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Large Strain Field Near a Crack Tip in a Rubber Sheet

Y C Gao* S H Chen**

* Northern Jiaotong University, 100044, Beijing, China

** LNM, Institute of Mechanics, Chinese Academy of Sciences, 100080, Beijing, China

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Abstract

The distribution of stress-strain near a crack tip in a rubber sheet is investigated by employing the constitutive relation given by Gao (1997) It is shown that the crack tip field is composed of two shrinking sectors and one expanding sector The stress state near the crack tip is in uniaxial tension The analytical solutions are obtained for both expanding and shrinking sectors

Key words. large strain, rubber sheet, crack

1. Introduction

The large strain elastic field near a crack tip is a difficult but very typical problem that can examine various elastic laws and analytical methods The feature of the field depends on the elastic law Knowles and Sternberg (1973) proposed an elastic law for rubber materials, and analyzed the mode I crack tip field under plane strain condition Gao (1990, 1997) proposed two elastic laws and gave a sector division method, those are used to analyze the mode I plane strain crack tip field The interesting fact is that according to the three different elastic laws mentioned above, the very similar stress fields near a crack tip are obtained, but Knowles and Sternberg (1973, 1974) did not consider the expanding sector Gao and Gao (1999) analyzed the large strain notch tip (crack is a special case) field and compared the three elastic laws It is found that for notch tip field, the three elastic laws can become equivalent if the constitutive parameters are related by some conditions It is also found that when the expanding sector is taken into account the restriction on parameter n (given by Knowles and Sternberg) is not needed

Comparing with plane strain problem, the plane stress crack tip field is more complicated because of the thickness shrinkage An asymptotic analysis of the plane stress crack was given by Gao and Durban (1995) where the thickness shrinkage can be expressed by strain invariants For the elastic law given by Gao (1997), the thickness shrinkage can only be given by a differential equation that must be solved numerically The purpose of this paper is to reveal the crack tip feature in a sheet that obeys the elastic law given by Gao (1997)

2. Basic Equations

Consider a three-dimensional elastic body Let $\mathbf{P}$ and $\mathbf{Q}$ denote the position vectors of a point before and after deformation, respectively x^i $(i = 1,2,3)$ denote Lagrangian coordinates Two sets of triads are defined as

$$\mathbf{P}_i = \frac{\partial \mathbf{P}}{\partial x^i} \qquad \mathbf{Q}_i = \frac{\partial \mathbf{Q}}{\partial x^i} \tag{1}$$

The displacement gradient is

$$\mathbf{F} = \mathbf{Q}_i \otimes \mathbf{P}^i \tag{2}$$

Note that $\boldsymbol{P}^i$ is the conjugate of $\boldsymbol{P}_i$, while $\otimes$ is the dyadic symbol and summation rule is implied

The Green and Cauchy deformation tensors are

$$D = F^T\ F \qquad d = F\ F^T \tag{3}$$

* Email ycgao@center njtu edu cn

where the superscript T denotes the transpose Let $\mathbf{E}$ stand for the unit tensor, then the following invariants will be used,

$$I_1 = \boldsymbol{D} \;\; \boldsymbol{E} = \boldsymbol{d} \;\; \boldsymbol{E}\,, \quad I_{-1} = \boldsymbol{D}^{-1} \;\; \boldsymbol{E} = \boldsymbol{d}^{-1} \;\; \boldsymbol{E}\,, \quad J = V_Q / V_P \tag{4}$$

where $V_Q = (\boldsymbol{Q}_1, \boldsymbol{Q}_2, \boldsymbol{Q}_3)$, $V_P = (\boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_3)$, and the brackets denote the mixed product

A kind of strain energy per undeformed unit volume was proposed in Gao (1997),

$$U = a(I_1^n + I_{-1}^n) \tag{5}$$

where a and n are material constants Then the Kirchhoff stress is

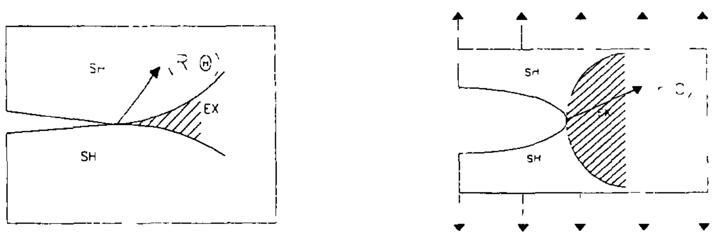
$$\boldsymbol{\sigma} = 2\frac{\partial U}{\partial \boldsymbol{D}} = 2na(I^{n-1}\boldsymbol{E} - I_{-1}^{n-1}\boldsymbol{D}^{-2}) \tag{6}$$

From Eq (6), the Cauchy stress can be obtained

$$\boldsymbol{\tau} = J^{-1}\boldsymbol{F} \;\; \boldsymbol{\sigma} \;\; \boldsymbol{F}^T = 2naJ^{-1}(I_1^{n-1}\boldsymbol{d} - I_{-1}^{n-1}\boldsymbol{d}^{-1}) \tag{7}$$

The equilibrium equation is

$$\frac{\partial(V_Q \; \boldsymbol{\tau} \; \boldsymbol{Q}^i)}{\partial x_i} = 0 \tag{8}$$



(a) (b)

Fig 1 The sector division near the crack tip, (a) before loading, (b) after loading

3. Shrinking Sector SH

Fig 1 (a) and (b) show the cracked rubber sheet before and after loading respectively Since the strain near the crack tip is very large, as analyzed in Gao (1990), the deformation cannot be described by an uniformed mapping function for the whole region According to the sector division method, the whole crack tip field is divided into one expanding sector EX and two shrinking sectors SH and SH' Before loading EX is very narrow while SH and SH' occupy almost the whole crack tip field After loading EX occupies the whole crack tip field while SH and SH' become very narrow Two Lagrangian coordinates are introduced, (R, Θ, Z) is the cylindrical coordinate before deformation and (r, θ, z) is the cylindrical coordinate after loading

For the shrinking sectors SH, the mapping function from (R, Θ, Z) to (r, θ, z) is made as follows,

$$r = R^{1-\delta}\varphi(\Theta), \qquad \theta = \frac{\pi}{2} - R^{\gamma}\psi(\Theta), \qquad z = R^t Z\eta(\Theta) \tag{9}$$

in which $\delta, \gamma, t > 0$, $0 < \Theta \le \pi$ $t > 0$ indicates that the thickness of the sheet shrinks tremendously From Eq (9), the local triads are

$$\begin{cases} \boldsymbol{Q}_R = R^{-\delta}\varphi[(1-\delta)\boldsymbol{e}_r - \gamma R^{\gamma}\psi\boldsymbol{e}_\theta] + R^{t-1}tZ\eta\boldsymbol{e}_z \\ \boldsymbol{Q}_\Theta = R^{1-\delta}(\varphi'\boldsymbol{e}_r - R^{\gamma}\varphi\psi'\boldsymbol{e}_\theta) + R^t Z\eta'\boldsymbol{e}_z \\ \boldsymbol{Q}_Z = R^t\eta\boldsymbol{e}_z \end{cases} \tag{10}$$

where

$$\boldsymbol{e}_r = \boldsymbol{Q}_r = \frac{\partial \boldsymbol{Q}}{\partial r}, \qquad \boldsymbol{e}_\theta = \frac{1}{r}\boldsymbol{Q}_\theta = \frac{1}{r}\frac{\partial \boldsymbol{Q}}{\partial \theta}, \qquad \boldsymbol{e}_z = \boldsymbol{Q}_z = \frac{\partial \boldsymbol{Q}}{\partial z} \tag{11}$$

It is assumed that the thickness of the sheet is much smaller than the size of considered domain, therefore the terms with Z in Eq (10) can be neglected

Substituting Eqs (10) and (2) into (3) and noting that

$$\boldsymbol{P}^R\ \boldsymbol{P}^R = 1,\quad \boldsymbol{P}^\Theta\ \boldsymbol{P}^\Theta = R^{-2},$$
$$\boldsymbol{P}^Z\ \boldsymbol{P}^Z = 1,\quad \boldsymbol{P}^i\ \boldsymbol{P}^j = 0,\quad (i, j = R, \Theta, Z \text{ and } i \neq j) \tag{12}$$

we obtain the dominant terms of $\boldsymbol{d}$ and $\boldsymbol{d}^{-1}$,

$$\begin{cases} \boldsymbol{d} = R^{-2\delta}[T\boldsymbol{e}_r\boldsymbol{e}_r - R^\gamma S(\boldsymbol{e}_r\boldsymbol{e}_\theta + \boldsymbol{e}_\theta\boldsymbol{e}_r) + R^{2\gamma}V\boldsymbol{e}_\theta\boldsymbol{e}_\theta] + R^{2t}\eta^2\boldsymbol{e}_z\boldsymbol{e}_z \\ \boldsymbol{d}^{-1} = R^{2\delta-2\gamma}q^{-2}[T\boldsymbol{e}_\theta\boldsymbol{e}_\theta + R^\gamma S(\boldsymbol{e}_r\boldsymbol{e}_\theta + \boldsymbol{e}_\theta\boldsymbol{e}_r) + R^{2\gamma}V\boldsymbol{e}_r\boldsymbol{e}_r] \\ \qquad + R^{-2t}\eta^{-2}\boldsymbol{e}_z\boldsymbol{e}_z \end{cases} \tag{13}$$

in which

$$\begin{cases} q = \varphi[\gamma\varphi'\psi - (1-\delta)\varphi\psi'] & T = \varphi'^2 + (1-\delta)^2\varphi^2 \\ S = \varphi[\gamma(1-\delta)\varphi\psi + \varphi'\psi'] & V = \varphi^2(\gamma^2\psi^2 + \psi'^2) \end{cases} \tag{14}$$

The strain invariants are

$$\begin{cases} I_1 = R^{-2\delta}T, & I_{-1} = R^{2\delta-2\gamma}q^{-2}T + R^{-2t}\eta^{-2} \\ J = R^{\gamma+t-2\delta}\eta q \end{cases} \tag{15}$$

in the coordinate R, Θ, Z, we have

$$V_Q = R^{1+\gamma+t-2\delta}\eta q \tag{16}$$

We assume that the shrinkage in the thickness direction and in Θ direction is the same order, then the two terms in the expression of I_{-1} must be the same order, so

$$t = \gamma - \delta \tag{17}$$

For the plane stress problem, since $\tau_{zz} = 0$, according to Eqs (13)-(15), (17) and (7) we obtain

$$t = \frac{n-1}{n+1}\delta,\qquad \gamma = \frac{2n}{n+1}\delta \tag{18}$$

and

$$T^{n-1}\eta^4 = (q^{-2}T + \eta^{-2})^{n-1} \tag{19}$$

then

$$\begin{aligned} \tau = 2na\eta^{-1}q^{-1}T^{n-1}R^{-\lambda}[T\boldsymbol{e}_r \otimes \boldsymbol{e}_r - R^\gamma S(\boldsymbol{e}_r \otimes \boldsymbol{e}_\theta + \boldsymbol{e}_\theta \otimes \boldsymbol{e}_r) \\ + R^{2\gamma}(V - T\eta^4 q^{-2})\boldsymbol{e}_\theta \otimes \boldsymbol{e}_\theta] \end{aligned} \tag{20}$$

where $\lambda = 2n\delta + 2\gamma - 3\delta$ (21)

Eq (20) indicates that the dominant component of stress is τ_{rr}

Substituting (20) and (16) into (8), noting that

$$\begin{cases} \dfrac{\partial \boldsymbol{e}_r}{\partial R} = -\gamma R^{\gamma-1}\psi\boldsymbol{e}_\theta, & \dfrac{\partial \boldsymbol{e}_r}{\partial \Theta} = -R^\gamma\psi'\boldsymbol{e}_\theta \\ \dfrac{\partial \boldsymbol{e}_\theta}{\partial R} = \gamma R^{\gamma-1}\psi\boldsymbol{e}_r, & \dfrac{\partial \boldsymbol{e}_\theta}{\partial \Theta} = R^\gamma\psi'\boldsymbol{e}_r \end{cases} \tag{22}$$

and

$$\begin{cases} \boldsymbol{Q}^R = -R^{\delta-\gamma}q^{-1}(R^\gamma\varphi\psi'\boldsymbol{e}_r + \varphi'\boldsymbol{e}_\theta) \\ \boldsymbol{Q}^\Theta = R^{\delta-1-\gamma}q^{-1}\varphi[R^\gamma\gamma\psi\boldsymbol{e}_r + (1-\delta)\boldsymbol{e}_\theta] \end{cases} \tag{23}$$

$$\begin{cases} \varphi\psi'T - u\varphi' = -(1-\delta)\varphi q, & T\gamma\psi - (1-\delta)S = \varphi' q/\varphi \\ -S\gamma\psi + (1-\delta)V = -q\psi', & -\varphi\psi' S + \varphi' V = \gamma\varphi\psi q \end{cases} \tag{24}$$

After extensive manipulation of these equations we obtain

$$[1 + \frac{2(n-1)}{T}\varphi'^2][\varphi'' + (1-\delta)^2\varphi] - (1-\delta)(1-2\delta)\varphi = 0 \tag{25}$$

and

$$
\begin{aligned}
&[\varphi+3(1-\delta)^2T\varphi^3\eta^4q^{-4}]\psi''+[2(n-1)\varphi\varphi'\psi'T^{-1}\\
&+2n(1-\delta)\varphi\varphi'\eta^4q^{-3}-3(1-\delta)\gamma T\varphi^2\psi\eta^4q^{-4}]\varphi''\\
&+2(1-\delta)^2\varphi\varphi'[(n-1)\varphi\psi'T^{-1}+(1-\delta)n\varphi\eta^4q^{-3}]\\
&+\gamma(1+\gamma)\varphi\psi-(2\delta+\gamma-1)q^{-3}T\eta^4\varphi'+2\varphi'\psi'\\
&+4(1-\delta)T\varphi\eta^3q^{-3}\eta'\\
&-3(1-\delta)T\varphi\eta^4q^{-4}[\gamma\varphi'^2\psi-(2-2\delta-\gamma)\varphi\varphi'\psi']=0
\end{aligned}
\tag{26}
$$

For the calculation of η, Eq (19) can be rewritten as

$$
\begin{aligned}
&(n-1)(q^{-2}T+\eta^{-2})^{n-2}[-2q^{-3}q'T+q^{-2}T'-2\eta^{-3}\eta']\\
&-(n-1)T^{n-2}T'\eta^4-4T^{n-1}\eta^3\eta'=0
\end{aligned}
\tag{27}
$$

in which

$$
\begin{cases}
q'=\gamma\varphi\varphi''\psi-(1-\delta)\varphi^2\psi''+\gamma\varphi'^2\psi-(2-2\delta-\gamma)\varphi\varphi'\psi'\\
T'=2\varphi'\varphi''+2(1-\delta)^2\varphi\varphi'
\end{cases}
\tag{28}
$$

In order to match the displacements in sectors SH and EX, Eqs (25)-(27) should meet the natural boundary conditions

$$
\varphi(0)=0, \qquad \psi(0)=\infty \tag{29}
$$

At $\Theta=\pi$, the traction free conditions can be reduced to

$$
\varphi'(\pi)=0 \qquad \psi'(\pi)=\frac{-\eta(\pi)}{\varphi(\pi)} \tag{30}
$$

Eq (19) gives,

$$
\eta(\pi)=[\frac{(1-\delta)^2\varphi_{(\pi)}^2}{2}]^{\frac{1-n}{2(1+n)}} \tag{31}
$$

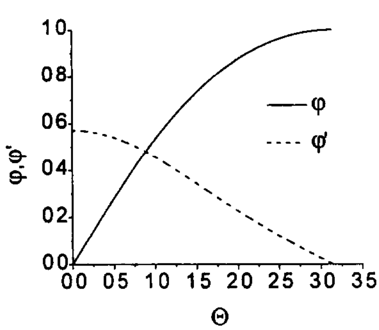


Fig 2 The curves of $\varphi(\Theta)$ and $\varphi'(\Theta)$

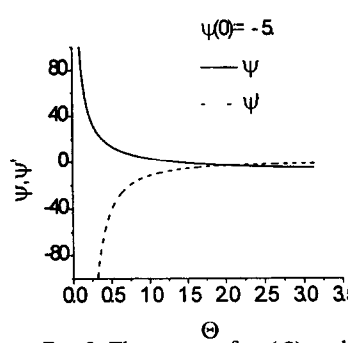


Fig 3 The curves of $\psi(\Theta)$ and $\psi'(\Theta)$

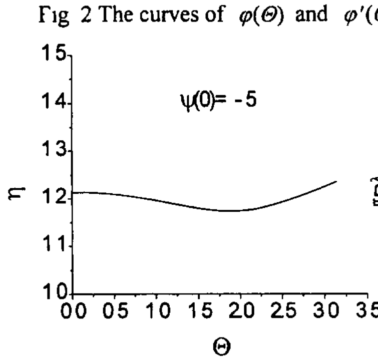


Fig 4 The curve of $\eta(\Theta)$

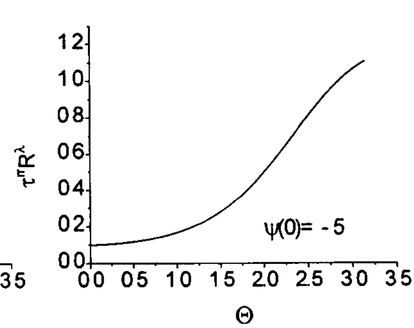


Fig 5 The curves of normalized stress $\tau^{rr}R^{\lambda}$

The detailed solution of Eq (25) subjected to the conditions (29) and (30) was given in Gao and Gao (1999) The eigenvalue δ is

$$\delta = \frac{1}{2n} \quad (32)$$

The analytical solution of φ is

$$\varphi = \frac{\sqrt{2}}{2}\varphi_\pi n^{\frac{1}{2}-\frac{1}{2n}}(\Omega - \cos\Theta)^{1/2}[\Omega + (1-\frac{1}{n})\cos\Theta]^{\frac{1}{2}-\frac{1}{2n}} \quad (33)$$

where

$$\Omega = [1-(1-\frac{1}{n})^2 \sin^2\Theta]^{1/2} \quad (34)$$

where $\varphi_\pi = \varphi(\pi)$ is a parameter to indicate the amplitude of the field The value of φ_π depends on the load at far field When φ_π is given, the function ψ and η can be solved numerically from Eqs (26)-(30) The functions φ, φ', ψ, ψ', η, $\tau_{rr}R^\lambda$ are shown in Fig 2 ~ 5 for the case $n = 2\ 0$, $\varphi(\pi) = 1$, $\psi(\pi) = -5\ 0$

Eq (32) is consistent with an analysis based on energy considerations Actually, at the crack tip, the energy density must be of the order R^{-1}, then from Eqs (5), (15) we have

$$2n\delta = 1 \quad (35)$$

4. Expanding Sector

The deformation pattern (9) is not valid when $\Theta \to 0$ because $\varphi \to 0$ and $\psi \to \infty$ Therefore, the problem must be considered in EX sector ($\Theta \sim R^\alpha$), where the mapping functions are assumed to be

$$\begin{cases} r = R^{1+\beta}\rho(\xi), & \theta = \frac{\pi}{2} - \omega(\xi) \\ z = R^b Z\varsigma(\xi), & \xi = \Theta R^{-\alpha} \end{cases} \quad (36)$$

where α, β, b are positive constants to be determined From Eq (1) and (36), we obtain

$$\begin{cases} \boldsymbol{Q}_R = R^\beta\{[(1+\beta)\rho - \alpha\xi\rho']\boldsymbol{e}_r + \alpha\xi\rho\omega'\boldsymbol{e}_\theta\} + R^{b-1}(b\varsigma - \alpha\xi\varsigma')Z\boldsymbol{e}_z \\ \boldsymbol{Q}_\Theta = R^{\beta-\alpha+1}(\rho'\boldsymbol{e}_r - \rho\omega'\boldsymbol{e}_\theta) + R^{b-\alpha}Z\varsigma'\boldsymbol{e}_z \\ \boldsymbol{Q}_Z = R^b\varsigma\boldsymbol{e}_z \end{cases} \quad (37)$$

Since the thickness of the sheet is assumed to be small, the terms with Z in Eq (37) can be neglected

Combining Eqs (37), (2), (3), (12), we can obtain the dominant terms of $\boldsymbol{d}$ and $\boldsymbol{d}^{-1}$,

$$\begin{aligned} \boldsymbol{d} = R^{-2\alpha+2\beta}[\rho'^2\boldsymbol{e}_r\boldsymbol{e}_r + \rho^2\omega'^2\boldsymbol{e}_\theta\boldsymbol{e}_\theta \\ - \rho\rho'\omega'(\boldsymbol{e}_r\boldsymbol{e}_\theta + \boldsymbol{e}_\theta\boldsymbol{e}_r)] + R^{2b}\varsigma^2\boldsymbol{e}_z\boldsymbol{e}_z \end{aligned} \quad (38)$$

$$\begin{aligned} \boldsymbol{d}^{-1} = R^{-2\beta}v^{-2}[\rho^2\omega'^2\boldsymbol{e}_r\boldsymbol{e}_r + \rho'^2\boldsymbol{e}_\theta\boldsymbol{e}_\theta + \\ \rho\rho'\omega'(\boldsymbol{e}_r\boldsymbol{e}_\theta + \boldsymbol{e}_\theta\boldsymbol{e}_r)] + R^{-2b}\varsigma^2\boldsymbol{e}_z\boldsymbol{e}_z \end{aligned} \quad (39)$$

in which

$$v = -(1+\beta)\rho^2\omega' \quad (40)$$

The invariants are

$$I_1 = R^{2\beta-2\alpha}u, \quad I_{-1} = R^{-2\beta}uv^{-2} + R^{-2b}\varsigma^{-2}, \quad J = R^{2\beta+b-\alpha}\varsigma v \quad (41)$$

where

$$u = \rho'^2 + \rho^2\omega'^2 \quad (42)$$

in the coordinate (R, Θ, Z)

$$V_Q = R^{2\beta+b-\alpha+1}\varsigma v \quad (43)$$

Assuming the shrinkage along the thickness direction and $\Theta = 0$ direction are the same order then the two terms in I_{-1} should be the same order, so

$$b = \beta \tag{44}$$

Substituting Eqs (38)-(41) into (7) and noting (44) and that $\tau_{zz} = 0$, it follows

$$\alpha = \frac{2n}{n-1}\beta \tag{45}$$

$$u^{n-1}\varsigma^2 - (uv^{-2} + \varsigma^{-2})^{n-1}\varsigma^{-2} = 0 \tag{46}$$

Noting the following relations

$$\frac{\partial\xi}{\partial\Theta} = R^{-\alpha}, \qquad \frac{\partial R}{\partial\Theta} = 0, \qquad \frac{\partial\theta}{\partial\Theta} = -\omega' R^{-\alpha} \tag{47}$$

$$\frac{\partial e_r}{\partial\Theta} = -R^{-\alpha}\omega' e_\theta, \qquad \frac{\partial e_\theta}{\partial\Theta} = R^{-\alpha}\omega' e_r \tag{48}$$

$$\begin{cases} Q^R = -R^{-\beta}v^{-1}(\rho\omega' e_r + \rho e_\theta) \\ Q^\Theta = R^{\alpha-1-\beta}v^{-1}\{\alpha\xi\rho\omega' e_r - [(1+\beta)\rho - \alpha\xi\rho']e_\theta\} \end{cases} \tag{49}$$

then $\qquad \tau\, \mathbf{Q}^R << \tau\, \mathbf{Q}^\Theta \qquad (50)$

Eq (8) can be reduced to

$$\frac{\partial(V_Q\, \tau\, Q^\Theta)}{\partial\Theta} = 0 \tag{51}$$

finally, it is obtained that,

$$\begin{cases} \rho\omega'' + 2\rho'\omega' = 0 \\ \rho'' - \rho\omega'^2 = 0 \end{cases} \tag{52}$$

The boundary conditions for (52) at $\xi = 0$ are

$$\rho'(0) = 0, \qquad \rho(0) = \rho_0 \tag{53}$$

$$\omega(0) = \frac{\pi}{2}, \qquad \omega'(0) = -c \tag{54}$$

ρ_0 and c are constants to be determined The solution of (52) is

$$\rho = \rho_0(c^2\xi^2 + 1)^{1/2}, \qquad \omega = \frac{\pi}{2} - \mathrm{arctg}(c\xi) \tag{55}$$

It is easy to prove that

$$\rho' = \frac{\rho_0^2}{\rho}(c^2\xi^2 + 1)^{1/2}, \qquad \omega' = -\frac{c}{\rho^2}\rho_0^2 \tag{56}$$

$$u = c^2\rho_0^2, \qquad v = (1+\beta)c\rho_0^2 \tag{57}$$

then according to (7) and (38) we have

$$\begin{aligned} \tau &\sim \rho'^2 e_r e_r + \rho^2\omega'^2 e_\theta e_\theta - \rho\rho'\omega'(e_r e_\theta + e_\theta e_r) \\ &= c^2\rho_0^2[\sin^2\theta e_r e_r + \cos^2\theta e_\theta e_\theta + \sin\theta\cos\theta(e_r e_\theta + e_\theta e_r)] \end{aligned} \tag{58}$$

From Eq (58) we conclude that the expanding sector is in uniaxial tension state Now we analyze the parameter ρ_0 and c From the mechanical point of view, the solution can only contain one free parameter, so there must be a relation between ρ_0 and c We consider the ratios of the shrinkage in thickness direction and in R direction at $\Theta = 0$ Since $\tau_{rr} = \tau_{r\theta} = \tau_{zz} = 0$ at $\Theta = 0$, it is required that $\frac{\partial r}{\partial R} = \frac{\partial z}{\partial Z}$ at $\Theta = 0$, then according to (36), we have

$$(1+\beta)\rho_0 = \varsigma(0) \tag{59}$$

Since u, v are constants as given in (57), ς is also a constant determined by the algebraic equation (46), Eq (59) and (46) give

$$c = \sqrt{2}(1+\beta)^{\frac{-2n}{2(n-1)}}\rho_0^{\frac{2-4n}{2(n-1)}} \tag{60}$$

therefore ρ_0 becomes the unique parameter for the expanding sector

5. Matching conditions

The solution for the shrinking sector SH when $\Theta \to 0$ must match with the solution for the expanding sector EX when $\xi \to \infty$

We know that when $\zeta \to \infty$ Eq (55) has the asymptotic expression,

$$\rho = c\rho_0 \xi \qquad \omega = \frac{1}{c\xi} \tag{61}$$

From (36) and (62), we obtain

$$r = R^{1+\beta-\alpha} \rho_0 c\Theta \qquad \theta = \frac{\pi}{2} - \frac{R^\alpha}{c\Theta} \tag{62}$$

In the shrinking sector, when $\Theta \to 0$, we have

$$\varphi = C_\varphi \Theta \tag{63}$$

in which C_φ is an arbitrary coefficient that is permitted since (25) is homogeneous

Similarly, when $\Theta \to 0$, we have

$$\psi = C_\psi \frac{1}{\Theta} \tag{64}$$

where C_ψ is an arbitrary constant

Using (9), (63) and (64) we have

$$r = R^{1-\delta} C_\varphi \Theta, \qquad \theta = \frac{\pi}{2} - R^\gamma C_\psi \frac{1}{\Theta} \tag{65}$$

Comparing (62) - (65) we find that if

$$\delta = \alpha - \beta, \qquad \alpha = \gamma \tag{66}$$

$$C_\varphi = c\rho_0, \qquad C_\psi = \frac{1}{c} \tag{67}$$

then functions r, θ are matched at the boundary between sectors EX and SH

From (66) and (17) we also find

$$\beta = t = b \tag{68}$$

Eq (68) shows that for sectors EX and SH, the shrinking ratio in the thickness direction has the same order

Now, we check the stress singularities for both kinds of sectors From Eq (21) we know that the singularity of the dominant stress (τ_{rr}) in shrinking sector is

$$\tau \sim R^{3\delta-2\gamma-2n\delta} \tag{69}$$

In expanding sector, the singularity of the stress is

$$\tau \sim R^{\alpha-2\beta-b-2n(\alpha-\beta)} \tag{70}$$

Comparing the order of (69) with (70) we find that the stresses have the same order in sector EX and SH

6. Conclusions

- The mode I crack tip in a rubber like material obeying the elastic law of Gao (1997) under plane stress condition is composed of two shrinking sectors and one expanding sector When the crack tip is approached, the dominant stress possesses the singularity of order $R^{-\lambda}$, ($\lambda = 1 + \frac{2}{1+n} - \frac{3}{2n}$)
- The thickness shrinkage near the crack tip is tremendous After deformation the thickness becomes the order of $R^{\frac{n-1}{2n(n+1)}}$ both in expanding and shrinking sectors
- Although the elastic law used in this paper is different from that in Gao and Durban (1995), the crack tip is still in uniaxial tension state, but the equations for minor quantities are different
- Both elastic law, Gao (1990) and Gao (1997) are valid to analyze the crack tip in rubber sheet, but Gao (1997) is simpler than Gao (1990)

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