Nonlinear Dynamic Responses of the Tensioned Tether under Parametric Excitations

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ABSTRACT

The nonlinear dynamic responses of the tensioned tether subjected to combined surge and heave motions of floating platform are investigated using 2-D nonlinear beam model. It is shown that if the transverse-axial coupling of nonlinear beam model and the combined surge-heave motions of platform are considered, the governing equation is not Mathieu equation any more, it becomes nonlinear Hill equation. The Hill stability chart is obtained by using the Hill's infinite determinant and harmonic balance method. A parameter M, which is the function of tether length, the surge and heave amplitude of platform, is defined. The Hill stability chart is obviously different from Mathieu stability chart which is the specific case as $M=0$. Some case studies are performed by employing linear and nonlinear beam model respectively. It can be found that the results differences between nonlinear and linear model are apparent.

KEY WORDS: Tensioned tether; nonlinear dynamic responses; Hill equation; harmonic balance method

INTRODUCTION

Tensioned tethers are an important type of components connecting hull of floating platform and foundations, whose dynamic responses are an important problem.

Tethers at low tensions were studied by Patel and Park (1991), the Mathieu stability charts were derived at large parameters. Patel and Park (1995) investigated the combined axial and transverse response of tethers of a tensioned buoyant platform using a semi-analytical method; the tether is modeled as a simply supported Euler-Bernoulli beam under the action of combined axial and lateral forces. Yigit and Christoforou (1996) studied the coupled vibration of the oil well drillstrings, the equations of motion were solved using the assumed mode method. Han and Benaroya (2000a,b) studied the coupled axial and transverse vibrations of a compliant tower, derived nonlinear coupled equations of motion and obtained the free and forced responses using the finite difference approach. Chatjigeorgiou and Mavrakos (2002, 2005), Chatjigeorgiou (2004) studied the transverse vibration of the cable excited by platform heave motion, the nonlinear coupling between different modes was considered. Park and Jung (2002) reported that the

parametric excitation alters the response pattern of a long slender marine structure, a linear beam model was taken and a finite element method was implemented in the time domain. Chandrasekaran, Chandak and Anupam (2006) presented the dynamic analysis of tethers considering the linearly varying tension along the tether length.

It can be found that many researches on the dynamic response of tensioned tether are performed by using linear beam model, in which the nonlinear coupling between the axial and transverse motion is not considered. Some other scholars considered the parametric excitation induced by the heave of platform employing nonlinear model. Nevertheless, it seems that there is hardly any study on the responses of tether subjected to prescribed combined platform surge and heave motions using nonlinear beam model. As shown later, if the nonlinear transverse-axial coupling and the surge-heave motions are considered, the governing equation is not nonlinear Mathieu equation any more, it becomes nonlinear Hill equation. The first aim of the paper is to obtain the Hill stability chart up to the large parameters. The second aim is to investigate dynamic responses of tether and compare the obtained results by using linear and nonlinear beam model respectively.

In this paper the nonlinear coupled motion of tether is examined in 2D space, assuming that the end connections of tether are hinged. The equations are reduced to nonlinear Hill equation by applying the Galerkin's method and the modes superposition principle. The stability of the Hill equation without damping is investigated based on the infinite determinant and harmonic balance method. Some example studies are performed for various cases, a fourth-order Runge-Kutta method is employed to obtain the time history of tether responses.

NONLINEAR COUPLED EQUATIONS OF TETHER

The tether oscillates in both x and y directions in the vertical plane. Then the 2D nonlinear beam equations of motions for the tether are (Han and Benaroya, 2000a, b)

$$\rho A \ddot{u} - \left(EA\left(u' + \frac{1}{2}v'^2 \right) \right)' = f_x \tag{1}$$

$$\rho A \ddot{v} - \left(EA\left(u' + \frac{1}{2}v'^2 \right)v' \right)' + \left(EIv'' \right)'' = f_y \tag{2}$$

The prime and dot denote the derivatives with respect to x and t respectively. ρ is the density of the tether, A is the cross-sectional area, E is the modulus of elasticity, I is the moment of inertia, u and v are the