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AN IMPROVEMENT AND PROOF OF OGY METHOD*

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Abstract

OGY method is the most important method of controlling chaos. It stabilizes a hyperbolic periodic orbit by making small perturbations for a system parameter. This paper improves the method of choosing parameter, and gives a mathematics proof of it.

Key words dynamical system, chaos, controlling chaos, hyperbolic periodic point, stable manifold, unstable manifold

I. Introduction

Recently controlling chaos becomes an interesting subject. In 1990, E. Ott, C. Greogi and J. A. Yorke[1] gave a conception of controlling chaos, and gave a controlling method known as OGY method. There were many results that showed OGY method is effective. After OGY, many methods of controlling chaos were given, most of them were based on OGY[3, 4, 5, 6, 7]. So OGY method is the fundament of controlling chaos. To achieve a proof of OGY is an important task for mathematics theory of controlling chaos. This paper changes the method of choosing parameter, and proves OGY method in mathematics.

OGY method considers the map $X_{n+1}=F_p(X_n)$ in R^2, where p is parameter. When $p=0$, system has a hyperbolic fixed point $\xi=0$, and it has stable manifold and unstable manifold. OGY method's aim is to stabilizie the fixed point. In controlling, for every time n, a proper parameter p_n is chosen, then the iterate $X_{n+1}=F(p_n, X_n)$ is used with initial value $X=X_0$ (near $\xi=0$) so that the sequence $\{X_n\}$ converges to $\xi=0$. Thus $\xi=0$ become stable. The key of method is how the parameter p_n is chosen. In OGY, they try to make $X_{n+1}=F(p_n, X_n)$ fall on the tangent direction of the stable manifold of $\xi=0$ by adjusting p_n. Follow this idea, they gave an approximate formula $p_n=\lambda_u(\lambda_u-1)^{-1}(X_n\cdot f_u)/(g\cdot f_u)$, where λ_u is the unstable eigenvalues of $\xi=0$, f_u is the unstable direction, $\cdot$ is dot product, $g=\partial\xi(p)/\partial p|_{p=0}$, $\xi(p)$ is the fixed point with parameter p. g is the tangent vector of $\xi(p)$. Because every $X_{n+1}=F(p_n, X_n)$ with the above p_n is near the direction of stable manifold, and not fall on the tangent direction strictly, so it can leave $\xi=0$ just like the statement in [1]. In fact, we can change the method of choosing p_n, so that there is an invariant region of $X_{n+1}=F(p_n, X_n)$. If the initial value X_0 is in this region, then for every n, X_n is in this region and the sequence $\{X_n\}$ converge to $\xi=0$ with velocity of $a^n (0<a<1)$. Thus we

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give an improvement of the choosing of parameter p_n, and complete the proof of OGY method in mathematics.

II. Main Results

We study the iterate in R^2:

$$X_{n+1} = F_p(X_n) \tag{2.1}$$

where $X = (x, y), p$ is the system parameter. For (2.1) we assume that:

H1. $F_p(X) \in C^2$. $\xi = 0$ is the hyperbolic fixed point, its stable direction is x axis and its unstable direction is y axis. In fact, we can realize it by a coordinate transformation. λ_s is the stable eigenvalues and λ_u is the unstable eigenvalues.

H2. $\xi(p) = (x(p), y(p))$ is the hyperbolic fixed point when the system parameter is $p(|p| < P^*)$, $\xi(p) \in C^2$, denote $\partial\xi(p)/\partial p|_{p=0} = (a, b)^T, \partial^2\xi(p)/\partial p^2|_{p=0} = (c, d)^T$.

H3. Without loss of generality, $b > 0, a \geqslant 0, \lambda_u > 1$ are assumed.

When H1. H2. H3 are satisfied, the following theorem is proved.

Theorem 1 $\exists K > 0, \forall k > K, \exists U$ is a neighborhood of $\xi = 0$ denoted, $\forall X_0 \in U$, choose p_n that satisfy

$$bp_n = y_n + 2y_n/(k\lambda_u) \tag{2.2}$$

the sequence $\{X_n\}$ generated by $X_{n+1} = F(p_n, X_n)$ converges to $\xi = 0$, and for every n, $X_n \in U$.

The proof of the theorem is given by some Lemmas:

Lemma 1 $\forall \delta_1 > 0, \exists P^* > 0$, for $\forall p$ satisfy $|p| < P^*$,

$$\xi(0) = \begin{pmatrix} ap + bp^2 + \tilde{e}p^2 \\ bp + cp^2 + \tilde{f}p^2 \end{pmatrix}$$, where $\tilde{e}, \tilde{f}$ are related to p, and $|\tilde{e}| < \delta_1$, $|\tilde{f}| < \delta_1$.

Proof It's obvious. Q. E. D.

Lemma 2 $\forall \delta_2, \delta_3 > 0, \exists N, M \in R, \exists \varepsilon > 0$, such that if X_0 is in the ε-neighborhood of $\xi(0) = 0$, then

$$x_1 = \lambda_s x_0 + \alpha(x_0^2 + y_0^2), y_1 = \lambda_u y_0 + \beta(x_0^2 + y_0^2) \qquad (N/2 < \alpha, \beta < M/2)$$

Specifically, if $|x_0/y_0| > \delta_2$, then $(1 - \delta_3)\lambda_s < |x_1/x_0| < (1 + \delta_3)\lambda_s$,

if $|y_0/x_0| > \delta_2$, then $(1 - \delta_3)\lambda_u < |y_1/y_0| < (1 + \delta_3)\lambda_u$.

And $\exists P_1^* < P^*$, when $|p| < P_1^*$, if X_0 is in the $\varepsilon/2$-neighborhood of $\xi(p)$, then

$$(1 - \delta_3/2)|\lambda_s[x_0 - x(p)]| + Nr^2 < |x_1 - x(p)| < (1 + \delta_3/2)|\lambda_s[x_0 - x(p)]| + Mr^2,$$
$$(1 - \delta_3/2)|\lambda_u[y_0 - y(p)]| + Nr^2 < |y_1 - y(p)| < (1 + \delta_3/2)|\lambda_u[y_0 - y(p)]| + Mr^2,$$

where $r^2 = \{[x_0 - x(p)]^2 + [y_0 - y(p)]^2\}$.

Specifically, if $|(x_0 - x(p))/(y_0 - y(p))| > \delta_2$, then

$$(1 - \delta_3)\lambda_s < |(x_1 - x(p))/(x_0 - x(p))| < (1 + \delta_3)\lambda_s,$$

if $|(y_0 - y(p))/(x_0 - x(p))| > \delta_2$, then

$$(1 - \delta_3)\lambda_u < |(y_0 - y(p))/(x_0 - y(p))| < (1 + \delta_3)\lambda_u.$$

where $(x_1, y_1) = X_1 = F_p(X_0)$.

Proof The results are proved easily. Q. E. D.

Denote $l_p = bp/(k\lambda_u\delta_2)$ where $k > 1, \delta_2 > 0$ can be determined later. and curves

$$\Gamma^+ : \begin{cases} x = x(p) + l_p/2 \\ y = y(p) \end{cases}$$

$$\Gamma^- : \begin{cases} x = x(p) - l_p/2 \\ y = y(p) \end{cases}$$

$U(p_0)$ is defined as the region enclosed by Γ^+, Γ^-, and $y = y(p_0)$, see Fig. 1.

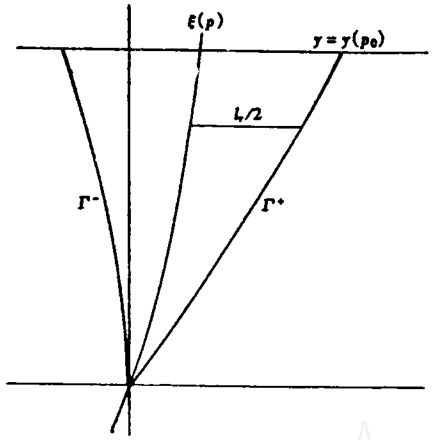

Fig. 1

Lemma 3 For $\forall \delta_1, \delta_2, \delta_3 > 0$, ε and P_1^* are determined by Lemma 1, Lemma 2, then $\forall \eta > 0, k > 1, \exists 0 < P_2^* < P_1^*$, such that $l_p < \varepsilon/2$, in $U(P_2^*)$. and $\forall 0 < p_1, p_2 < P_2^*$

$$(a - \eta)|p_1 - p_2| < |x(p_1) - x(p_2)| < (a + \eta)|p_1 - p_2|$$

$$(b - \eta)|p_1 - p_2| < |y(p_1) - y(p_2)| < (b + \eta)|p_1 - p_2|$$

Proof The results are natural. Q. E. D.

Lemma 4 $\forall 0 < \delta_1$, $\delta_3 < 1$, $k > 3$, $\eta > 0$, $0 < \delta_2 < (b - \eta)/5(a + \eta)$, then $\exists 0 < P_3^* < P_2^*$, such that $\forall X_n = (x_n, y_n) \in U(P_3^*)$ satisfies $|(y_n - y(p_n))/(x_n - x(p_n))| > \delta_2$, therefore $(1 - \delta_3)\lambda_u < |(y_n - y(p_n))/(x_n - y(p_n))| < (1 + \delta_3)\lambda_u$, and $1.5y_n/k\lambda_u < |(y_n - y(p_n)| < 2.5y_n/k\lambda_u$. Specifically, $y_{n+1} > 0$, where p_n is determined by (2.2).

Proof See Fig. 2, choose a horizontal line $\xi(p)E$, $|\xi(p)E| = l_p$, EF is perpendicular line, the slope of $\xi(p)F$ is $-\delta_2$. Denote $h_p = |EF|$.

When $p < P_2^*$, from Lemma 3, $l_p < \varepsilon/2$. It's easy to check that $h_p/bp = 1/k\lambda_u$.

Now we prove that $\exists P_3^* < P_2^*$, $\forall X_n \in U(P_3^*)$, y_n is situated down $\xi(p)F$. For the same reason y_n is situated down $\xi(p)F'$. (See Fig. 3.) It's to say that $|(y_n - y(p_n))/(x_n - x(p_n))| > \delta_2$.

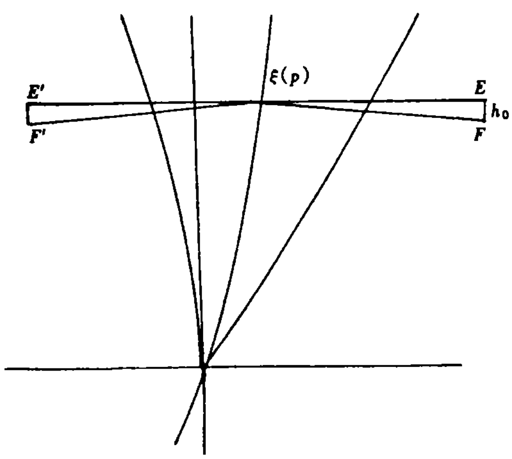

Fig. 2

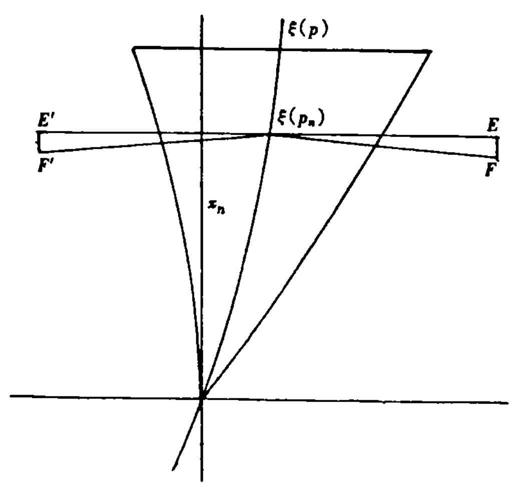

Fig. 3

Firstly, y_n is in $U(P_2^*)$, from (2.2) $bp_n = y_n + 2y_n/(k\lambda_u)$, p_n is determined. Then the following relations are considered

$$y(p_n) = bp_n + (d + \tilde{f})p_n^2$$
$$y(p_n) - y_n = 2y_n/k\lambda_u + (d + \tilde{f})[(1 + 2/k\lambda_u)y_n]^2/b^2$$

therefore, $\exists P_3^* < P_2^*$. such that $1.5y_n/k\lambda_u < |(y_n - y(p_n))| < 2.5y_n/k\lambda_u$.
Specifically,

$$y_{n+1} > y(p_n) - |(y_{n+1} - y(p_n))| > y(p_n) - |(y_n - y(p_n))|(1 + \delta_3)\lambda_u$$
$$> y(p_n) - [1.5y_n/k\lambda_u](1 + \delta_3)\lambda_u$$

As $k > 3$, $y_{n+1} > y(p_n) - 0.5y_n(1 - \delta_3) > y_n - 0.5y_n(1 + \delta_3) > 0$.

While

$$\begin{aligned} y_F - y_n &= [y_F - y(p_n)] + [y(p_n) - y_n] \\ &> -h_p + 1.5y_n/k\lambda_u \\ &> -by_n/k\lambda_u + 1.5y_n/k\lambda_u \\ &= -(1 + 2/k\lambda_u)y_n/k\lambda_u + 1.5y_n/k\lambda_u > 0 \end{aligned}$$

and $y_{F'} - y_n > 0$.

The remain problem is to prove $|x_n| < |x_F|$, that is $|x_n - x(p_n)| < l_{pn}$.

$|x_n - x(p_n)| < |x_n - x(\tilde{p})| + |x(\tilde{p}) - x(p_n)|$ where $\tilde{p}$ satisfy $y(\tilde{p}) = y_n$

Because

$$|x(\tilde{p}) - x(p_n)| < (a + \eta)|\tilde{p} - p_n| < (a + \eta)|y_n - y(p_n)|/(b - \eta)$$
$$< 2.5y_n(a + \eta)/(b - \eta)k\lambda_u$$

from $X_n \in U(P_3^*)$ and l_p is monotonic increasing with p, the following result is obtained

$$|x_n - x(\tilde{p})| < l_p/2 < l_{pn}/2$$

so

$$|x_n - x(p_n)| < l_{pn}/2 + 2.5y_n(a + \eta)/(b - \eta)k\lambda_u$$
$$< l_{pn}/2 + [(a + \eta)/(b - \eta)](l_{pn}/2)(2 \times 2.5\delta_2 y_n/bp_n)$$

Because $y_n/bp_n < 1$,

$$|x_n - x(p_n)| < l_{pn}/2 + 5\delta_2[(a + \eta)/(b - \eta)](l_{pn}/2)$$

from the hypothesis about δ_2 in Lemma,

$$|x_n - x(p_n)| < l_{pn}/2 + l_{pn}/2 = l_{pn}$$

So $|x_n| < |x_F|$. For the same reason, $|x_n| < |x_{F'}|$.

Therefore, X_n is situated down $\xi(p)F$, and situated down $\xi(p)F'$ and $|y_n - y(p_n))/(x_n - x(p_n))| > \delta_2$. Further, from Lemma 3, $(1 - \delta_3)\lambda_u < |(y_n - y(p_n))/(x_n - y(p_n))| < (1 + \delta_3)\lambda_u$ is given.

Q. E. D.

δ_1, δ_3 and η are chosen by the following conditions: $\delta_1 < 1, \delta_3 < \min\{(1/|\lambda_s| - 1)/2, (\lambda_u - 1)/(\lambda_u + 1)\}$, i. e. $|\lambda_s|(1 + 2\delta_3) < 1, \lambda_u(1 - \delta_3) > (1 + \delta_3)$. and $[(b + \eta)/(b - \eta)](1 + \eta) < 1/[|\lambda_s|(1 + 2\delta_3)]$ then $\exists K > 0$, when $k < K, (1 + 2.5/k\lambda_u)/[1 + 1.5/k\lambda_u - 2.5(1 + \delta_3)/k] < 1 + \eta$. For this k, δ_2 is chosen as follows $\delta_2 < |\lambda_s|/20$, and

$$\{[1+5\delta_2(a+\eta)/(b-\eta)]|\lambda_s|(1+\delta_3)+5\lambda_u\delta_2(1+\delta_3)(a+\eta)/(b-\eta)\}\cdot < |\lambda_s|(1+2\delta_3) \qquad (\bigstar)$$

Lemma 5 $\forall \delta_1, \delta_2, \delta_3, k, \eta$ satisfy ($\bigstar$), then $\exists P_4^* < P_3^*$, $\forall X_n \in U(P_4^*)$, for p_n determined by (2.2), $X_{n+1} = F(p_n, X_n) \in U(P_4^*)$.

Proof $\tilde{p}$ satisfies equation $y(\tilde{p}) = y_n$, $\tilde{\tilde{p}}$ satisfies equation $y(\tilde{\tilde{p}}) = y_{n+1}$. Obviously $p_n > \tilde{p} > \tilde{\tilde{p}}$. It is sufficient that $|x_{n+1} - x(\tilde{\tilde{p}})| < l_{\tilde{\tilde{p}}}/2$ is proved. First from Lemma 4: $1.5y_n/k\lambda_u < |(y_n - y(p_n))| < 2.5y_n/k\lambda_u$, and $|(y_n - y(p_n))/(x_n - x(p_n))| > \delta_2$, therefore $(1-\delta_3)\lambda_u < |(y_n - y(p_n))/(y_n - y(p_n))| < (1+\delta_3)\lambda_u$. Now two different cases are discussed:

1) If $|(x_n - x(p_n)]/[y_n - y(p_n)]| > \delta_2$. From Lemma 2, the following relation is obtained

$$|x(p_n) - x_{n+1}| < |x_n - x(p_n)|(1+\delta_3)|\lambda_s|$$

while, by the proof of Lemma 4, it is given

$$|x_n - x(p_n)| < l_{pn}/2 + 5\delta_2[(a+\eta)/(b-\eta)](l_{pn}/2)$$

hence

$$|x(p_n) - x_{n+1}| < (l_{pn}/2)\{1+5\delta_2[(a+\eta)/(b-\eta)]\}(1+\delta_3)|\lambda_s|$$

On the other hand,

$$\begin{aligned} |x(p_n) - x(\tilde{\tilde{p}})| &< (a+\eta)|p_n - \tilde{\tilde{p}}| \\ &< [(a+\eta)/(b-\eta)]|y(p_n) - y_{n+1}| \\ &< [(a+\eta)/(b-\eta)](1+\delta_3)\lambda_u|y_n - y(p_n)) \\ &< [(a+\eta)/(b-\eta)](1+\delta_3)\lambda_u 2.5bp_n/k\lambda_u \\ &< [(a+\eta)/(b-\eta)](1+\delta_3)\lambda_u 5\delta_2(l_{pn}/2) \end{aligned}$$

Therefore

$$\begin{aligned} |x_{n+1} - x(\tilde{\tilde{p}})| &< |x(p_n) - x(\tilde{\tilde{p}}| + |x(p_n) - x_{n+1}| \\ &< (l_{pn}/2)\{[1+5\delta_2(a+\eta)/(b-\eta)](1+\delta_3)|\lambda_s| \\ &\quad + [(a+\eta)/(b-\eta)](1+\delta_3)\lambda_u 5\delta_2\} \end{aligned}$$

It's easy to check by ($\bigstar$)

$$\{[1+5\delta_2(a+\eta)/(b-\eta)](1+\delta_3)|\lambda_s| + [(a+\eta)/(b-\eta)](1+\delta_3)\lambda_u 5\lambda_u\delta_2\} < |\lambda_s|(1+2\delta_3)$$

$$|x_{n+1} - x(\tilde{\tilde{p}})| < (l_{pn}/2)|\lambda_s|(1+2\delta_3)$$

Now, the estimation of l_{pn}, is studied

$$l_{pn} = (l_{pn}/l_{\tilde{\tilde{p}}})l_{\tilde{\tilde{p}}} = [(l_{pn}-0)/(l_{\tilde{\tilde{p}}}-0)]l_{\tilde{\tilde{p}}} < [(b+\eta)y(p_n)/(b-\eta)y_{n+1}]l_{\tilde{\tilde{p}}}$$

where

$$\begin{aligned} y_{n+1} &> y(p_n) - \lambda_u(1+\delta_3)|(y_n - y(p_n))| \\ &> 1.5y_n/k\lambda_u - \lambda_u(1+\delta_3)2.5y_n/k\lambda_u \\ y(p_n) &< (1+2.5/k\lambda_u)y_n \end{aligned}$$

Therefore $l_{pn} < [(b+\eta)/(b-\eta)][(1+2.5/k\lambda_u)y_n/\{1.5y_n/[k\lambda_u - \lambda_u(1+\delta_3)2.5y_n/k\lambda_u]\}]l_{\tilde{\tilde{p}}}$

while from the hypothesis about k, it can be derived

$$[(1+2.5/k\lambda_u)y_n/\{1.5y_n/[k\lambda_u-\lambda_u(1+\delta_3)2.5y_n/k\lambda_u]\}]<1+\eta$$

Therefore $l_{pn}<[(b+\eta)/(b-\eta)](1+\eta)l_{\bar p}<[1/|\lambda_s|(1+2\delta_3)]l_{\bar p}$

Hence

$$|x_{n+1}-x(\bar p)|<(l_{pn}/2)|\lambda_s|(1+2\delta_3)<l_{\bar p}/2$$

$$X_{n+1}\in U(P_4^*)$$

2) If $|[x_n-x(p_n)]/[y_n-y(p_n)]|\leqslant\delta_2$. The formula

$$(1-\delta_3/2)|\lambda_s[x_n-x(p_n)]|+N\{[x_n-x(p_n)]^2+[y_n-y(p_n)]^2\}<|x_{n+1}-x(p_n)|$$
$$<(1+\delta_3/2)|\lambda_s[x_n-x(p_n)]|+M\{[x_n-x(p_n)]^2+[y_n-y(p_n)]^2\}$$

is used to study this case.

$$\begin{aligned}|x(p_n)-x_{n+1}|&<(1+\delta_3/2)|\lambda_s[x_n-x(p_n)]|+M\{[x_n-x(p_n)]^2+[y_n-y(p_n)]^2\}\\&<(1+\delta_3/2)|\lambda_s[x_n-x(p_n)]|+M\{\delta_2[y_n-y(p_n)]^2+[y_n-y(p_n)]^2\}\\&<(1+\delta_3/2)|\lambda_s[x_n-x(p_n)]|+M(\delta_2+1)[y_n-y(p_n)]^2\\&<(1+\delta_3/2)|\lambda_s\delta_2[y_n-y(p_n)]|+M(\delta_2+1)[y_n-y(p_n)]^2\\&<(1+\delta_3/2)|[y_n-y(p_n)]|\{\lambda_s\delta_2+M(\delta_2+1)[y_n-y(p_n)]\}\\&<(1+\delta_3/2)(2.5y_n/k\lambda_u)\{(\lambda_s\delta_2+M(\delta_2+1)(2.5y_n/k\lambda_u)\}\\&<(1+\delta_3/2)(2.5y_n/k\lambda_u)\{\lambda_s\delta_2+M(\delta_2+1)(2.5y_n/k\lambda_u)\}\\&<(1+\delta_3/2)(l_{pn}/2)(5\delta_2)\{\lambda_s\delta_2+M(\delta_2+1)(2.5y_n/k\lambda_u)\}\end{aligned}$$

$\exists\, P_4^*<P_3^*$, when $X_n=(x_n,y_n)\in U(P_4^*)$, then $y_n<k\lambda_u/[2.5M(\delta_2+1)]$. Further when $M(\delta_2+1)<(2.5y_n/k\lambda_u)$

$$\begin{aligned}|x(p_n)-x_{n+1}|&<(1+\delta_3/2)(l_{pn}/2)(5\delta_2)\{\lambda_s\delta_2+1\}\\&<(1+\delta_3/2)(l_{pn}/2)(5\delta_2)2\\&<(1+\delta_3/2)(l_{pm}/2)10\delta_2\\&<(\lambda_s/2)(1+\delta_3/2)(l_{pn}/2)\end{aligned}$$

Just like 1), the following estimation is given

$$|x(p_n)-x_{n+1}|<(l_{pn}/2)(1+\delta_3)|\lambda_s|\{1+5\delta_2[(a+\eta)/(b-\eta)]\}$$

While, from 1), other estimations are derived

$$|x(p_n)-x(\bar p)|<[(a+\eta)/(b-\eta)](1+\delta_3)\lambda_u5\lambda_u\delta_2(l_{pn}/2)$$
$$|x_{n+1}-x(\bar p)|<|x(p_n)-x(\bar p)|+|x(p_n)-x_{n+1}|<l_{\bar p}/2$$

Therefore $X_{n+1}\in U(P_4^*)$. Q. E. D.

Lemma 6 Under the hypothesis (★), $\exists\, P_5^*<P_4^*$, $\forall X_0\in U(P_5^*)$, $\{X_n\}$ is generated by (2.2), then $X_n\to 0$.

Proof

As
$$y(p_n)-y_n=2y_n/k\lambda_u+(d+\tilde f)[(1+2/k\lambda_u)y_n]^2$$
$$\exists\, P_5^*<P_4^*,\ \forall X_n=(x_n,y_n)\in U(P_5^*)$$
$$(2-\delta_3/2)y_n/k\lambda_u<|y(p_n)-y_n|<(2+\delta_3/2)y_n/k\lambda_u$$

Therefore

$$y_{n+1} < y(p_n) - \lambda_u(1-\delta_3)|y(p_n) - y_n|$$
$$< y(p_n) - \lambda_u(1-\delta_3)(2-\delta_3/2)y_n/k\lambda_u$$

Further, from condition(★)and $\lambda_u(1-\delta_3) > (1+\delta_3)$ the estimation of y_{n+1} can be derived

Therefore
$$\begin{aligned} y_{n+1} &< y(p_n) - (1+\delta_3)(2-\delta_3/2)y_n/k\lambda_u \\ &< [1+(2+\delta_3/2)/k\lambda_u]y_n - (1+\delta_3)(2-\delta_3/2)y_n/k\lambda_u \\ &< \{1+[2+\delta_3/2-(1+\delta_3)(2-\delta_3/2)]/k\lambda_u\}y_n \\ &< \{1+[2+\delta_3/2-(1+\delta_3)(2-\delta_3/2)]/k\lambda_u\}y_n \\ &< \{1+[2+\delta_3/2-(2+1.5\delta_3-\delta_3^2/2)]/k\lambda_u\}y_n \\ &< \{1+[-\delta_3+\delta_3^2/2]/k\lambda_u\}y_n < [1-(\delta_3/2)/k\lambda_u]y_n \end{aligned}$$

Hence, y_n monotonicly converge to 0, obviously x_n converge to 0, then X_n converge to 0.

Proof of Theorem 1

In Theorem 1, taking neighborhood $U = U(P_5^*)$, by the six lemmas, the proof of Theorem 1 can be finished.

Remark When the H3. can not be satisfied, but $b \neq 0$, the result is still correct. If H1, H2. were satisfied, the follow theorem can be proved by similar method.

Theorem 1′ If $b \neq 0$. $\exists K > 0$, $\forall k > K$, $\exists U$ is a region of $\xi = 0$, $\forall X_0 \in U$, and parameter p_n is determined by the following equation

$$bp_n = y_n + 2y_n/(k\lambda_u)$$

$\{X_n\}$ generated by $X_{n+1} = F(p_n, X_n)$ converge to $\xi = 0$, and for every n, $X_n \in U$.

III. Example

Generally speaking, it is eary to check the condition of Theorem 1. Now we study Lauwerier attractor[2] as an example. The Lauwerier map is:

$$\begin{cases} x_{n+1} = x_n(1-2y_n)/2 + y_n \\ y_{n+1} = 4y_n(1-y_n) \end{cases}$$

Denote $X_{n+1} = F_p(X_n)$ as

$$\begin{cases} x_{n+1} = x_n(1-2y_n)/2 + y_n \\ y_{n+1} = 4(1-p)y_n(1-y_n) \end{cases}$$

When $p=0$, it has the fixed point (3/5, 3/4), and $\lambda_u = -2, \lambda_s = 1/2$.
x axes is the direction of stable manifold of the fixed pointed and Y axis is the direction of unstable manifold of the fixed pointed.
When $p \neq 0$, the fixed point is $\xi(p) = (x(p), y(p))$, where $y(p) = (3-4p)/(4-4p)$

$$b = \partial y(p)/\partial p|_{p=0} = -1/4 \neq 0$$

And obviously $\xi(p) \in C^2, F_p(X) \in C^2$.
Therefore it can be controlled by this method.

We will report our results about chaotic controlling in higher-dimensional systems in elsewhere.

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