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A Time Domain Computation Method for Dynamic Behavior of Mooring System ①

Liu Yingzhong[1], Miao Guoping[1], Li Yile[2], Liu Ziyuan[3]
Liu Hedong[4], Huang Qingyu[5] and Ma Qingjiu[5]

Abstract — A quasi-steady time domain method is developed for the prediction of dynamic behavior of a mooring system under the environmental disturbances, such as regular or irregular waves, winds and currents. The mooring forces are obtained in a static sense at each instant. The dynamic feature of the mooring cables can be obtained by incorporating the extended 3–D lumped-mass method with the known ship motion history. Some nonlinear effects, such as the influence of the instantaneous change of the wetted hull surface on the hydrostatic restoring forces and Froude- Krylov forces, are included. The computational results show a satisfactory agreement with the experimental ones.

Key words: *mooring systems*; *dynamic characteristics*; *time domain analysis*

1. Introduction

Mooring systems are widely used in offshore oil exploration, whose working condition is vehement due to the combined effects of waves, winds and currents. The structure's motion induced by environmental forces, will lead to large tension in mooring cables. To accurately predict the tension of mooring cable is very important in the reasonable design and safe operation of those offshore oil systems.

The mechanism of the interactive effects among waves, winds, currents and mooring bodies is so complex that the present calculating techniques cannot give the strict solution including all nonlinear effects, though computer techniques have developed so rapidly in the last decade. From the viewpoint of engineering, it is necessary to develop an approach to predict the motions and tensions of mooring systems during the initial design or case comparison. Using the quasi-steady time domain method, the authors have developed a practical method to predict the motion of moored structures and the dynamic responses of mooring cables under the combined effects of waves (regular or irregular), winds and currents. In the present approach, a three dimensional singularity distribution method is employed to obtain the hydrodynamic coefficients and wave loads without taking into account the effects of currents since the current velocity is usually low. The instantaneous variation of wetted hull surface is considered in the determination of the hydrostatic restoring forces and Froude-Krylov forces, which was shown to contribute much to the nonlinear nature of ship motions (Fang, 1993). Some empirical methods

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1 Professor, School of Naval Architecture and Ocean Engineering, Shanghai Jiao Tong University, Shanghai 200030
2 Assistant, School of Naval Architecture and Ocean Engineering, Shanghai Jiao Tong University, Shanghai 200030
3 Ph. D. Candidate, Dept. of Naval Architecture and Ocean Engineering, University of Michigan, Ann Arbor, USA
4 Postdoctor, Institute of Mechanics, China Academy of Sciences, Beijing 100080
5 Senior Engineer, China Offshore Oil Bohai Corporation, Tianjin 300452

are introduced to estimate the wind and current loads as well as the rolling damping (Tang, 1982; Ovokata, 1981; Ikeda, 1978) In the prediction of Ship motion, a quasi-static method (Nakajima, 1982; Huang, 1994) for composite mooring cables is adopted since it is commonly regarded that the dynamic behavior of the mooring cables have less influence on ship motions. An extended 3–D lumped-mass method is then used for the detailed dynamic analysis of the mooring cables with the known ship motion history.

2. Theoretical Model

2.1 Equations of Ship Motion

A space-fixed coordinate system $O'-x'y'z'$ and a ship-fixed coordinate $G-xyz$ are used to describe the ship motion, as shown in Fig. 1. The equation set of ship motions can be formulated in the $G-xyz$ system as:

$$\left.\begin{aligned} & M\frac{d\vec{u}_G}{dt}+\vec{\omega}\cdot\vec{u}_G=\vec{F}_E+\vec{F}_M-D^T\left(Mg\vec{j}\right) \\ & I_G\frac{d\vec{\omega}}{dt}+\vec{\omega}\cdot I_G\vec{\omega}=\vec{T}_E+\vec{T}_m \\ & \frac{d\vec{x}_G}{dt}=D\vec{u}_G \\ & \frac{d\vec{\alpha}}{dt}=B^{-1}\vec{\omega} \end{aligned}\right\} \tag{1}$$

in which M and I_G are the mass and inertia matrix respectively; g is the gravitational acceleration; $\vec{j}$ is the upward unit vector in $O'-x'y'z'$; $\vec{u}_G$, $\vec{\omega}$ are the translational and rotational veloicty respectively; $\vec{F}$ and $\vec{T}$ are external force and moment respectively; Subscripts E and M mean the loads of environment and mooring system, defined in the ship-fixed frame; $\vec{x}_G$ is the displacement of G with respect to the origin of $O'-x'y'z'$; $\vec{\alpha}=(\theta,\varphi,\psi)$, denoting the Eulerian angles of roll, yaw and pitch; D and B^{-1} are the transformation matrices of translational and rotational vectors.

2.2 Environmental Loads

2.2.1 *Wind and Curent Loads*

Wind and current loads in different directions are defined as follows:

longitudinal force $F_{Xcu}=\frac{1}{2}C_{Xcu}\rho_0 V^2_{Cur}L_{bp}d$, $\quad F_{Xwd}=C_{Xwd}\rho_a V^2_{wdr}A_T$ (2a)

lateral force $F_{Xcu}=\frac{1}{2}C_{Zcu}\rho_0 V^2_{Cur}L_{bp}d$, $\quad F_{Xwd}=C_{Zwd}\rho_a V^2_{wdr}A_L$ (2b)

yawing moment $M_{Xcu}=\frac{1}{2}C_{XZcu}\rho_0 V^2_{Cur}L^2_{bp}d$, $\quad M_{XZwd}=C_{Xwd}\rho_a V^2_{wdr}A_L L^2_{bp}$ (2c) in which the subscripts *cu* and *wd* denote the corresponding variables for currents and winds; ρ_a and ρ_0 are the density of air and water; A_T and A_L the longitudinal and transverse area of the ship above the water surface; L_{bp} and d are the ship length between perpendiculars and the draft, respectively; C_x, C_z and C_{xz} are the non-dimensional coefficients of wind or current loads, which can be obtained in some empirical ways (Tang, 1982; Obokata, 1981). The apparent wind and current velocity can be calculated as follows:

$$\left.\begin{aligned}\vec{V}_{wdr} &= \vec{V}_{wd} - \vec{u}_G ,\\ \vec{V}_{cur} &= \vec{V}_{cu} - \vec{u}_G\end{aligned}\right\} \tag{3}$$

in which $\vec{V}_{wd}$ and $\vec{V}_{cu}$ are the absolute veloicties in $O' - x'y'z'$.

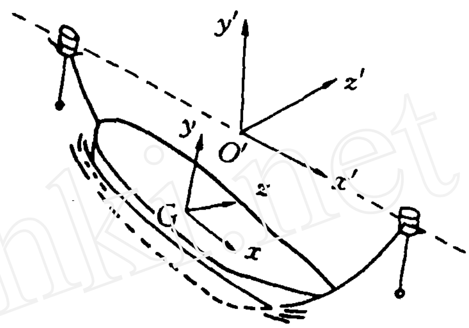

Fig. 1. Coordinate framework.

2.2.2 *Wave Loads*

— Hydrostatic Restoring Force and Froude-Krylov Force

To include the nonlinear effects due to the change of wetted hull surface, the hydrostatic restoring force and Froude-Krylov force may be obtained by integrating the pressure over the instantaneous wetted hull surface S_b

$$\left.\begin{aligned}\vec{F}_k &= \int_{S_b} P_k \vec{n} ds ,\\ \vec{T}_k &= \int_{S_b} P_k \left(\vec{r} \cdot \vec{n}\right) ds\end{aligned}\right\} \tag{4}$$

The upper integral limit of z is the wave surface ζ rather than static water surface $z = 0$. This fact necessitates some sorts of approximation to evaluate the pressure in the free surface zone since the linear wave theory implies as infinitesimal amplitude which does not extend above or below the static water surface. The calculation of pressure is written as:

$$p_k (x', y', z', t) = -\rho_0 g \left(z' - \sum_{m=1}^{N} \zeta_m\right)$$

$$+ \rho_0 g \sum_{m=1}^{N} \zeta_m \cdot \frac{\cosh k_m (z' + h) - \cosh k_m (\zeta_m + h)}{\cosh(k_m h)} - \frac{1}{2} \rho_0 u^2 \tag{5}$$

where u is the velocity of wave particle and h is the water depth.

— Radiation Forces

The added mass and damping coefficient can be obtained by the 3–D singularity distribution method (Liu, 1987). This method is based on linear theory and the hydrodynamic coefficients are frequency-dependent. For irregular ship motions, frequency should be used in calculation. One available approach is based on a scheme that samples the previous two zero-crossing periods of six free-degree motions such as surge, heave, sway, roll, yaw and pitch. The two values are averaged and six characteristic frequencies are derived for the evaluation of the coefficients.

$$F_{Ri} = -\sum_{j=1}^{6} \left(\mu_{ij}\ddot{x}_j + \lambda_{ij}\dot{x}_j\right) \qquad i = 1, 2, \dots 6 \tag{6}$$

in which μ and λ are the added mass and damping coefficient respectively, depending on the instant frequency, $\ddot{x}$ and $\dot{x}$ are the instantaneous acceleration and velocity of motions of different modes.

— Diffraction Forces

The regular wave exciting forces are calculated in the frequency domain by 3–D method. For different frequencies a transfer function $H_j(\omega_m, \chi_{rx})$ is obtained for each motion mode and heading angle related to the incident wave. Because of the smallness of the averaged motion of the ship's gravity center, the natural frequency of the incident wave rather than the encounter frequency is used in the transfer function. The yaw angle from its initial position can be determined approximately by

$$\chi_s(t) = \chi_{s0} + \int_0^t \Omega'_3(t)\,dt = \int_0^t (\psi \sin\theta + \varphi \cos\theta \cos\varphi)\,dt + \chi_{s0} \tag{7}$$

and the relative heading angle is given by

$$\chi_{rx} = \left|\chi_w - \chi_s(t)\right| \tag{8}$$

The instantaneous values of the forces are thus calculated by

$$F_{dj}(t) = \sum_{m=1}^{N} H_j(\omega_m, \chi_{rx})\,\zeta_m \tag{9}$$

— Drift Forces

Drift forces play an unnegligible role to the horizontal motion of the ship, especially in a mooring system. The far field method (Newman, 1967) is exended to calculate the transfer function of drift forces for finite water depth (Liu, 1996). The function is derived from the regular incident wave case. When the ship is encountered with irregular waves, the second-order wave forces can be calculated as follows

$$F_j^{(2d)} = \sum_{m=1}^{N} \sum_{n=1}^{N} \left\{ H_j^{(2)} (\omega_m, \omega_n, \chi_{rx}) \zeta_m \zeta_n \exp\left[-i (\omega_m - \omega_n) t + i (\varepsilon_m - \varepsilon_n) \right] \right\} \tag{10}$$

where, according to Newman, the transfer function is approximated to

$$H_j^{(2)} (\omega_m, \omega_n, \chi_{rx}) \approx \overline{H}_j^{(2)} \left[\frac{1}{2} (\omega_m + \omega_n), \chi_{rx} \right] \tag{11}$$

in which the $H^{(2)}$ is the transfer function of second order wave force for a harmonic wave with a frequency of $\omega = \frac{1}{2} (\omega_m + \omega_n)$.

2.3 Mooring Forces

The cables are made up of chain, rope or a combination of both, linking with surface or submerged buoys. The 2–D method supposes that the cable is kept in one plane and the effects of wave, current and the motion of itself are negligible.

In the dynamic analysis, a 3–D lump-mass-and-spring model is established. The cable is replaced by a discretized model consisting of many point masses and massless segments. The mass of each point equals to half of the mass of the two neighboring elements. Only three translational added masses are taken into account. The equations of motion of a mass point are:

$$\left.\begin{aligned} (M + A_{11}) \ddot{x} + A_{12} \ddot{y} + A_{13} \ddot{z} &= F_x \\ A_{21} \ddot{x} + (M + A_{22}) \ddot{y} + A_{23} \ddot{z} &= F_y \\ A_{31} \ddot{x} + A_{32} \ddot{y} + (M + A_{33}) \ddot{z} &= F_z \end{aligned}\right\} \tag{12}$$

where M is the mass of the point, A_{ij} is the added mass and F_j is the external force acting on the segment. The forces include the tension forces in the two segments, drag force, the gravitational force and the buoyant force.

The drag force, according to Morrison's formula, can be written as:

$$\left.\begin{aligned} f_t &= \frac{1}{2} \rho_0 C_{Dt} D \cdot l \cdot U_t |U_t| \\ f_s &= \frac{1}{2} \rho_0 C_{Ds} D \cdot l \cdot U_s |U_s| \\ f_n &= \frac{1}{2} \rho_0 C_{Dn} D \cdot l \cdot U_n |U_n| \end{aligned}\right\} \tag{13}$$

where t, x, n are the three unit vectors in the local coordinate system; D *and* l are the diameter and length of the segment; C_{Dt}, C_{Ds} and D_{Dn} are the drag coefficients for three directions; U_t, U_s and U_n are the three components of the relative velocity of the mass point.

The solution of dynamic behavior of a 3-D cable can only be executed in the time domain by numerical method. The Houbolt scheme is used in the descritization of the equations of motion. The second and first derivatives are expressed respectively as:

$$\left.\begin{aligned} \ddot{s}_j^{n+1} &= \frac{1}{\Delta t^2}\left(2S_j^{n+1} - 5s_j^n + 4s_j^{n-1} - s_j^{n-2}\right) \\ \dot{s}_j^{n+1} &= \frac{1}{6\Delta t}\left(11S_j^{n+1} - 18s_j^n + 9s_j^{n-1} - 2s_j^{n-2}\right) \end{aligned}\right\} \quad (14)$$

where superscript n represents time step $n\Delta t$ and subscript j represents node number.

2.4 Integration of Equations of Motion

The fourth-order Runge-Kutta method is utilized to solved the equation set of motion to obtain the time history of ship motions. It is assumed that the dynamic effects of the mooring cables on the ship motions are negligible and the mooring force acting on the ship are obtained in a static sense at each instant. With the known history of ship motion, the extended 3-D lumped-mass method is used, taking exterior fluid forces and elastic characteristics into consideration. It can be found from our numerical and experimental results that this assumption is reasonable and the nonlinear effects of changeable wetted hull surface are important from the numerical and experimental results.

3. Numerical Results and Discussions

In order to exemplify the feasibility of the present method, a series of computations for a 5000 T oil tanker are carried out, under different combinations of waves, winds and currents with various directions and velocities. The ship is moored in fore and aft directions between two mooring cables, each with a nylon rope at the ship end and one or two steel cables at the sea bottom end which connect each other with a cylindrical buoy. The JONSWAP spectrum is used to simulate irregular waves.

The L.B.P. of the oil tanker is 104.00 m and its width is 16.4 m. The draft and volume are $D = 5.70$ m, Volume $= 7621\ \mathrm{m}^3$ for full condition and $D = 3.31$ m, Volume $= 4185\ \mathrm{m}^3$ for ballast condition. The tanker's cubic coefficient is $C_b = 0.749$.

The geometry and weight of the buoy is $H * D *$ Weight $= 4.0\ \mathrm{m} * 1.5\ \mathrm{m} * 44.145$ kN. For nylon cable, $L * D = 80.0\ \mathrm{m} * 0.146$ m, and for the steel cable, $L * D = 8.4\ \mathrm{m} * 0.144$ m. The distance between the force and aft anchoring points on the sea bottom is 268.00 m.

Fig. 2 shows the time history of the six translational and rotational motions of the ship. It can be found that the time history of motion includes two main components: the low frequency component and the high frequency one. The former is induced by the mooring system, whose characteristic frequency is low and the characteristic period is about 100～200 seconds, the latter is induced by the wave forces, whose period is about 5～10 seconds. The time history of the tension of the mooring cable is shown in Fig. 3. The 3-D results are slightly larger than the 2-D results. The difference of the two results is very small and it is considered that the 2−D method is accurate enough to satisfy practical engineering requirement. The tension is mainly from the elastic action of the nylon rope. The elongation of the rope is about 10~30% and the effects of fluid drag forces and dynamic action are not important compared with the elastic effects.

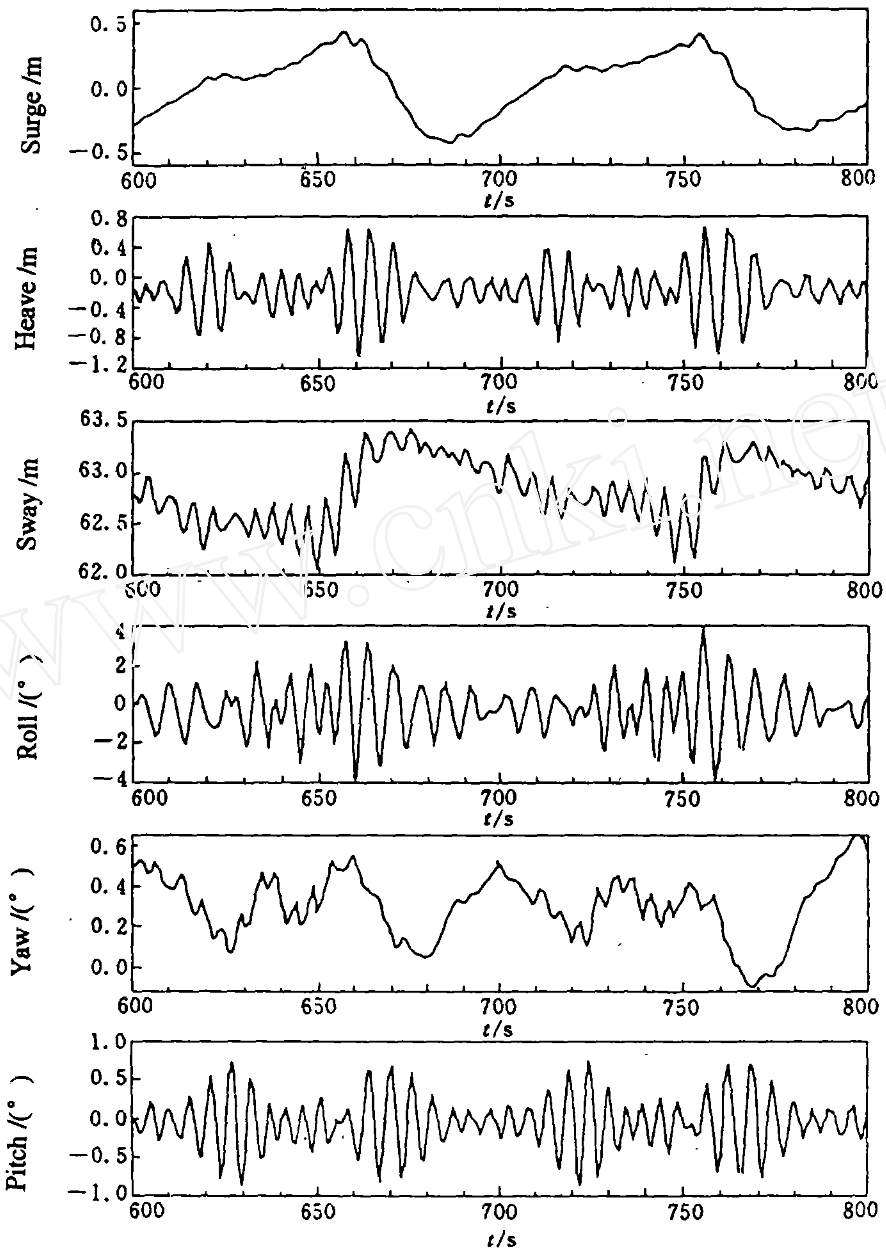

Fig. 2. Motions of oil tanker.
water depth = 15 m, wave height = 1.5 m, beam waves
wind velocity = 15 m / s, current velocity = 1.03 m / s

Table 1 shows the motions and mooring forces under the conditions of significant wave height $H = 1.5$ m and the wave direction $\beta = 90$, 180, 135 degrees (the wind and current directions are the same as that of the wave). The maximum fore cable tension is 406.3 kN (beam wave), 86.5 kN (heading wave) and 278.3 kN (heading oblique wave) respectively, showing that the fore cable reaches its maximum tension in beam wave / wind / current. In the case of heading wave, the ship goes back from its initial position and the fore cable is tight and the aft one is so loose that its tension may be zero. In the design of mooring systems, the strength of mooring cable must be satisfied when the ship is under combined effects of beam waves, winds and currents.

Table 1 Motions of oil tanker and force of mooring cables

		F_{head} (kN)	F_{tail} (kN)	Roll (°)	Pitch (°)	Yaw (°)	Heave (m)	Sway (m)	Surge (m)	Wave (°)
Mean	Exp.	304.77	314.13	−0.064	0.187	0.074	0.681	41.994	2.887	90
	Cal.	368.22	381.04	−0.294	−0.059	0.278	−0.190	62.900	0.001	
$A_{1/3+}$	Exp.	344.98	355.09	3.053	0.904	0.123	2.664	41.362	3.627	
	Cal.	395.87	402.08	2.130	0.514	0.243	0.484	0.938	0.338	
A_{max+}	Exp.	437.14	430.54	8.257	1.756	0.258	3.625	44.610	5.279	
	Cal.	406.30	416.00	4.177	0.796	0.738	0.719	64.492	0.507	
Mean	Exp.	57.32	30.22	0.799	1.713	4.310	0.320	1.763	−9.815	180
	Cal.	55.03	6.42	0.0	−0.053	0.0	−0.189	0.0	−2.671	
$A_{1/3+}$	Exp.	84.48	38.41	1.738	3.710	4.340	0.794	2.861	−10.236	
	Cal.	82.82	32.11	0.0	0.470	0.0	0.141	0.0	2.726	
A_{max+}	Exp.	111.20	58.54	2.556	5.663	4.402	1.117	11.867	−11.207	
	Cal.	86.50	35.00	0.0	0.801	0.0	0.075	0.0	0.302	
Mean	Exp.	235.62	181.76	1.005	0.423	0.319	0.673	42.095	−46.709	135
	Cal.	253.78	218.18	−0.070	−0.196	−4.216	−0.189	38.079	−40.364	
$A_{1/3+}$	Exp.	317.75	233.90	3.102	2.897	1.416	2.433	42.559	−46.206	
	Cal.	270.83	228.43	0.642	0.632	0.209	0.157	0.313	0.621	
A_{max+}	Exp.	402.82	331.48	5.827	4.950	2.545	3.433	43.770	−48.779	
	Cal.	278.30	230.00	1.046	0.849	−4.740	0.073	38.522	−40.770	

Notes: water depth = 15 m, sign. wave height = 1.5 m, wind velocity = 15 m / s, current velocity = 1.03 m / s.

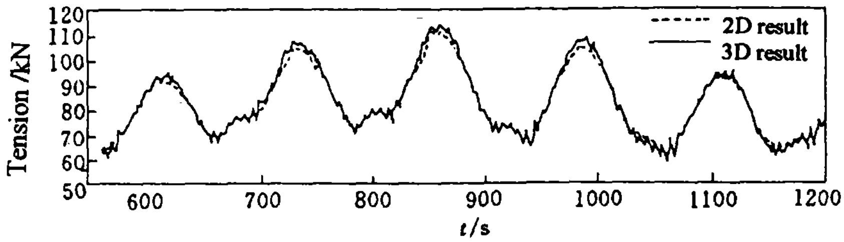

Fig. 3. Tension of fore mooring cable.
water depth = 15 m, wave height = 2.5 m, heading waves
wind velocity = 20 m / s, current velocity = 1.03 m / s

Unlisted numerical results show that the wave height makes most contribution to tension of the cable. Under beam environment loads, the average of the fore cable's tensions are 368.0 kN (H = 1.5 m), 656.0 kN (H = 2.5 m) and 594.0 kN (H = 3.5m), which are almost equal to the tension of the aft cable. This phenomenon is due to the increment of second-order force and the motion response of the mooring system with the increment of incident waves. When the motions are large enough, the nonlinear effects will make main contribution and the average cable tension may decrease, though the fluctuation will become more serious.

The mooring forces increase when the water depth decreases. With the same wave height, current and wind velocity, the mean value of the force of the mooring cable are 347.0 kN (20m), 368.0 kN (15m) and 550.0 kN (10m). This is because the drift forces and current forces increase with the decrease of water depth.

The comparison of different loading status is also done. The mean value of the mooring force is from 274.0 (ballast) and 368.0 kN (full). The maximum value is 322.0 kN (ballast) and 406.0 kN (full). The difference in motion is not significant.

4. Conclusions

A practical method to calculate the mooring system is presented and the computational results show satisfactory agreement with the experimental ones although some simplifications are introduced in the present method. From the results, one may conclude that:

(1) The current force and drift force play important roles in the mooring system. Their effects influence the distance between the relative balance position of the ship and its initial position and are the most important factors to the mooring system.

(2) The mooring force and ship motion increase when the wave height increases. When the wave height is large enough, the nonlinear effects are obvious and the result will differ from that of the linear theory.

(3) The mooring forces increase on the decreasing water depth. The reason may be the character of current force and drift force.

(4) The dynamic behavior of mooring systems is not very dependent on the loading status.

References

Fang, M. C., Lee, M. L. and Lee, C. K., 1993. Time simulation of water shipping for a ship advancing in large longitudinal waves, *Journal of Ship Research*, **37**(2), 126–137.

Huang S., 1994. Dynamic analysis of three dimensional marine cables, *Ocean Engng.*, **21**, 587–605.

Ikeda, Y., Himeno, Y. and Tanaka, N., 1978. A prediction method for ship roll damping, *Report of Univ. Osaka Prefecture*, No. 00405.

Liu, Y. Z. and Miao, G. P., 1987. *The theory on ship motion in waves*, Shanghai, Shanghai Jiao Tong Univ. Press. (in Chinese)

Liu, Y. Z. and Miao, G. P., 1996. Influence of water depth on the second-order steady wave forces, *Journal of Shanghai Jiao Tong Univ.* **30**(10), 147–151. (in Chinese)

Nakajima, T., 1982. On the dynamic analysis of multi-component mooring lines, *Proc. 14th Offshore Tech. Conf.*, Houston, OTC 4309, 105 ~ 121.

Newman, J. N., 1967. The drift force and moment on ships in waves, *Journal of Ship Research*, **11**(1), 51 ~ 60.

Ovokata, J., Sasaki, N. and Nagashima, J., 1981. On the estimation of current force induced on a ship hull by some model tests, *J. Kansai Society of Naval Architectures*, **180**, 47 ~ 57. (in Japanese)

Tang, Z. G. and Shi, L. R., 1982. Aerodynamic force ship, *J. of Wuhan Institute of Water Transportation Engineering*, **4**, 79 ~ 89. (in Chinese)