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Shear Instability of Saturated Soil Considering the Gradients of Strain and Pore Pressure

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Abstract The dynamic instability of saturated soil considering the gradients of strain and pore pressure was discussed in this paper. The inertia and the drainage effects were considered and the Perturbation method was used. It is shown that the strain gradient and pore pressure gradient are positive factors for the instability of saturated soil.

Key words saturated soil, instability, gradient.
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Nomenclature

- σ_{ey} effective stress in y direction
- τ shear stress
- γ shear strain
- $\dot{\gamma}$ shear Strain ratio
- p pore pressure
- $\zeta = \frac{\partial^2 p}{\partial x^2}$ pore pressure gradient
- $\eta = \frac{\partial^2 \gamma}{\partial x^2}$ strain gradient
- ρ density of saturated soil
- μ viscosity of water
- k physical permeability
- C_1 material parameter
- E_r rebound modulus of grains
- α frequency of perturbation
- β wave number of perturbation
- R_0 strain hardening coefficient
- Q_0 pore pressure softening coefficient
- H_0 strain ratio hardening coefficient
- S_0 strain gradient of coefficient
- P_0 pore pressure gradient coefficient
- t_c characteristic time

1 Introduction

The localization caused by shear instability can develop in saturated soils. Shear localization may appear as a single zone, several zones or a regular pattern[1]. A single shear zone is created when boundaries of the specimen can move freely and the material can thus dilate freely[2]. In turn, the pattern of shear zones occurs if non-deforming boundaries move against each other or if saturated soils are subject to rehardening[3]. The importance to understanding the mechanism of shear instability is first they usually signifies the failure of saturated soils, and second, they influence the magnitude of forces transferred from soil bodies to surrounding structures[1,4]. The issue of the inception of shear instability in saturated soils has been the object of both theoretical, numerical as well as experimental researches. Theoretical contributions are related mainly to the stability and bifurcation analysis of diffused and localized failure models[5-7]. Typically, the stability problem is formulated by considering small perturbations in field variables (*e.g.*, displacement and pore pressure). Classical continuum approach leads, in this case, to the ordinary diffusion equation for the perturbation in pore pressure[5,6]. Alternatively, the instability in fluid-infiltrated soil may be considered as a bifurcation problem[8]. The experimental evidence on deformation instabilities and the failure models come from conventional simple shear apparatus[9], triaxial[10,11], plain strain biaxial tests[12-16], in earth pressure test[17], as well as granular flow in a hopper[18]. The experimental results indicated that the uniform response is often followed by

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the onset of a diffused, nonhomogeneous deformation model, after which distinct shear bands or liquefaction areas form. The development of shear localization increases with increasing grain diameter[1]. The effect of the pressure level on the thickness of a localization zone has not been clarified. And, the influences of the gradients of strain or stress were almost not discussed in the instability of saturated soil[5,19,20].

The localization of saturated soils has been numerically investigated within continuum mechanics[21-24] and discontinuous mechanics[25]. In order to overcome the difficulties including ill-posedness of the governing equations in the softening regime caused by not equipped with an internal length[26], higher order strain gradients were introduced into a few classes of standard constitutive behavior[27,28]. It is shown that the gradient approach has the ability to provide the characteristic lengths of deformation for soils. In fact, the pore pressure plays important roles in the development of localization[13] and the gradient in saturated soils is high, but its influence to the localization has not been classified.

In view of the above analysis, an attempt of this paper is to discuss the instability of saturated soil by considering the effects of strain gradient and pore pressure so as to obtain a comprehensive and precise picture of the instability phenomenon.

2 Formulation of Problem

Consider a sample of saturated soil which is subjected to a partial drained deformation under simple shear. The pore water and grains are both incompressible. The x-axis is in horizontal direction while the y-axis is in vertical direction. Shear loading is applied in the x-axis. The deformation can only occur in the x direction but may have a gradient in the other directions.

3 Constitutive Relation

The skeleton of soil is taken as visco-plastic and is influenced by the gradients of strain and pore pressure. Therefore, the constitutive relations is assumed to be expressed as follows under shear loading[29-32].

$$\tau = f_3(\gamma, \dot{\gamma}, p, \zeta, \eta) \qquad (1)$$

in which τ is the shear stress; γ is the shear strain; $\dot{\gamma}$ is the shear strain ratio; p is the pore pressure and is equal to $\sigma - \sigma_{ey}$, σ is the total stress in the y direction and is a constant, σ_{ey} is the effective stress in the y direction, Thus, p denotes σ_{ey}; $\zeta = \frac{\partial^2 p}{\partial y^2}$; $\eta = \frac{\partial^2 \gamma}{\partial y^2}$.

As an example, based on the model presented in Refs. [34,35]by considering the gradient of pore pressure as a linear factor, there holds

$$\tau = A\gamma^m \dot{\gamma}^a p^b - c\zeta - d\eta \qquad (2)$$

in which A, m, a, b, c, d are parameters which may be determined by experiments. We can see that it is a nonlinear equation. Its application will be given in Section 6.

4 Controlling Equations

Here, the momentum equations of water and grains are given according to the mixture theory[33,34]:

$$\begin{cases} \frac{\partial p}{\partial t} - \frac{E_r}{K}\frac{\partial^2 p}{\partial y^2} = C_1 E_r \tau \frac{\partial \gamma}{\partial t} \\ \rho \frac{\partial^2 \gamma}{\partial t^2} - \frac{\partial^2 \tau}{\partial y^2} = 0 \end{cases} \qquad (3)$$

in which p is the pore pressure, E_r is the rebound modulus, $K = \mu / k$ and μ is the viscosity of water and k is the physical permeability whose dimension is $[L^2]$, C_1 is a material parameter, and ρ is the density of saturated soil.

The solutions of these equations are difficult to seek because of the non-linearity. Since the aim of this paper is so seek for the condition of instability. The perturbation method is adopted here. Hence, a smooth developing deformation state γ_0, τ_0, p_0 is taken as the base state which is a solution of Eq. (3). When perturbation is applied to the governing equations, we will be able to analyze the factors and criteria of instability.

5 Perturbation Analysis[33,34]

We study the solutions in the following form:

$$\begin{cases} \gamma = \gamma_0 + \gamma' ; & |\gamma'| << |\gamma_0| \\ p = p_0 + p' ; & |p'| << |p_0| \\ \tau = \tau_0 + \tau' ; & |\tau'| << |\tau_0| \end{cases} \qquad (4)$$

where (γ_0, p_0, τ_0) is a solution of Eq. (3), and

$$\begin{cases} \gamma' = \gamma^* e^{\alpha t + i\beta y} \\ p' = p^* e^{\alpha t + i\beta y} \\ \tau' = \tau^* e^{\alpha t + i\beta y} \end{cases} \qquad (5)$$

in which α, β are respectively the frequency and the wave number.

Differentiating the constitutive relations (1), we may obtain

$$d\tau = R_0 d\gamma - Q_0 dp + H_0 d\dot{\gamma} - P_0 d\zeta - S_0 d\eta \quad (6)$$

where

$$R_0 = \left(\frac{\partial\tau}{\partial\gamma}\right)_0, Q_0 = -\left(\frac{\partial\tau}{\partial p}\right)_0, H_0 = \left(\frac{\partial\tau}{\partial\dot{\gamma}}\right)_0,$$
$$P_0 = -\left(\frac{\partial\tau}{\partial\zeta}\right)_0, S_0 = -\left(\frac{\partial\tau}{\partial\eta}\right)_0 \quad (7)$$

Therefore

$$\tau^{\cdot} = R_0\gamma^{\cdot} - Q_0 p^{\cdot} + \alpha H_0\gamma^{\cdot} - P_0\beta^2 p^{\cdot} - S_0\beta^2\gamma^{\cdot} \quad (8)$$

Substituting (3), (4) and (7) into (2) leads to a homogeneous system of equations

$$\begin{cases} [\rho\alpha^2 + \beta^2(R_0 + \alpha H_0) - S_0\beta^4]\gamma^{\cdot} - (\beta^4 P_0 + \beta^2 Q_0)p^{\cdot} = 0 \\ (E_r C_1 \tau_0 \alpha + E_r C_1 \dot{\gamma}_0 R_0 + E_r C_1 \dot{\gamma}_0 H_0 \alpha - \\ \quad E_r C_1 \dot{\gamma}_0 S_0 \beta^2)\gamma^{\cdot} - A_1 p^{\cdot} = 0 \end{cases} \quad (9)$$

in which $A_1 = C_1 E_r \dot{\gamma}_0 Q_0 + C_1 E_r \dot{\gamma}_0 P_0 \beta^2 + \alpha + \frac{E_r}{K}\beta^2$

As we all know, the determinant of the coefficients should be equal to zero such that the system has nonzero solutions, which leads to

$$\rho\alpha^3 + \left(\rho\left(C_1 E_r \dot{\gamma}_0 (Q_0 + P_0\beta^2) + \frac{E_r}{K}\beta^2\right) + \beta^2 H_0\right)\alpha^2 + A_2\alpha + \frac{E_r R_0}{K}\beta^4 - \frac{E_r S_0}{K}\beta^6 = 0 \quad (10)$$

where $A_2 = R_0\beta^2 - S_0\beta^4 + \frac{E_r H_0}{K}\beta^4 - \beta^4 P_0 C_1 E_r \tau_0 - \beta^2 Q_0 C_1 E_r \tau_0$. It is a spectral equation and if α has a positive real root, instability is possible.

If the influences of ζ, η are neglected and we proceed in the same way as in Ref. [31], we can obtain the following solution.

By $\frac{\partial\alpha}{\partial\beta^2} = 0$, the following equation about β^2 may be obtain as:

$$\beta^2 = \frac{\left(\rho C_1 E_r \dot{\gamma}_0 P_0 + \rho\frac{E_r}{K} + H_0\right)\alpha^2 + (R_0 - Q_0 C_1 E_r \tau_0)\alpha}{\left(2S_0 - 2H_0\frac{E_r}{K} + 2P_0 C_1 E_r \tau_0\right)\alpha - 2R_0\frac{E_r}{K}} \quad (11)$$

Substituting Eq. (11) into Eq. (10), we can obtain the following conditions under which α has positive roots:

$$8\rho C_1 E_r \dot{\gamma}_0 Q_0 R_0 \frac{E_r}{K} > R_0 - Q_0 C_1 E_r \tau_0 \quad (12)$$

In the case of small strain ratio, the above inequality becomes

$$\frac{Q_0 C_1 E_r \tau_0}{R_0} > 1 \quad (13)$$

It is the same as that in Ref. [31] for this case. It means that no matter whether there is the influence of the pore pressure gradient, the instability criteria may be the same.

Next, we mainly discuss the influences of ξ, η.

(1) $S_0 = 0$

In this case, the spectral equation (10) becomes

$$\rho\alpha^3 + \left\{\rho\left[C_1 E_r \dot{\gamma}_0 (Q_0 + P_0\beta^2) + \frac{E_r}{K}\beta^2\right] + \beta^2 H_0\right\}\alpha^2 + A_2\alpha + \frac{E_r R_0}{K}\beta^4 = 0 \quad (14)$$

in which $A_2 = R_0\beta^2 - S_0\beta^4 + \frac{E_r H_0}{K}\beta^4 - \beta^4 P_0 C_1 E_r \tau_0 - \beta^2 Q_0 C_1 E_r \tau_0$.

Since the coefficients of the terms α^2, α^0 are positive, if only $A_2 < 0$, there might be a positive real root of α which leads to

$$\frac{\beta^4 P_0 C_1 E_r \tau_0 + E_r C_1 \tau_0 Q_0 \beta^2}{R_0\beta^2 + \frac{E_r H_0}{K}\beta^4} > 1 \quad (15)$$

It means that once the softening effect of pore pressure and its gradient overcomes the hardening effect of the strain and the strain ratio, instability will occur. It means that the gradient of pore pressure has the positive influence on the instability of saturated soil.

(2) $P_0 = 0$

In this case, the spectral equation (10) becomes

$$\rho\alpha^3 + \left(\rho\left(C_1 E_r \dot{\gamma}_0 Q_0 + \frac{E_r}{K}\beta^2\right) + \beta^2 H_0\right)\alpha^2 + A_2\alpha + \frac{E_r R_0}{K}\beta^4 - \frac{E_r S_0}{K}\beta^6 = 0 \quad (16)$$

where $A_2 = R_0\beta^2 - S_0\beta^4 + \frac{E_r H_0}{K}\beta^4 - \beta^4 P_0 C_1 E_r \tau_0 - \beta^2 Q_0 C_1 E_r \tau_0$

The conditions that α may has a positive real root are as follows:

$$\frac{E_r R_0}{K}\beta^4 - \frac{E_r S_0}{K}\beta^6 < 0 \quad \text{or} \quad A_2 < 0 \tag{17}$$

which might be written as

$$R_0 - S_0\beta^2 < 0,$$

$$R_0 + \beta^2 \frac{E_r H_0}{K} < E_r C_1 \tau_0 (Q_0 + P_0\beta^2) + \beta^2 S_0 \tag{18}$$

It is shown that S_0 is a positive factor for the instability of saturated soil.

If $P_0 = 0$, $S_0 = 0$, the criterion $\frac{Q_0 C_1 E_r \tau_0}{R_0} > 1$ is valid.

If $P_0 = 0$, the criterion $R_0 - S_0\beta^2 < 0$, $R_0 + \beta^2 \frac{E_r H_0}{K} < E_r C_1 \tau_0 (Q_0 + P_0\beta^2) + \beta^2 S_0$ is valid.

6 Practical Criterion

Now we concentrate to the practical criterion. It is desirable to establish a criterion connecting state parameters with material constants in the criterion. And, if the constitutive relation of the soil is formulated explicitly, the criterion is easy to obtain.

Inserting Eq. (2) into the criterion (15), we can obtained the criterion considering the influence of pore pressure gradient

$$\frac{C_1 E_r \left(c\beta^2 - \frac{b\tau_0}{p_0} \right)}{\frac{m}{\gamma_0} + \frac{E_r a}{K \dot{\gamma}_0}\beta^2} > 1 \tag{19}$$

Generally, the parameters of soils have the following orders:

$K \sim 10^{10\sim 11}\,\text{kg/m}^3\text{s}$, $E_r = 10^{7\sim 8}\,\text{Pa}$, $\beta \sim 10^0/\text{m}$, $\gamma_0 \sim 10^{-1\sim -1}$, $\dot{\gamma}_0 \sim 10^1/\text{s}$, $n_0 \sim 10^0$, $b \sim -10^{1\sim 2}$, $m \sim 10^{-1}\,\text{Pa}$, $a \sim 10^{-1}$, $c \sim 10^{1\sim 2}\,\text{m}^2$, $\tau_0 \sim 10^6\,\text{Pa}$, $C_1 \sim 1.0$, $p_0 \sim 10^{5\sim 6}\,\text{Pa}$

Then by neglecting the term $\frac{E_r}{K}\frac{a}{\dot{\gamma}_0}\beta^2$ Eq. (19) may be reduced as to

$$\gamma > \frac{m}{C_1 E_r \left(c\beta^2 - \frac{b\tau_0}{p_0} \right)} \tag{20}$$

If $c = 0$ and $d = 0$, then the criterion becomes

$$\gamma > -\frac{m p_0}{b C_1 E_r \tau_0} \tag{21}$$

The above analysis is just theoretical and compared with the results in some special cases. The experimental verification of the criterion is being processed.

7 Conclusions

It has been shown that the gradients of strain and pore pressure have both the positive influences on the instability of saturated soil. When the effects of the gradients of strain or pore pressure overcomes the hardening effect of strain and strain ratio, instability may occur. The solutions may be the same as that in Ref. [31] when neglected ζ, η.

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