Cross-relaxation dynamics on anomalous saturation processes in low pressure supersonic cw HF chemical laser amplifier *

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Abstract It is assumed that both translational and rotational nonequilibrium cross-relaxations play a role simultaneoulsy in low pressure supersonic cw HF chemical laser amplifier. For two-type models of gas flow medium with laminar and turbulent flow diffusion mixing, the expressions of saturated gain spectrum are derived respectively, and the numerical calculations are performed as well. The numerical results show that turbulent flow diffusion mixing model is in the best agreement with the experimental result.

Keywords: chemical laser, translational and rotational nonequilibrium, laminar and turbulent flow mixing, saturated gain spectrum.

In early years, it was found[1] that the saturated gain profile related to frequency in low pressure CO_2 molecular laser exhibits some mixed phenomena of inhomogeneous and homogeneous broadening. Subsequently the cross-relaxation effect of Doppler broadening CO_2 laser (CO_2 : N_2 : He = 0.1 : 0.1 : 0.8, 133.32 × 10 Pa) was investigated in experiment[2]. The operation pressure of cw HF chemical laser is generally maintained at the order of (1—10) × 133.32 Pa. At this pressure level, the gain medium should be inhomogeneously broadening, but it is not so in experiment[3]. In early stages, laser output power was investigated using the model of the translational and rotational equilibrium of lasing medium[4]. The models, including either rotational nonequilibrium[5] or translational nonequilibrium[6], were developed, and source flow effect in cw chemical laser was also investigated[7,8], in terms of which, some properties of cw chemical laser were interpreted reasonably. Stepanov et al.[9] have studied the saturation intensity of homogeneous and inhomogeneous broadening for several laser systems, such as the discharged CO_2 laser, the pulse and cw HF chemical laser, etc. Mirels[10] intended to study the saturated gain spectrum of HF chemical laser amplifier and saturated properties of laser oscillator using a model including both translational and rotational nonequilibriums. But in fact, the calculation performed and conclusion obtained were in the case of the translational nonequilibrium and rotational equilibrium. In refs. [11] and [12], the gain saturated law and saturated gain spectrum for two-type cases of laminar and turbulent flow mixing in gas flow medium have been studied using a model of

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"convection-nonequilibrium cross-relaxation". In the practical operation condition of supersonic cw HF chemical laser, it has not been reported that both translational and rotational nonequilibriums are considered simultaneously.

In this paper, the gain saturation theory of Doppler broadening low pressure cw chemical laser is further developed. Considering both translational and rotational nonequilibrium cross-relaxation effects, the general expressions of saturated gain spectrum are derived for the two-type models of laminar and turbulent flow diffusion mixing. The precise numerical calculation and illustration are performed. Comparison of calculation result with the experiment shows that the present theory is in good agreement with the experimental measurement.

1 Experimental alignment

In this paper, the calculation example is the experimental result in ref. [3]. The experimental alignment in ref. [3] is the supersonic diffusion cw HF chemical laser amplifier, which is illustrated in fig. 1. Experimental conditions are as follows: the gases of H_2 and F diffused from standard 36-slit supersonic nozzles and reacted to produce HF, and caused vibrational excitation of HF. The nozzle produced HF medium of 18 cm width and 2.5 cm height normal to the flow direction (x-direction in fig. 1). The operating pressure of an active medium was maintained at 6 ×133.32 Pa. This amplifier was illuminated by two cw HF lasers that were perpendicular to the HF flow direction. The beams from the two lasers were arranged concentrically in the amplifying medium, so that both interacted with the same molecules of the medium. One of the two lasers produced a high-power density up to 85 W/cm^2, so that it was sufficient for saturation of HF. The other was of much lower power, which probed the active medium from opposite side, 0.4 cm downstream of the slit nozzle exit. The probing laser swept frequency to determine the frequency dependent gain-profile of the investigated transition on the $P_2(8)$ line of HF. In order to distinguish the low-power gain signal from the colinear high-power saturating beam, the two beams were polarized orthogonally to each other. It can be seen in detail from ref. [3] that the total experimental appartus was complex and precise.

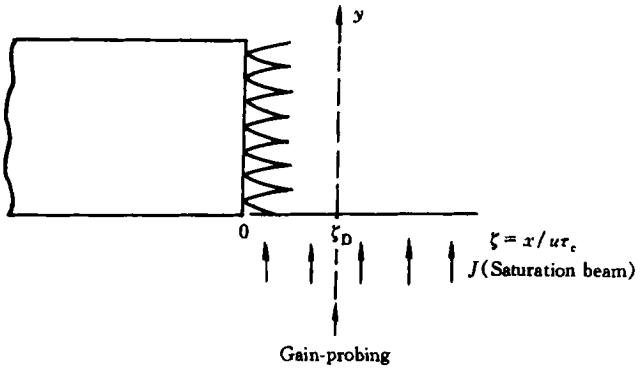

Fig. 1. Schematic diagram of the supersonic diffusion cw HF chemical laser amplifier.

2 Theoretical model

We denote the lower and upper levels of laser transition in the active medium by subscripts 1 and 2, respectively. ν and $\phi(\nu)$ are the laser frequency and the photon number passing through unit sectional area per unit time for frequency ν, respectively. It is assumed that an incident laser radiation is perpendicular to the gas flow direction (x-direction). ν' is the Doppler-shifted frequency of particles with thermal velocity v_y, and

$$\nu' = \nu_0\left(1 + \frac{v_y}{c}\right), \tag{1}$$

where ν_0 denotes the line center frequency of the active medium. Let $n_v(J, \nu')$ denote the num-

ber density of particles with vibrational evergy level v, rotational energy level J, and Doppler-shifted frequency ν'. The particles with Doppler frequency ν' emit homogeneous broadening profile with center frequency ν' and homogeneous width $\Delta\nu_h$. The transition cross-section $\sigma(\nu,\nu')$ is

$$\sigma(\nu,\nu') = \sigma_0 L(\nu,\nu'), \tag{2}$$

where

$$L(\nu,\nu') = \frac{(\Delta\nu_h/2)^2}{(\nu-\nu')^2+(\Delta\nu_h/2)^2}. \tag{3}$$

In the case of translational equilibrium, for the particles with a Maxwell thermal velocity distribution, the normalized Doppler profile is

$$g(\nu',\nu_0) = \frac{n_v(J,\nu')}{n_v(J)} = \frac{1}{\Delta\nu_D}\sqrt{\frac{4\ln 2}{\pi}}\exp\left[-4\ln 2\left(\frac{\nu'-\nu_0}{\Delta\nu_D}\right)^2\right], \tag{4}$$

where $\Delta\nu_D$ is Doppler width. In the case of rotational equilibrium, we neglect the effect of vibrational energy level on the rotational energy temperature T_R. The fraction $f(J)$ of particles at rotational evergy level J is

$$f(J) = \frac{n_v(J)}{n_v} = \frac{g(J)\exp(-E_J/kT)}{\sum_J g(J)\exp(-E_J/kT)}, \tag{5}$$

where $E_J = (2J+1)kT_R$ is the rotational energy. For HF, $T_R = 30.16$ K, $g(J) = 2J+1$ is the degeneracy of rotational energy level, and

$$n_v(J) = \int n_v(J,\nu)\,d\nu, \quad n_v = \sum_J n_v(J). \tag{6}$$

We consider the P-type laser transition,

$$v+1, \quad J-1 \rightarrow v, \quad J. \tag{7}$$

$P_2(8)$-line indicates $v+1=2$, $J=8$. It is assumed that the chemical reaction creates only upper level particles and they are in translational and rotational equilibriums. Generally, the effect of chemical pumping is related to diffusion streamwise distance of the gas medium. Assume that in the medium diffusion process, the medium particles are either in translational nonequilibrium or in rotational nonequilibrium, and there are translational cross-relaxation and rotational cross-relaxation effects. These effects cause the variation of the particle number of laser level which is proportional to the departure from translational equilibrium and rotational equilibrium, respectively. Then the master equations of active medium which diffuses in x-direction with a velocity u interacting with laser radiation are expressed as

$$\begin{aligned}\frac{\partial n_2(J_2,\nu')}{\partial t} + u\frac{\partial n_2(J_2,\nu')}{\partial x} &= n_p(x)\tau_p^{-1}g(\nu',\nu_0)f(J_2) - \tau_c^{-1}n_2(J_2,\nu')\\ &+ \tau_t^{-1}[g(\nu',\nu_0)n_2(J_2) - n_2(J_2,\nu')] + \tau_r^{-1}[f(J_2)g(\nu',\nu_0)n_2 - n_2(J_2,\nu')]\\ &- \sigma_0 L(\nu,\nu')\phi(\nu)[n_2(J_2,\nu') - \beta n_1(J_1,\nu')],\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\partial n_1(J_1,\nu')}{\partial t} + u\frac{\partial n_1(J_1,\nu')}{\partial x} &= \tau_c^{-1}n_2(J_2,\nu') + \tau_t^{-1}[g(\nu',\nu_0)n_1(J_1) - n_1(J_1,\nu')]\\ &+ \tau_r^{-1}[f(J_1)g(\nu',\nu_0)n_1 - n_1(J_1,\nu')]\\ &+ \sigma_0 L(\nu,\nu')\phi(\nu)[n_2(J_2,\nu') - \beta n_1(J_1,\nu')].\end{aligned} \tag{9}$$

The terms on the right-hand side of eq. (8) represent the effects of chemical pumping, collisional deactivation, translational cross-relaxation, rotational cross-relaxation and stimulated emission and absorption for the upper laser level. τ_p^{-1} is the pumping rate of chemical reaction, τ_p is the

pumping time, τ_c^{-1} is the rate of collisional deactivation of the upper laser level, τ_t^{-1} and τ_r^{-1} are the rate of translafional cross-relaxation and rotational cross-relaxation respectively, $\beta = g(J_2)/g(J_1)$. The terms on the right-hand side of eq. (9) are the corresponding expression for the lower laser level.

The following nondimensional quantities are introduced:

$$\zeta = x/u\tau_c, \quad R_t = \tau_c/\tau_t, \quad R_r = \tau_c/\tau_r, \quad R_p = \tau_c/\tau_p, \tag{10}$$

where R_p, R_t and R_r are the pumping intensity, translational cross-relaxation intensity and rotational cross-relaxation intensity, respectively. ζ is nondimensional diffusion streamwise distance. In the case of steady state, eqs. (8) and (9) become

$$\begin{aligned}\frac{\partial}{2\zeta}\Delta n(J_1,J_2,\nu) &= n_p(\zeta)\,R_p\,g(\nu',\nu_0)\,[f(J_2) - \Delta n(J_1,J_2,\nu) - \beta N(J_1,J_2,\nu)\\ &+ R_t\,g(\nu',\nu_0)\Delta n(J_1,J_2) - R_t\Delta n(J_1,J_2,\nu)]\\ &+ R_r\,g(\nu',\nu_0)\,[f(J_2)\,n_2 - \beta f(J_1)\,n_1] - R_t\Delta n(J_1,J_2,\nu)]\\ &- R_t I_\nu L(\nu,\nu)\Delta n(J_1,J_2,\nu)\end{aligned} \tag{11}$$

and

$$\begin{aligned}\frac{\partial}{2\zeta}N(J_1,J_2,\nu) &= n_p(\zeta)\,R_p\,g(\nu',\nu_0)\,f(J_2) + R_t\,g(\nu',\nu_0)\,N(J_1,J_2)\\ &- R_t N(J_1,J_2,\nu) + R_r\,g(\nu',\nu_0)\,[f(J_2)\,n_2 + f(J_1)\,n_1]\\ &- R_r N(J_1,J_2,\nu),\end{aligned} \tag{12}$$

where

$$\Delta n(J_1,J_2,\nu) = n_2(J_2,\nu) - \beta n_1(J_1,\nu), \quad \Delta n(J_1,J_2) = \int_0^\infty \Delta n(J_1,J_2,\nu)\,\mathrm{d}\nu', \tag{13a}$$

$$N(J_1,J_2,\nu) = n_2(J_2,\nu) + n_1(J_1,\nu), \quad N(J_1,J_2) = \int_0^\infty N(J_1,J_2,\nu)\,\mathrm{d}\nu',$$

$$N = \sum_{J_i} N(J_1,J_2); \tag{13b}$$

$$I_\nu = (1+\beta)\,\sigma_0\tau_t\phi(\nu) = J_\nu/J_s, \qquad J_\nu = \phi(\nu)\,h\nu, \quad J_s = h\nu/(1+\beta)\,\sigma_0\tau_t, \tag{13c}$$

J_s is laser intensity which characterizes line shape distortion resulting from saturation effect. I_ν is nondimensional intensity. Eqs. (11) and (12) are integrated for ν' and summed for J_i, we obtain

$$\begin{aligned}\frac{\partial}{2\zeta}\Delta n(J_1,J_2) &= n_p(\zeta)\,R_p f(J_2) - \Delta n(J_1,J_2) - \beta N(J_1,J_2) - R_r[f(J_2)\,n_2 - \beta f(J_1)\,n_1]\\ &- R_t\Delta n(J_1,J_2) + R_t I_\nu\int_0^\infty L(\nu,\nu)\Delta n(J_1,J_2,\nu)\,\mathrm{d}\nu',\end{aligned} \tag{14}$$

$$\frac{\partial}{2\zeta}N(J_1,J_2) = n_p(\zeta)\,R_p f(J_2) + R_r[f(J_2)\,n_2 + f(J_1)\,n_1] - R_r N(J_1,J_2), \tag{15}$$

$$\frac{\partial}{2\zeta}N = n_p(\zeta)\,R_p. \tag{16}$$

For HF chemical laser, the operation temperature is 400 K, the pumping time $\tau_p \approx 4\times10^{-7}$ s[9], collisional deactivation time $\tau_c = 1/k_c \approx 7\times10^{-6}$ s (see Appendix). The order of pumping intenisty is 10, the order of translational cross-relaxation R_t and rotational cross-relaxation R_r are 10^2 (see sec. 3). Thus it can be assumed that $R_t, R_r \gg 1$; $R_t, R_r \gg R_p$, then we can get from eqs. (15) and (16)

$$N(J_1,J_2) = Nf(J_2). \tag{17}$$

Eqs. (11) and (14) can be simplified. We use $R_t^{-1} \times (11)$, $R_r^{-1} \times (14)$, and neglect the terms with R_t^{-1} and R_r^{-1}, getting,

$$\frac{\Delta n(J_1, J_2, \nu')}{\Delta n(J_1, J_2)} = \frac{g(\nu', \nu_0)}{1 + \frac{R_r}{R_t} + I_\nu L(\nu, \nu')} \left[1 + \frac{R_r}{R_t} \cdot \frac{f(J_2)\, n_2 - \beta f(J_1)\, n_1}{\Delta n(J_1, J_2)} \right], \tag{18}$$

$$\frac{f(J_2)\, n_2 - \beta f(J_1)\, n_1}{\Delta n(J_1, J_2)} = 1 + \frac{R_t}{R_r} I_\nu \int_0^\infty L(\nu, \nu') \frac{\Delta n(J_1, J_2, \nu')}{\Delta n(J_1, J_2)}. \tag{19}$$

Substituting eq. (19) into eq. (18), the self-consistency equation for $\Delta n(J_1, J_2, \nu')$ is obtained as

$$\frac{\Delta n(J_1, J_2, \nu')}{\Delta n(J_1, J_2)} = \frac{g(\nu', \nu_0)}{1 + \frac{R_r}{R_t} + I_\nu L(\nu, \nu')} \left[1 + \frac{R_r}{R_t} + I_\nu \int_0^\infty L(\nu, \nu') \frac{\Delta n(J_1, J_2, \nu')}{\Delta n(J_1, J_2)} \mathrm{d}\nu' \right]. \tag{20}$$

In order to obtain the analytic solution of eq. (20), using the approximation in eq. (18)

$$f(J_2)\, n_2 - \beta f(J_1)\, n_1 = \Delta n(J_1, J_2), \tag{21}$$

and considering eqs. (16) and (17), we find respectively from eqs. (20) and (14)

$$\Delta n(J_1, J_2, \nu') = \frac{g(\nu', \nu_0)}{1 + \frac{R_r}{R_t} + I_\nu L(\nu, \nu')} \Delta n(J_1, J_2) \left[\left(1 + \frac{R_r}{R_t} \right) (1 + I_\nu P(\nu)) \right], \tag{22}$$

and

$$\frac{\partial}{\partial \zeta} \Delta n(J_1, J_2) = \left(\frac{\partial N}{\partial \zeta} - \beta N \right) f(J_2) - \left[1 + R_t I_\nu \left(1 + \frac{R_r}{R_t} \right) P(\nu) \right] \Delta n(J_1, J_2), \tag{23}$$

where

$$P(\nu) = \int_0^\infty \frac{L(\nu, \nu') g(\nu', \nu_0)}{1 + \frac{R_r}{R_t} + I_\nu L(\nu, \nu')} \mathrm{d}\nu' = \frac{1}{\sqrt{\pi}} \int_{-\infty}^\infty \frac{\eta^2 \mathrm{e}^{-t^2}}{(\xi - t)^2 \left(1 + \frac{R_r}{R_t} \right) + \left(1 + \frac{R_r}{R_t} + I_\nu \right) \eta^2} \mathrm{d}t, \tag{24}$$

where, ξ, t and η are respectively as follows:

$$\xi = \sqrt{4\ln 2} \frac{\nu - \nu_0}{\Delta \nu_D}, \quad t = \sqrt{4\ln 2} \frac{\nu' - \nu_0}{\Delta \nu_D}, \quad \eta = \sqrt{\ln 2} \frac{\Delta \nu_h}{\Delta \nu_D}, \tag{25}$$

where ξ and η are respectively tunning parameter and broadening parameter. $\Delta n(J_1, J_2)$ in eq. (22) can be obtained by solving eq. (23). To solve eq. (23), we must know the character of flow-field of the gas medium with supersonic velocity. There are two types of diffusion flow, the laminar flow and the turbulent flow.

Diffusion flow model I is for the laminar flow diffusion mixing:

$$N = n_p R_p \zeta^{1/2}. \tag{26}$$

Diffusion flow model II is for the turbulent flow diffusion mixing:

$$N = n_p R_p \zeta. \tag{27}$$

For the models of laminar and turbulent flow diffusion mixing, the solutions of eq. (23) are respectively

$$\Delta n_L(J_1, J_2) = n_p R_p f(J_2) \frac{1}{\left[1 + \left(1 + \frac{R_r}{R_t} \right) S_\nu \right]^{3/2}}$$

$$\cdot\left\{\left[1+\beta+\left(1+\frac{R_r}{R_t}\right)S_\nu\right]D\left(\sqrt{1+\left(1+\frac{R_r}{R_t}\right)S_\nu}\,\zeta\right)-\beta\sqrt{1+\left(1+\frac{R_r}{R_t}\right)S_\nu}\,\zeta\right\} \tag{28}$$

and

$$\Delta n_T(J_1,J_2)=n_p R_p f(J_2)\frac{1}{\left[1+\left(1+\frac{R_r}{R_t}\right)S_\nu\right]^2}\left\{\left[1+\beta+\left(1+\frac{R_r}{R_t}\right)S_\nu\right]\cdot\left[1-\exp\left(-\left[1+\left(1+\frac{R_r}{R_t}\right)S_\nu\right]\zeta\right)\right]-\beta\left[1+\left(1+\frac{R_r}{R_t}\right)S_\nu\right]\zeta\right\}, \tag{29}$$

where $S_\nu = R_t I_\nu P(\nu)$, $D(x) = e^{-x^2}\int e^{t^2}dt$ is Dawson integration[13].

3 Saturated gain spectrum

Suppose now that ν_j is the frequency of high power laser which causes saturation in HF medium, $\phi(\nu_j)$ is the photon number passing through unit sectional area per unit time. $I_j = J_j/J_s(J_j = \phi(\nu_j)h\nu_j)$ is the nondimentional intensity, simultaneously an incident lower power laser (i.e. probe laser) which is tunable in frequency ν and is perpendicular to the gas flow direction, is used at the diffusion streamwise station ζ_D. Then the gain of the probe laser characterizes the frequency characteristics of HF saturation effect caused by saturation laser. The gain of the probe laser is

$$G(\nu, I_j) = \int\sigma(\nu,\nu')\Delta n_j(J_1,J_2,\nu')d\nu'. \tag{30}$$

According to eq. (22), $\Delta n_j(J_1,J_2,\nu')$ in eq. (30) should be expressed as

$$\Delta n_j(J_1,J_2,\nu') = \frac{g(\nu',\nu_0)}{1+\frac{R_r}{R_t}+I_jL(\nu_j,\nu')}\Delta n_j(J_1,J_2)\left[\left(1+\frac{R_r}{R_t}\right)(1+I_jP(\nu_j))\right], \tag{31}$$

where, $P(\nu_j)$ is the value for $\nu=\nu_j$, $I_\nu = I_j$ in eq. (24), i.e.

$$P(\nu_j) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{\eta^2 e^{-t^2}}{(\xi_j-t)^2\left(1+\frac{R_r}{R_t}\right)+\left(1+\frac{R_r}{R_t}+I_j\right)\eta^2}dt. \tag{32}$$

For the models of laminar and turbulent flow diffusion mixing, we use $\Delta n_j(J_1,J_2)$ in eqs. (28) and (29), respectively, where ζ is ζ_D, S_ν is S_j, $S_j = R_t I_j P(\nu_j)$. Then for the two-type models of diffusion flow, we obtain respectively

$$[G(\nu,I_j)/G^0(\nu_0)]_L = \left(1+\frac{R_r}{R_t}\right)[1+I_jP(\nu_j)]\frac{1}{\left[1+\left(1+\frac{R_r}{R_t}\right)S_j\right]^{3/2}}\cdot\frac{1}{(1+\beta)D(\sqrt{\zeta_D})-\beta\sqrt{\zeta_D}}$$
$$\cdot\left\{\left[1+\beta+\left(1+\frac{R_r}{R_t}\right)S_j\right]D\left(\sqrt{1+\left(1+\frac{R_r}{R_t}\right)S_j}\,\zeta_D\right)-\beta\sqrt{1+\left(1+\frac{R_r}{R_t}\right)S_j}\,\zeta_D\right\}$$
$$\cdot\frac{1}{\int_{-\infty}^{\infty}\frac{e^{-t^2}}{t^2+\eta^2}dt}\cdot\int_{-\infty}^{\infty}\frac{e^{-t^2}}{(\xi-t)^2+\eta^2}\cdot\frac{(\xi_j-t)^2+\eta^2}{\left(1+\frac{R_r}{R_t}\right)(\xi_j-t)^2+\left(1+\frac{R_r}{R_t}+I_j\right)\eta^2}dt, \tag{33}$$

$$[\,G(\nu, I_j)/\,G^0(\nu_0)\,]_{\mathrm{T}} = \left(1+\frac{R_{\mathrm{r}}}{R_{\mathrm{t}}}\right)[1+I_jP(\nu_j)]\frac{1}{\left[1+\left(1+\frac{R_{\mathrm{r}}}{R_{\mathrm{t}}}\right)S_j\right]^2}\cdot\frac{1}{(1+\beta)[1-\exp(-\zeta_{\mathrm{D}})]-\beta\zeta_{\mathrm{D}}}$$
$$\cdot\left\{\left[1+\beta+\left(1+\frac{R_{\mathrm{r}}}{R_{\mathrm{t}}}\right)S_j\right]\left[1-\exp\left(-\left[1+\left(1+\frac{R_{\mathrm{r}}}{R_{\mathrm{t}}}\right)S_j\right]\zeta_{\mathrm{D}}\right)\right]-\beta\left[1+\left(1+\frac{R_{\mathrm{r}}}{R_{\mathrm{t}}}\right)S_j\right]\zeta_{\mathrm{D}}\right\}$$
$$\cdot\frac{1}{\int_{-\infty}^{\infty}\frac{\mathrm{e}^{-t^2}}{t^2+\eta^2}\mathrm{d}t}\cdot\int_{-\infty}^{\infty}\frac{\mathrm{e}^{-t^2}}{(\xi-t)^2+\eta^2}\cdot\frac{(\xi_j-t)^2+\eta^2}{\left(1+\frac{R_{\mathrm{r}}}{R_{\mathrm{t}}}\right)(\xi_j-t)^2+\left(1+\frac{R_{\mathrm{r}}}{R_{\mathrm{t}}}+I_j\right)\eta^2}\mathrm{d}t. \quad (34)$$

In order to compare the present results with experimental ones, we calculated $P_2(8)$ line of HF chemical laser. For HF chmeical laser, the average temperature $T = 400$ K, wavelenth $\lambda = 2.7 - 3.2$ μm. For $P_2(8)$ line $\lambda = 2.911$ μm, $\beta = 1.13$, Doppler width is

$$\Delta\nu_{\mathrm{D}} = 2\left(\frac{2kT\ln 2}{mc^2}\right)^{1/2}\nu_0 = (960.3/\lambda)(T/400)^{1/2}(20/M)^{1/2}\ \mathrm{s}^{-1}, \quad (35)$$

where λ is the wavelenth in meter, M is the molecular weight in gram, T is the absolute temperature. It is calculated that $\Delta\nu_{\mathrm{D}} = 330\times10^6$ Hz. The collision broadening plays a dominant part in homogeneous broadening of molecular laser. Generally, the collision width of HF laser spectrum is assumed[9] to be

$$\Delta\omega_{\mathrm{L}} = 2\times10^8 P/(133.32\sqrt{T}), \quad (36)$$

where $\Delta\omega_{\mathrm{L}}$ is in radian per second, P is the gas pressure with Pa, T is the absolute temperature. When $P = 6\times133.32$ Pa and $T = 400$ K, $\Delta\omega_{\mathrm{L}} = 6\times10^7$ rad/s, $\Delta\nu_{\mathrm{L}} = \Delta\omega_{\mathrm{L}}/2\pi = 9.55\times10^6$ Hz, the broadening parameter $\eta = 0.02$. When $P = 10\times133.32$ Pa, $\eta = 0.04$. The collisional deactivation rate (see Appendix) $k_{\mathrm{c}} = 1.433\times10^5$/s, $\tau_{\mathrm{c}} = k_{\mathrm{c}}^{-1} = 6.98\times10^{-6}$ s. The translational cross-relaxation time $\tau_{\mathrm{t}} = 1/(2\Delta\nu_{\mathrm{L}}) = 5.24\times10^{-8}$ s. The rotational cross-relaxation time $\tau_{\mathrm{r}} = 1/\Delta\nu_{\mathrm{L}} = 1.05\times10^{-7}$ s. $R_{\mathrm{t}} = \tau_{\mathrm{c}}/\tau_{\mathrm{t}} = 133$, $R_{\mathrm{r}} = \tau_{\mathrm{c}}/\tau_{\mathrm{r}} = 0.5R_{\mathrm{t}}$. In experimental measurement, $\xi_j = 0.4$, and the saturated gain was measured at the nondimensional diffusion streamwise distance $\zeta_{\mathrm{D}} = 0.305$ (0.4 cm downstream of the nozzles' exit). The numerical calculations were made for two-type models of laminar and turbulent flow mixing in terms of eqs. (33) and (34) respectively, the results are illustrated in figs. 2 and 3. The experimental results in ref. [3] are illustrated in fig. 4. It has been shown that the theoretical calculations are in good agreement with the measured results, particularly for the model of turbulent flow diffusion mixing.

4 Conclusion and discussion

The saturated gain of low pressure supersonic cw HF chemical laser amplifier is not only related to saturation intensity, broadening parameter and laser frequency but also to diffusion streamwise distance of the gas medium. The saturated gain profile exhibits an "anomalous" phenomenon with a mixing of inhomogeneous and homogeneous broadening profiles. Both translational and rotational nonequilibrium relaxations make some contribution to the "anomalous" mixing profile.

It seems puzzle that the result of turbulent flow mixing model is in better agreement with the experimental measurement than that of laminar flow mixing model. We think that even if supersonic nozzles and channels are ideally designed and aligned, and the H_2 and F are in ideal laminar

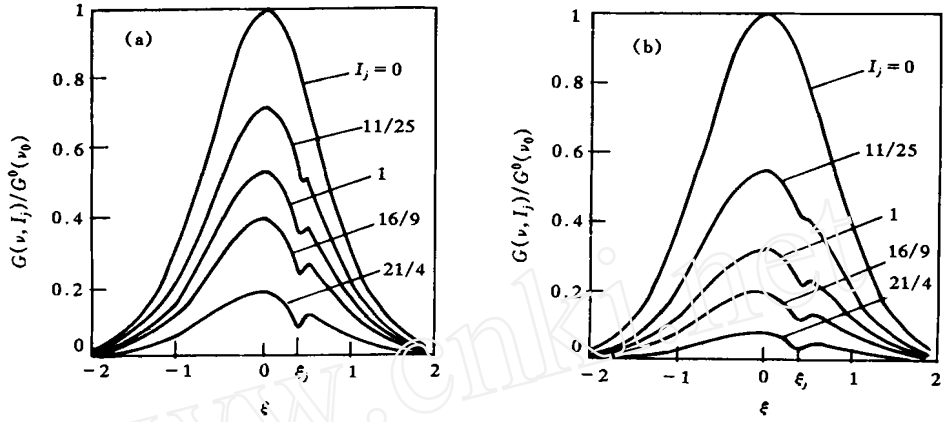

Fig. 2. Profiles of saturated gain spectra of supersonic cw HF chemical laser amplifier with laminar flow diffusion mixing. $\zeta_D = 0.305$, $\xi_j = 0.4$. (a) $\eta = 0.02$, (b) $\eta = 0.04$.

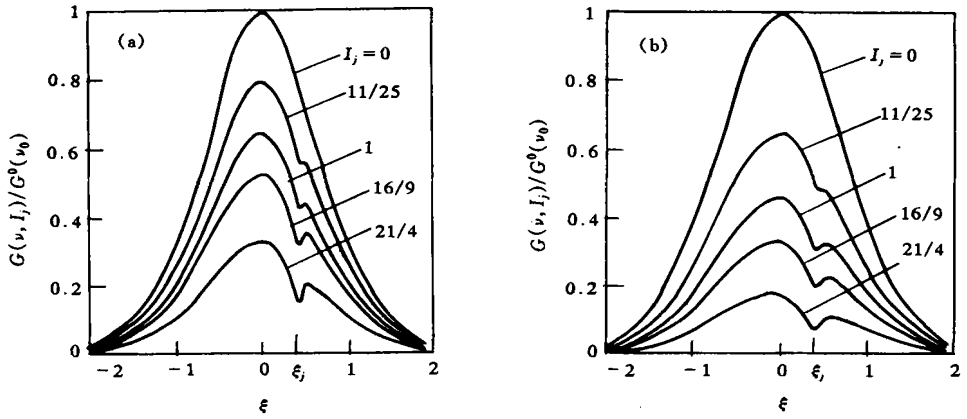

Fig. 3. Profiles of saturated gain spectra of supersonic cw HF chemical laser amplifier with turbulent flow diffusion mixing. $\zeta_D = 0.305$, $\xi_j = 0.4$. (a) $\eta = 0.02$, (b) $\eta = 0.04$.

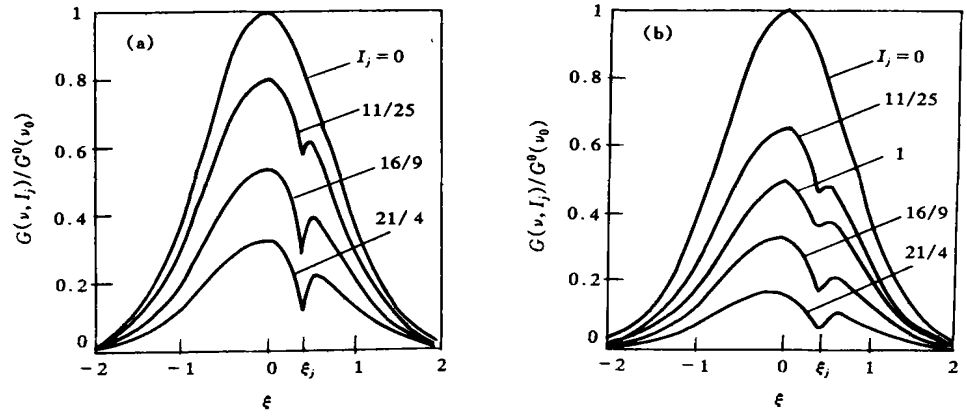

Fig. 4. The experiment results on the profiles of saturated gain spectra of supersonic cw HF chemical laser amplifier in ref. [3]. $\zeta_D = 0.305$, $\xi_j = 0.4$. (a) $\eta = 0.02$, (b) $\eta = 0.04$.

flow diffusion, respectively, it is probable that mixed gases are still the turbulent flow mixing, and there is a waveshape on the boundary layer of mixed gases. So the turbulent flow mixing model is in best agreement with the experimental results.

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Appendix

Calculation of the collisional deactivation rate of HF.

The collisional deactivation of HF chemical laser is in the form

$$\mathrm{HF}(v+1) + M_i \xrightarrow{\bar{k}_c^{\,i}} \mathrm{HF}(v) + M_i, \qquad (\mathrm{A}\text{-}1)$$

the net rate of collisional deactivation is

$$k_c = P \sum_i \frac{P_i}{P} \frac{\bar{k}_c^{\,i}}{R_0 T}, \qquad (\mathrm{A}\text{-}2)$$

the values of $\bar{k}_c^{\,i} / R_0 T$ are given in table 1. In table 1, $R_0 = 82.06$ (cm^3·atm)/(mole·K). The parameter of cw HF chemical laser is[10] HF : H_2 : He : O_2 = 0.12 : 0.47 : 0.39 : 0.02. The average temperature $T = 400$ K, when $P = 6 \times 133.32$ Pa, for $P_2(8)$ line $k_c = 1.433 \times 10^5$ s^{-1}; $P = 10 \times 133.32$ Pa, $k_c = 2.388 \times 10^5$ s^{-1}.

Table 1 Calculation values of $\bar{k}_c^{\,i} / R_0 T$ ($T = 400$ K)

M_i	$\bar{k}_c^{\,i}$		$\bar{k}_c^{\,i} / R_0 T$/(atm·s)$^{-1}$
	Calculation formula	value/cm^3·(mole·s)$^{-1}$	
HF	$(v+1)^{2.6}(3 \times 10^{14} T^{-1} + 3.5 \times 10^4 T^{2.26}$	4.71×10^{12}	1.43×10^8
F	$(v+1)^{2.7}(1.9 \times 10^{13} \mathrm{e}^{-1359/T})$	4.13×10^{12}	1.25×10^8
H_2	$(v+1)^{2.7}(6 \times 10^{11} T^{-1} + 10^4 T^{2.28})$	6.54×10^{10}	1.99×10^6
He	$(v+1)^{2.7}(3.7 \times 10^{-5} T^{4.5})$	3.30×10^7	1.01×10^3