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Some Advances in Study of Crack Identification Problems

C.T. Liu and W.J. Chen

Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080 China

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ABSTRACT

In the present paper, the crack identification problems are investigated. This kind of problems belong to the scope of inverse problems and are usually ill-posed on their solutions. The paper includes two parts: (1) Based on the dynamic BIEM and the optimization method and using the measured dynamic information on outer boundary, the identification of crack in a finite domain is investigated and a method for choosing the high sensitive frequency region is proposed successfully to improve the precision. (2) Based on 3-D static BIEM and hypersingular integral equation theory, the penny crack identification in a finite body is reduced to an optimization problem. The investigation gives us some initial understanding on the 3-D inverse problems.

1. INTRODUCTION

In order to estimate the safety of a structure, we not only need to determine if there exist defects but also need to decide their exact geometry. The problems belong to the aim of nondestructive testing in industry and the inverse problems mechanics in theory. Due to their high value in practice, several kinds of theories and numerical methods have been proposed[1-3]. These methods can be generally divided into two kinds. The first kind is based on the analytic or half analytic theories, in which the BORN approximation[4,5] and inverse scattering approach[6] are most representative. In the second kind of methods, the representative works are belonged to Tanaka[7], Nishimura[8] and Chen[9], where the Chen's Pulse-Spectrum Technique(PST) has been applied with some success to identify the medium parameters[9]. But due to the high difficulties in application and theory, much attention has to be given in this kind of method. The second kind is indirect method, which is based on the solution of direct problem and use the iterative optimization approach to identify the unknowns. The indirect method need to cost long CPU time of computer, but following the development of computer techniques, it has been believed to be a very hopeful in future. It has been pointed out that the solution of inverse problem is usually ill-posed and high nonlinear. The solving of inverse problem is generally much more difficult than solving of direct problem and there is not a perfect approach at present. Since the precision of flaw detection depends on both the quality of testing equipment and the level of software, to obtain a valid identification result one has to make full use of the testing information and analysis them correctly. This means the most important duty at present is to make further deep investigation and study on the theory of inverse problems.

In this paper, two kinds of indirect methods based on the theory of elastodynamics and elastostatics, the defect identification problems in both 2-D and 3-D are investigated. In the first part, using the similar ideal of PST, the 2-D inverse problem is reduced to solving an optimization problem in Laplace transform space where the square sum of differences between the computed displacements and measured ones at selected points on outer boundary should be minimized. In the iterative process the new type boundary integral equation method[10] is used to solve the direct problems and is proved to be effective in reduction of the calculation error. In the choice of frequency spectrum, a method for choosing high sensitive frequency region is proposed. The results show that the method can fully and reasonably use the rich information from the wide frequency region of the transient elastodynamic wave, and obtain high precision identification result with less selected points and calculation. The method proposed is significance in guiding the development of nondestructive testing techniques. In the second part, the 3-D crack identification problem is reduced to solving an optimization problem, where the direct problem is solved by 3-D static BIEM and hypersingular integral equation theory. The result shows that the method is successful when the initial guess has not great error.

2. THE MIXED-TYPE BIEM FOR DIRECT PROBLEMS

Since the optimization method needs to make a lot of computation about direct problem in each step of iteration, one of the keys to make successful inversion of the defect geometry parameters is based on the accurate and fast numerical method for direct problem.

2.1. The Dynamic BEM for 2-Dimensional Crack Problem

The mixed-type boundary integral equations[10] are used to solve the direct problem. These equations are different from the regular boundary integral equations, where we don't need to cut along the crack and the element divides can be complete automatically in each step of iteration, and are helpful to reduce the errors in iterative computation. For the plane strain crack problem shown in Fig.1, if the informations of crack are known, the displacement $u_i\,(y,t)$ must satisfy the governing equations

$$\mu u_{i,kk} + (\lambda + \mu) u_{k,ki} = \rho \ddot{u}_i \tag{1}$$

and boundary and initial conditions

$$\begin{cases} \sigma_{ij} n_j = t_i(x,t), x \in S_F \\ u_i = \bar{u}(x,t), \quad x \in S_u \end{cases}, \quad t \geq 0 \tag{2}$$

$$\begin{cases} u_i(x,t)\big|_{t=0} = u_i^0(x) \\ \dot{u}_i(x,t)\big|_{t=0} = \dot{u}_i^0(x) \end{cases}, \quad x \in \Omega + S \tag{3}$$

where λ and μ are elastic moduli, ρ is density of material, Ω is the region surrounded by boundary $S+\Gamma^{\pm}$, n_i is the outward normal of boundary. Suppose the structure is static before transient loading are applied, then we have $u_i^0(x) = \dot{u}_i^0(x) = 0$. Apply the Laplace transform about time t to Eq.1

$$\mu \bar{u}_{i,kk} + (\lambda + \mu)\bar{u}_{k,ki} - \rho p^2 \bar{u}_i = 0 \tag{4}$$

where the bar denotes the Laplace transform, is the transform parameter. The solution in Ω can be expressed by Somigliana formulae

$$\begin{aligned} \bar{u}_k(y,p) = & \int_S [\bar{t}_i(\eta,p)\bar{U}_{ik}(\eta-y,p) - \bar{u}_i(\eta,y,p)\bar{T}_{ik}(\eta,y,p)]d\Gamma(\eta) \\ & - \int_a^b [\int_{\eta_1}^b \bar{T}_{ik}(\eta^*,y,p)d\eta^*]\Delta\bar{u}_{i,1}(\eta_1,p)d\eta_1, \qquad y \in \Omega \end{aligned} \tag{5}$$

where $\bar{U}_{ik}$ and $\bar{T}_{ik}$ are the Green basic solution, $\bar{T}_{ik}^{+} = \bar{T}_{ik}\big|_{n(0,-1)}$, $\bar{t}_i$ and $\bar{u}_i$ are the traction and displacement on the outer boundary S, $\Delta\bar{u}_{i,1}$ is the dislocation density function along crack.

To determine the unknowns on boundary, by use of the techniques about Green basic solution and singularity analysis, following mixed-type integral equations were derived by authors[10]

$$\begin{aligned} \frac{1}{2}\bar{u}_k(y,p) = & \int_S [\bar{t}_i(\eta,p)\bar{U}_{ik}(\eta-y,p) - \bar{u}_i(\eta,p)\bar{T}_{ik}(\eta,y,p)]dS(\eta) \\ & - \int_a^b [\int_{\eta_1}^b \bar{T}_{ik}^{+}(\eta^*,y,p)d\eta^*]\Delta\bar{u}_{i,1}(\eta_1,p)d\eta_1, \qquad k=1,2, \qquad y \in S \end{aligned} \tag{6a}$$

$$\begin{aligned} & \int_S [\bar{t}_i(\eta,p)\bar{T}_{ki}^{+}(\eta,y_1,p) + \bar{K}_{ik}(\eta,y_1,p)\bar{u}_i(\eta,p)]dS(\eta) + \frac{A}{\pi}\int_a^b \frac{\Delta\bar{u}_{k,1}(\eta_1,p)}{\eta_1 - y_1}d\eta_1 \\ & + \int_a^b \bar{M}_k(\eta_1,y_1,p)\Delta\bar{u}_{k,1}(\eta_1,p)d\eta_1 = \bar{q}_k(y_1,p), \qquad k=1,2, \qquad y_1 \in (a,b) \end{aligned} \tag{6b}$$

where, $\bar{u}_k = \bar{\sigma}_{k2}\big|_{\Gamma^{\pm}}$ is the loading along crack, $A = \mu/2(1-\nu)$, $\bar{K}_{ik}$ and $\bar{M}_k$ are the integrable kernel whose expressions can be found in Ref [10].

The above Eqs. 6a, 6b are different from the regular integral equations, in which the traction-type BIE Eq. 6a along the crack are Cauchy singular integral equations with dislocation density functions as unknowns. Eqs. 6a, 6b can be reduced to solving a set of linear equations by combining the numerical method of singular integral equation with the boundary element method. Since this method can be easily used and has very high precision, it is more suitable for the iterative calculation.

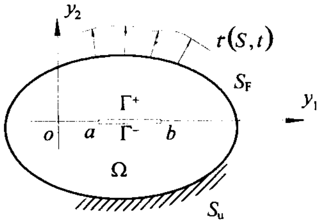

Fig. 1 The dynamic plane crack problem

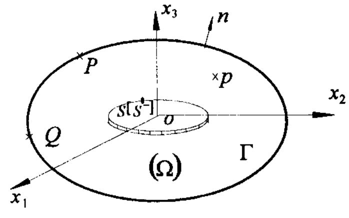

Fig. 2 Finite body with a plane crack

2.2 The Static BEM and Hypersingular Integral Equation for 3-Dimensional Crack Problem

Consider a planar crack $S(S^{\pm})$ of arbitrary shape inside of a finite body. Suppose that x_1 and x_2 are Cartesian coordinates in the crack plane and x_3 is normal to the crack. Using the Somigliana representation, the displacements at an internal point p can be expressed as follows[12]

$$u_k(p) = -\int_\Gamma T_{ki}(p,Q)u_i(Q)ds(Q) + \int_\Gamma U_{ki}(p,Q)t_i(Q)ds(Q) - \int_{S^+} T^+_{ki}(p,Q)\tilde{u}(Q)ds(Q) \qquad p\in\Omega, \quad k,i=1,2,3 \tag{7}$$

where u_i and t_i are displacement and traction boundary values respectively, $\tilde{u}_i = u_i^+ - u_i^-$ is the ith displacement discontinuity of the crack S, $T^+_{ki}(p,Q)$ is the value of $T_{ki}(p,Q)$ at point $Q \in S^+$, and $U_{ki}(p,Q)$ and $T_{ki}(p,Q)$ are Kelvin's point force solutions of 3-D elastostatics

$$U_{ki}(p,Q) = \frac{1+\nu}{8\pi E(1-\nu)r}[(3-4\nu)\delta_{ki} + r_{,k}r_{,i}] \tag{8}$$

$$T_{ki}(p,Q) = -\frac{1}{8\pi(1-\nu^2)r^2}\{\frac{\partial r}{\partial n}[(1-2\nu)\delta_{ki} + 3r_{,k}r_{,i}] - (1-2\nu)(r_{,k}n_i - r_{,i}n_k)\} \tag{9}$$

in which ν is Poisson's ratio, E is the elastic modulus, r is the distance from p to Q.

Using the Eq. 7 and the constitutive equations, the corresponding stresses at point p are obtained

$$\sigma_{ij}(p) = -\int_\Gamma S_{kij}(p,Q)u_k(Q)ds(Q) + \int_\Gamma D_{kij}(p,Q)t_k(Q)ds(Q) - \int_{S^+} S^+_{kij}(p,Q)\tilde{u}_k(Q)ds(Q) \qquad p\in\Omega, \quad k,i=1,2,3 \tag{10}$$

where $S^+_{kij}(p,Q)$ is the value of $S_{kij}(p,Q)$ at point $Q\in S^+$, and the integral kernels S_{kij} and D_{kij} are

$$S_{kij} = \frac{E}{8\pi(1-\nu^2)r^3}\{3\frac{\partial r}{\partial n}[(1-2\nu)r_{,k}\delta_{ij} + \nu(r_{,i}\delta_{jk} + r_{,j}\delta_{ik}) - 5r_{,k}r_{,i}r_{,j}] + 3\nu(r_{,i}n_j + r_{,j}n_i)r_{,k} + (1-2\nu)(3r_{,i}r_{,j}n_k + n_i\delta_{jk} + n_j\delta_{ik}) - (1-4\nu)n_k\delta_{ij}\} \tag{11}$$

$$D_{kij} = \frac{1}{8\pi(1-\nu^2)r^2}[(1-2\nu)(r_{,j}\delta_{jk} + r_{,i}\delta_{ik} - r_{,k}\delta_{ij}) + 3r_{,k}r_{,i}r_{,j}] \tag{12}$$

The integral equations for the crack problems can be written as [12,13]

$$\frac{E}{8\pi(1-\nu^2)}\int_{S^+}\frac{1}{r^3}[(1-2\nu)\delta_{\alpha\beta} + 3\nu r_{,\alpha}r_{\beta}]\tilde{u}_\beta d\xi_1 d\xi_2 - \int_\Gamma S_{k\alpha 3}(p,Q)u_k(Q)ds(Q) = -\int_\Gamma D_{k\alpha 3}(p,Q)t_k(Q)ds(Q), \qquad P\in S^+, \quad \alpha,\beta=1,2 \tag{13}$$

$$\frac{E}{8\pi(1-\nu^2)}\int_{S^+}\frac{1}{r^3}\tilde{u}_3 d\xi_1 d\xi_2 - \int_\Gamma S_{k33}(p,Q)u_k(Q)ds(Q) = -\int_\Gamma D_{k33}(p,Q)t_k(Q)ds(Q) \quad P\in S^+ \tag{14}$$

$$c_{ki}u_k(P) + \int_{\Gamma} T_{ki}(P,Q)ds(Q) + \int_{S^+} T^+_{ki}(P,Q)\tilde{u}_i(Q)ds(Q) = \int_{\Gamma} U_{ki}(P,Q)t_i(Q)ds(Q) \quad P \in \Gamma \tag{15}$$

where c_{ki} is the constant related to the boundary point P. Simultaneously solving these equations, we can obtain all the displacement discontinuities $\tilde{u}_i$ (i=1,2,3) and calculated stress intensity factors.

3. THE ITERATIVE OPTIMIZATION METHOD FOR CRACK IDENTIFICATION

For the problem shown in Fig.1, if the crack location and geometry are unknown and need to identify, then it is a fracture inverse problem. In the present paper the problem of straight line crack identification is considered. Because of the highly nonlinear and ill-posed on the solution it is impossible to solve the inverse problem directly from Eqs. 6a, 6b. In order to overcome the difficulties the indirect method is used, where we suppose that the displacements at some selected boundary points are measured to be the identification information. These selected points are usually called over-prescribed boundary points whose displacement and traction conditions are known at the same time. By use of the numerical method of direction problem and iterative optimization. The crack identification is reduced to minimizing the square sum of differences between the computed displacements and measured ones at selected points on outer boundary.

For 2-D problem, only 4 parameters $\vec{\xi} = (a_0, \theta, x_0, y_0)$ are needed to determine the crack, in which a_0 is the half length of crack, θ is the angle between crack and horizontal direction, (x_0, y_0) is the central point of crack. To identify these parameters, we choose the following objective function in iterative optimization

$$W(\vec{\xi}) = \sum_{k=1}^{P}\sum_{l=1}^{N}\sum_{i=1}^{2}[\bar{u}_i(X_l, p_k) - \bar{u}_i^*(X_l, p_k)]^2 \tag{16}$$

where P is the number of selected frequency points and p_k is the value of frequency. N is the number of selected measurement points. $\bar{u}_i(X_l, p_k)$ is the displacement at point X_l by calculation and $\bar{u}_i^*(X_l, p_k)$ is the displacement at point X_l by measurement.

Now the inverse problem is reduced to determining the best parameters $\vec{\xi} = (a_0, \theta, x_0, y_0)$ by minimizing the object function W ($W_{\min} = 0$). In order to choose a suitable convergence criterion of iterative computations, the non-dimensional objective function is defined as

$$Z = W \Big/ \sum_{k=1}^{P}\sum_{l=1}^{N}\sum_{i=1}^{2}[\bar{u}_i^*(X_l, p_k)]^2 \tag{17}$$

and the parameters $\vec{\xi} = (a_0, \theta, x_0, y_0)$ are iterated by following manner

$$\vec{\xi}^{(n+1)} = \vec{\xi}^{(n)} + l\vec{d}^{(n)} \tag{18}$$

where l is optimal step length where a method of multi-constant step is used, in which the scope of residual value is divided into $\varepsilon_1 > \varepsilon_2 > ... > \varepsilon_m > \varepsilon^*$ and the correspondence steps are chosen as $l_1 > l_2 > ... > l_m$. If $\varepsilon_j < Z < \varepsilon_{j+1}$, then $l = l_j$. $\vec{d}^{(n)}$ is the vector of search direction at the nth iteration and can be determined as follows [11]

$$\vec{d} = -\frac{gradW}{|gradW|}, \qquad gradW = (\frac{\partial W}{\partial a_0}, \frac{\partial W}{\partial \theta}, \frac{\partial W}{\partial x_0}, \frac{\partial W}{\partial y_0}) \tag{19}$$

and the convergence criterion used in this paper is[7]

$$|Z^{(n)}| < \varepsilon^*, \quad |Z^{(n)} - Z^{(n-1)}| < \varepsilon^{**} \tag{20}$$

where ε^* and ε^{**} are small positive real number.

The optimization method for 3-D is similar as above process and will not listed again.

4. SOME EXAMPLES

The identification precision is mainly related with measurement point X_l and frequency value

p_k. Although large number of X_l and p_k will offer more information, good identification precision does not always achieved and much computation has to be made. Thus, how to use the finite information reasonably is another key for successful identification. In this paper, the frequency point p_k are chosen only in the most sensitive frequency region of real half-axis $(0,\infty)$. The method proposed is based on following consideration: (1) It can use the information reasonably and effectively with less computation cost. (2) the computation is only in real space and the complex calculation is avoided. Many examples show this choosing method is successful and the expected aim is achieved.

4.1. Frequency choosing and two dimensional identification example

The materials constants are $\mu = 8\times10^{10} Pa$, $\nu = 0.29$, $\rho = 7800 Kg/m^3$. The structure is a rectangle with geometry of 2.0m × 1.0m and the length and location of the crack are unknown. The outer boundary is divided into 30 elements. The measurement points are selected at the central points of No7 and No14 elements and the impact loading are placed on the central part of the up side.

Example 1. The rectangle with a central horizontal crack of 0.1m half-length

The real frequency spectrums at No7 element are shown in Fig.3a, where the most sensitive frequency region is obviously between $0.25 < p/c_2 < 4.0$. The frequency points are selected as $p_k = kc_2/2, k = 1,2,3,4,5$. The spectrums obtained by identification are shown in Fig.3b which is very identical with Fig.3a. The identification process is given by Fig.4 and Table 1.

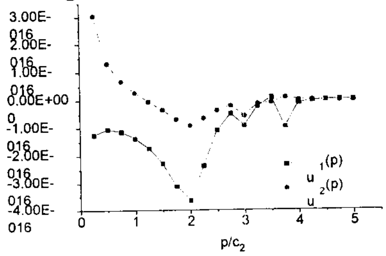

Fig.3a

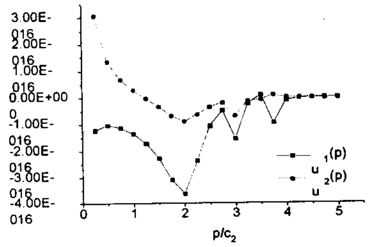

Fig.3b

Table 1. The numerical results of crack identification

n	$a_0^{(n)}$	$\theta^{(n)}$	$x_0^{(n)}$	$y_0^{(n)}$	$Z^{(n)}$
0	0.2000000	0.000000	0.500000	0.300000	5.068914%
4	0.108132	-0.068823	0.942502	0.328535	0.121692%
8	0.105963	-0.040041	0.969309	0.367314	0.061392%
12	0.104587	-0.033909	0.974467	0.414461	0.026143%
16	0.102758	-0.023796	0.979136	0.469794	0.008426%
FINAL	0.100869	-0.020680	0.982905	0.483193	0.004219%
REAL	0.100000	0.000000	1.000000	0.500000	/

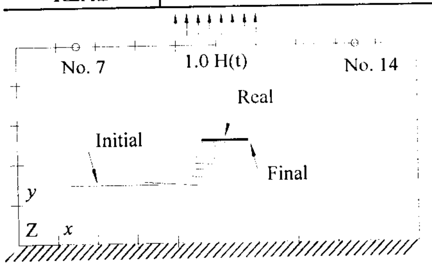

Fig. 4

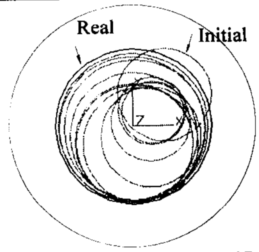

Fig. 5 The top wiew on the identification process of the central penny crack

4.2. Three Dimensional Identification Example

We take the identification of a central penny crack embedded in a circular cylinder as Fig. 5. Here the height is 2.0m, the radius of the cylinder is 1.0m and the radius of the penny crack is 0.5m.

Example 2. Suppose we do not know any parameters of the crack beforehand. Then 6 parameters are needed to be identified. They are: the angle α between crack and x coordinate; the angle β between crack and y coordinate; the penny crack radius R; the central point coordinates (x_0, y_0, z_0) of the crack. The identification results are listed in Table 2

Table2. The results on identification of a central penny crack

n	$\alpha^{(n)}$	$\beta^{(n)}$	$R^{(n)}$	$x_0^{(n)}$	$y_0^{(n)}$	$z_0^{(n)}$	$z^{(n)}$
0	0.000	0.000	0.600	0.100	0.100	0.100	101.997 %
4	6.990E-4	-8.592E-4	0.586	9.232E-2	8.357E-2	0.100	63.984 %
8	-1.757E-3	-2.348E-3	0.568	8.240E-2	5.804E-2	9.975E-2	30.716 %
12	-6.798E-3	-4.408E-3	0.553	7.255E-2	9.968E-3	9.872E-2	7.593 %
16	-1.466E-2	7.42E-3	0.547	3.612E-3	-3.589E-2	9.407E-2	0.685 %
20	-2.386E-3	4.294E-3	0.518	6.409E-3	-2.354E-2	8.296E-2	6.984E-2%
FINAL	-4.516E-3	2.26E-3	0.500	-3.195E-3	-3.212E-2	8.200E-2	3.695E-2%
REAL	0.000	0.000	0.500	0.000	0.000	0.000	/

5. CONCLUSION

In the present paper a crack identification method, based on the BIEM for direction problem and iterative optimization technique and method of high sensitive frequency region choosing, is proposed for both 2-D and 3-D inverse problems. The results of many number examples show that the method proposed is successful in using measurement information sufficiently and obtaining good identification effect with less computation. Its advantage includes: 1. The direct calculation reduces the errors effectively; 2. The wide frequency region is fully used and only few measurement points are needed; 3. The most sensitive information is used to reduce the computation cost and obtain good identification results by the method of high sensitive frequency region choosing.

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