 0, \tag{16}$$

where $T = P/2t_1$.

The Bessel equation of (16) has the general solution

$$V = AJ_{\frac{1}{3}}(\eta) + BJ_{-\frac{1}{3}}(\eta). \tag{17}$$

From $V = UT^{-\frac{1}{6}}L^{-\frac{1}{2}}$, Eq. (17) can be rewritten as

$$U = T^{\frac{1}{6}}L^{\frac{1}{2}}V = T^{\frac{1}{6}}L^{\frac{1}{2}}\left[AJ_{\frac{1}{3}}(\eta) + BJ_{-\frac{1}{3}}(\eta)\right]. \tag{18}$$

Eq. (18) could be divided into two formulae:

$$T^{\frac{1}{6}}L^{\frac{1}{2}}AJ_{\frac{1}{3}}(\eta) = \frac{T^{\frac{1}{3}}LA}{3^{\frac{1}{3}}\Gamma\left(\frac{4}{3}\right)}\left\{1 + \sum_{k=1}^{\infty}\frac{(-1)^k(TL^3)^k}{k!\cdot 3^k\cdot[4\cdot 7\cdots(3k+1)]}\right\}. \tag{19}$$

$$T^{\frac{1}{6}}L^{\frac{1}{2}}BJ_{-\frac{1}{3}}(\eta) = \frac{3^{\frac{1}{3}}B}{\Gamma\left(\frac{2}{3}\right)}\left\{1 + \sum_{k=1}^{\infty}\frac{(-1)^k(TL^3)^k}{k!\cdot 3^k\cdot[2\cdot 5\cdot 8\cdots(3k-1)]}\right\}. \tag{20}$$

Using the boundary conditions (14) in the Eqs. (18)–(20), coefficient A must satisfy the following equation

$$A = 0. \tag{21}$$

Substituting (19)–(21) into (13), we obtain

$$1 + \sum_{k=1}^{\infty}\frac{(-1)^k(TL^3)^k}{k!\cdot 3^k\cdot[2\cdot 5\cdot 8\cdots(3k-1)]} = 0. \tag{22}$$

Let $\lambda = T(l/R)^3$, the minimum λ satisfying Eq. (22) can be computed by numerical iteration method as

$$\lambda_{\min} = 7.84, \tag{23}$$

then

$$2\frac{\rho g R}{E}\frac{1}{1+\left(\frac{t}{R}\right)^2}\left(\frac{l_{cr}}{R}\right)^3 = 7.84,$$

or

$$\left(\frac{l_{cr}}{R}\right)^3 = 3.92\frac{E}{\rho g R}\left[1+\left(\frac{t}{R}\right)^2\right]. \tag{24}$$

The above equation is the required relationship among the dimensionless numbers in Eq. (2).

4. Examples and discussion

If $t/R = 1$, the SWCNT is equivalent to a solid rod with the radius ($2t$). In this case, Eq. (24) is reduced to

$$\left(\frac{l_{cr}}{R}\right)^3 = 7.84\frac{E}{\rho g R}, \tag{25}$$

which is the same as the result of [9].

The relationship between l_{cr}/R and t/R is schematically shown in Fig. 4. The thickness of SWCNT is taken as 0.34 nm and the Young's modulus is taken as 1 TPa. The value of t/R is physically confined to 0.05–0.85, corresponding to $R = 3.4$ and $R = 0.2$ nm, respectively. The curve only in this range is shown up. It is noted that the ratio l_{cr}/R increases monotonically with the ratio t/R. The relationship between l_{cr}/R and t/R is approximately linear except for the beginning part of curve. The beginning of the curve is steeper than the rest. It means that l_{cr}/R decreases dramatically as t/R (<0.1) reduces, i.e., R (>1.7 nm) increases. Fig. 5 shows that the ratio l_{cr}/R decreases monotonically with the increase of the dimensionless number $\rho gR/E$. The ratio $\rho gR/E$ in Fig. 5 is physically confined to 2.6068×10^{-18}–4.43156×10^{-17} according to the value of R. It is noted that the ratio t/R is taken as the same range for both Figs. 4 and 5.

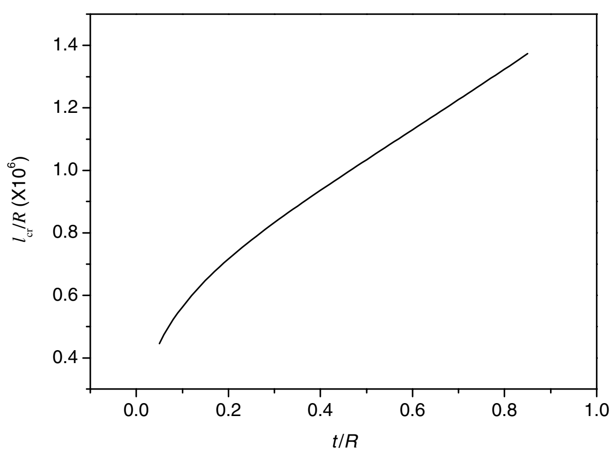

Fig. 4. The relationship between t/R and l_{cr}/R.

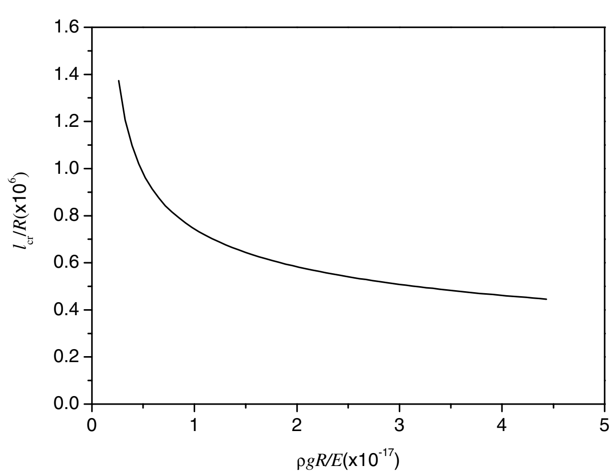

Fig. 5. The relationship between $\rho gR/E$ and l_{cr}/R.

It is found that the magnitude of critical ratio of altitude to radius of middle plane of SWCNT is about 10^6 in Figs. 4 and 5 because of the high Young's modulus and the low density.

Let us consider several actual examples. Eq. (24) can be rewritten as

$$l_{cr} = 1.576\sqrt[3]{\frac{E}{\rho g}(R^2 + t^2)}. \qquad (26)$$

Eq. (26) can be used to calculate the theoretical critical length of SWCNT. The density ρ of 1.33×10^3 Kg/m^3, the Young's modulus E of 1 TPa, the thickness of SWCNT $2t$ of 0.34 nm are used which are responsible for the SWCNTs. The different radii of middle plane are used during the computation. Sawada and Hamada [16] predicted a critical tube diameter of 0.4 nm by using Tersoff's potential method to the calculate cohesive energies of carbon nanotubes. Thus the radius of middle plane, less than 0.2 nm, is disregarded. These material parameters are substituted into Eq. (26) and the critical

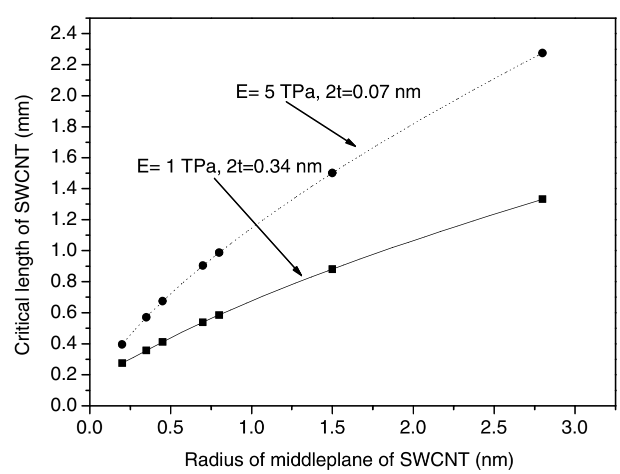

Fig. 6. The schematic of the radius of middle plane of SWCNT versus the critical length of SWCNT. The squares and dots from left to right present the calculating results of 0.2, 0.35, 0.45, 0.7, 0.8, 1.5 and 2.8 nm, respectively.

Table 1
The critical lengths calculated by Eq. (24) according to different radii of middle plane of SWCNT

R (nm)	0.2 [10]	0.35 [11]	0.45 [12]	0.7 [13]	0.8 [14]	1.5 [15]	2.8 [15]
l_{cr1} (mm)	0.2745	0.3569	0.4111	0.5381	0.5857	0.8813	1.332
l_{cr2} (mm)	0.3956	0.5706	0.6738	0.9035	0.9875	1.5008	2.275

The density ρ of SWCNT is 1.33×10^3 Kg/m^3.
l_{cr1}: $2t = 0.34$ nm and $E = 1$ TPA
l_{cr2} : $2t = 0.07$ nm and $E = 5$ TPa.

lengths of pure SWCNT are found. The different radii of middle plane and the critical lengths l_{cr1} of SWCNT are showed in Table 1 and Fig. 6.

Although most researchers have adopted the size of 0.34 nm as the wall thickness of SWCNT combined with Young's modulus of about 1 TPa [17], Yakobson et al. [18], Zhou et al. [19] and Vodenitcharova et al. [20] have suggested a much smaller thickness (about 0.07 nm) combined with Young's modulus of about 5 TPa. These parameters are used to calculate the critical length l_{cr2}, which is also showed in Table 1 and Fig. 6. It is found that l_{cr2} is about 50% larger than l_{cr1}.

5. Conclusion

The stability of pure SWCNT under its own weight is studied based on elastic column model, the application background is the NEMS nanotweezers. Dimensional analysis is applied to derive the governing dimensionless numbers for this problem. The theoretical analysis presents the relationship among the dimensionless numbers: l_{cr}/R, t/R and $\rho gR/E$. It is found that the ratio l_{cr}/R, which increases almost linearly with t/R, is very large because of the high Young's modulus and low density. The influence of the dimensionless number $\rho gR/E$ on the vertical stability of SWCNT is also discussed, and this dimensionless number is the ratio of gravity to elastic resilience. The analytical result shows that as the dimensionless number $\rho gR/E$ increases, which physically means that the compression (weight) per unit length becomes larger (with E being fixed), the critical ratio l_{cr}/R must decrease to maintain its stability of the vertical SWCNT. The formula for calculating the critical length of SWCNT is presented and several actual examples are calculated with different wall thickness and Young's modulus. It is believed that this analysis will assist the design for a NEMS tweezer with SWCNT as its arms.

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