Advanced Materials Research Vols. 41-42 (2008) pp 291-296
Online available since 2008/Apr/08 at www.scientific.net

doi:10.4028/www.scientific.net/AMR.41-42.291

Failure Process Analysis of Cement Under Explosive Loading with a Generalized Driven Nonlinear Threshold Model

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Keywords: cement, microstructual heterogeneity, stress fluctuation, generalized driven nonlinear threshold model, explosive loading

Abstract The microstructural heterogeneity and stress fluctuation play important roles in the failure process of brittle materials. In this paper, a generalized driven nonlinear threshold model with stress fluctuation is presented to study the effects of microstructural heterogeneity on continuum damage evolution. As an illustration, the failure process of cement material under explosive loading is analyzed using the model. The result agrees well with the experimental one, which proves the efficiency of the model.

Introduction

Brittle materials, like ceramics, rocks, and cement are widely used in armor engineering. Hence, it is important to understand the mechanical properties of these materials under strong, high rate impact loading. The response of brittle materials to impact loading has been extensively investigated experimentally [1-3]. The experiments showed that there existed many intrinsic defects in the materials, which lead to the stochastic strength properties of the bulk material. In addition, under dynamics loading, these defects may nucleate to form many micro-cracks, which result in the fragmentation of the material. Moreover, the high-speed crack growth process is always accompanied by the inertia effect and rate-dependent fracture process.

The complexity of the failure process of brittle materials under dynamic loading makes its computational simulation an elusive goal. Firstly, since the microscopic defects significantly influence the strength and failure mode of the material, it is necessary to include at least two length scales (mesoscopic scale and macroscopic scale) in the computational model. Secondly, since the intrinsic defects distribute heterogeneously in the material, we also need to describe the heterogeneity of the material properties and its effects on true stress distribution in the material. Thirdly, since the damage evolution is always coupled with the inertia effect and stress wave propagation, a coupled analysis should be employed in the simulation.

Motivated by these facts, in this paper, a generalized driven nonlinear threshold model with stress fluctuation is presented to study the effects of microstructural heterogeneity on continuum damage evolution. As an example, the failure process of cement material under explosive loading is analyzed with the model. The result agrees well with the experimental observation, which proves the efficiency of the model.

Model

We consider a macroscopic representative volume element (RVE) (at $\mathbf{x}$) comprised of a great number of interacting, nonlinear, mesoscopic units, that is, a driven nonlinear threshold model [4]. The heterogeneity of the mesoscopic units can be characterized by their broken threshold. The mesoscopic units are assumed to be statistically identical, and their broken threshold σ_{c} follows a statistical distribution function $\varphi(\sigma_{\mathrm{c}},t,\mathbf{x})$.

The RVE is subjected to nominal driving force $\sigma_0(t,\mathbf{x})$, which is adopted as macroscopic variable. In the RVE, a mesoscopic unit will have probability to break as the real driving force σ (true stress) on it becomes higher than its threshold. When a unit breaks, it will be excluded from the distribution function. Hence, we introduce a time-dependent distribution function of intact units $\varphi(\sigma_{\mathrm{c}},t,\mathbf{x})$ with initial condition

$$\varphi(\sigma_{\mathrm{c}},t=0,\mathbf{x})=h(\sigma_{\mathrm{c}}), \tag{1}$$

where $h(\sigma_{\mathrm{c}})$ is normalized as

$$\int_0^\infty h(\sigma_{\mathrm{c}})\,d\sigma_{\mathrm{c}}=1. \tag{2}$$

In Eq.(2), σ_{c} is non-dimensionalized and normalized by a parameter σ^* , the characteristic value of σ_{c}.

With function $\varphi(\sigma_{\mathrm{c}},t,\mathbf{x})$, the continuum damage of the RVE at time t can be defined by

$$D(t,\mathbf{x})=1-\int_0^\infty \varphi(\sigma_{\mathrm{c}},t,\mathbf{x})\mathrm{d}\sigma_{\mathrm{c}}. \tag{3}$$

Due to the heterogeneity, the true stress applied on the intact units in RVE varies. We assume that the true stress follows a statistical distribution function $\xi(\sigma;t,\mathbf{x})$. Hence, the probability that the real driving force (true stress) applied on intact units is σ can be denoted by $\xi(\sigma;t,\mathbf{x})$. Roughly speaking, function $\xi(\sigma;t,\mathbf{x})$ is determined by the nominal stress $\sigma_0(t,\mathbf{x})$ and continuum damage $D(t,\mathbf{x})$, that is,

$$\xi(\sigma;t,\mathbf{x})=\xi(\sigma;\sigma_0(t,\mathbf{x}),D(t,\mathbf{x})). \tag{4}$$

In addition, function $\xi(\sigma;t,\mathbf{x})$ should be normalized as

$$\int_0^\infty \xi(\sigma;t,\mathbf{x})\,d\sigma = 1 , \tag{5}$$

and the mean value of driving force on intact units follows

$$\int_0^\infty \sigma\xi(\sigma;t,\mathbf{x})\,d\sigma = \frac{\sigma_0(t,\mathbf{x})}{1-D(t,\mathbf{x})} \tag{6}$$

By assuming statistical independency between broken threshold σ_c and true stress σ, the evolution of distribution function $\varphi(\sigma_c;t,\mathbf{x})$ is suggested to follow an equation based on relaxation time model:

$$\frac{\partial\varphi(\sigma_c;t,\mathbf{x})}{\partial t} = -\int_0^\infty \frac{\varphi(\sigma_c;t,\mathbf{x})}{\tau(\sigma,\sigma_c)}\xi(\sigma;t,\mathbf{x})\,d\sigma , \tag{7}$$

where τ is the characteristic relaxation time of damage. In general, τ is determined by the true driving force and the threshold of mesoscopic units, $\tau = \tau(\sigma,\sigma_c)$.

Integrating Eq.(7) and substituting the definition of continuum damage (Eq.(3)) to the obtained equation, we obtain the evolution equation of continuum damage:

$$\frac{\mathrm{d}D(t,\mathbf{x})}{\mathrm{d}t} = f = -\int_0^\infty \frac{\partial\varphi(\sigma_c,t,\mathbf{x})}{\partial t}\mathrm{d}\sigma_c = \int_0^\infty\int_0^\infty \frac{\varphi(\sigma_c,t,\mathbf{x})}{\tau(\sigma,\sigma_c)}\xi(\sigma;t,\mathbf{x})\mathrm{d}\sigma_c\mathrm{d}\sigma \tag{8}$$

Where f is the dynamic function of damage (DFD), the agent linking mesoscopic microdamage relaxation and macroscopic damage evolution.

Similar to [5], in order to establish a closed, complete formulation, Eq.(8) should be associated with traditional, macroscopic equations of continuum, momentum, and energy, constitutive relationship and Eq.(4). This is a formulation with intrinsic trans-scale closure. Eq.(4) and Eq.(8) relate the macroscopic and mesoscopic scales.

With the abovementioned formulation, we numerically investigated the failure process of cement material under explosive loading and compared the results with experimental ones.

Experiment

We made thick-walled, cylindrical specimens with cement. After curing for 7 days, the specimens are used to test the fracture behavior of the material. In the experiment, the gas with high-pressure is imported to the inner surface of the cylinder, which cause the specimens to break. The pressure on the inner surface is recorded during the experiments. The details of the experiments are described in [6]. Fig.1 shows a typical pressure curve obtained in the experiments. Fig.2 is the photo of the specimen after test shown in Fig.1. The experiments show that the breaking of the specimen was caused by tensile stress. In addition, the counts of cracks in the specimen vary with the rate, amplitude and duration of stress wave. In general, the higher rate and amplitude, the longer duration result in more cracks in the specimen.

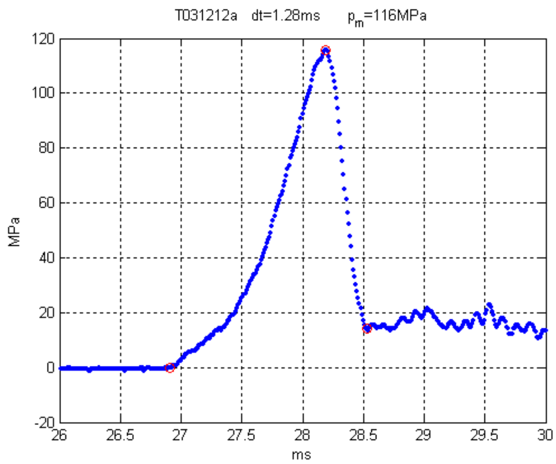


Fig.1 A typical pressure vs. time curve

Fig.2 The broken specimen after experiment

Simulation

Similar to Weibull's statistical strength theory [7], the initial distribution of threshold $h(\sigma_c)$ of the material is expressed as:

$$h(\sigma_c) = m\frac{(\sigma_c - \sigma^*)^{m-1}}{(\sigma^*)^m}\exp\left[-\left(\frac{\sigma_c - \sigma^*}{\sigma^*}\right)^m\right], \tag{9}$$

where m is the Weibull modulus and σ^* the characteristic value of σ_c. The smaller Weibull modulus (m), the broader the distribution becomes, and the material is more heterogeneous. On the other hand, larger m value represents a homogeneous material in which the stress threshold is almost constant.

There are various ways to determine the characteristic relaxation time of damage τ [5] . For simplicity, we may assume that if $\sigma \geq \sigma_c$, the damage relaxation time is a fixed value τ_D. That is,

$$\tau(\sigma, \sigma_c) = \begin{cases} \infty & \text{if } \sigma > \sigma_c \\ \tau_D & \text{otherwise} \end{cases}, \tag{10}$$

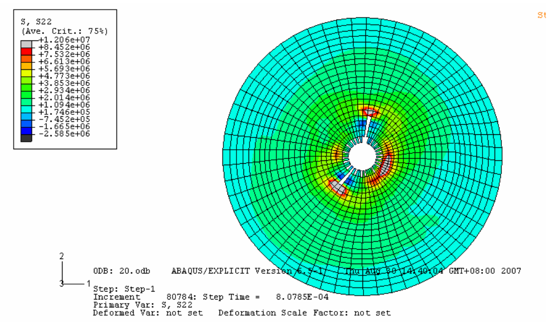
The fluctuation of true stress exerted on intact mesoscopic units can be dealt with using different approaches. We assume the statistical distribution function of true stress $\xi(\sigma, t, \mathbf{x})$ as follows:

$$\xi(\sigma,T,X)=\begin{cases}\dfrac{2F_0}{\sigma_2-\sigma_1}(\sigma-\sigma_1), & \text{as } \sigma_1<\sigma\le\dfrac{\sigma_1+\sigma_2}{2}\\ \dfrac{2F_0}{\sigma_2-\sigma_1}(\sigma_2-\sigma), & \text{as } \dfrac{\sigma_1+\sigma_2}{2}<\sigma<\sigma_2\,,\\ 0, & \text{otherwise.}\end{cases} \tag{11}$$

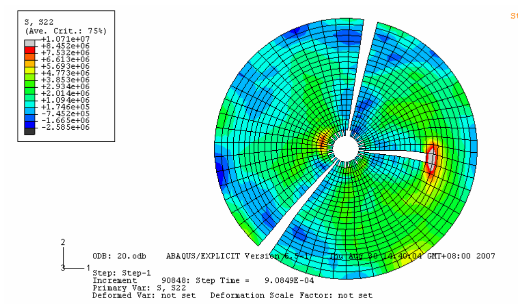
where $\sigma_1=\dfrac{1-\sqrt{6kD}}{1-D}\sigma_0$, $\sigma_2=\dfrac{1+\sqrt{6kD}}{1-D}\sigma_0$, $F_0=\dfrac{1-D}{\sigma_0\sqrt{6kD}}$, and $k=\overline{(\sigma-\bar{\sigma})^2}\Big/\left(D\left(\dfrac{\sigma_0}{1-D}\right)^2\right)$ is the stress fluctuation parameter.

We use ABAQUS/Explicit to simulate the breaking process of the specimen. The abovementioned driven nonlinear threshold model is included in the analysis with User Subroutine. In the simulation, we assume that there exists initial damage in the sample by seeding random damage in elements, and that the element fails when the damage of the elements reach a criteria D_{cr}. In the simulation, the parameters are as follows: $m=10, \eta=2.3*10^6\,\text{Pa}, \tau_D=4*10^{-6}s, k=0.3$ and $D_{cr}=0.1$.

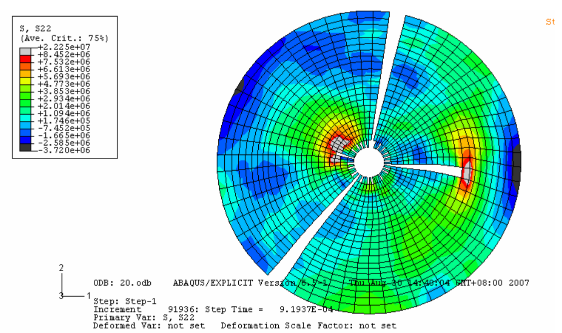
It is noticeable that, the characterized damage evolution time τ_D is about several microseconds, which is comparable with the characterized stress wave propagation time. Therefore, it is necessary to simultaneously couple the analysis of stress wave propagation and damage evolution.



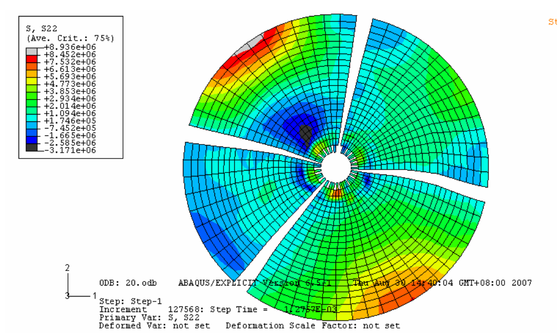
(a). t=8.079×10^{-4} s



(b).t=9.085×10^{-4} s



(c). t=9.194×10^{-4} s



(d).t=1.276×10^{-3} s

Fig.3 The evolution of crack pattern in the specimen

Fig.3 shows the evolution of crack pattern that we obtained. Obviously, the model and simulation can reproduce the breaking process of the specimen. The figures also show that, the characterization time for cracks to go through the specimen is about $10^{-3} s$, which is comparable with the duration of stress wave.

Summary

In this paper, a generalized driven nonlinear threshold model is presented to study the effects of microstructural heterogeneity and stress fluctuation on continuum damage evolution. The breaking process of cement under explosive loading is analyzed with this model. The results show that the model can reproduce the damage pattern evolution observed in experiments, which proves the efficiency of the model.
The characterized damage evolution time is comparable with the characterized stress wave propagation time, which necessitates the coupled analysis of stress wave propagation and damage evolution.

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Structural Integrity and Failure, 2008
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doi:10.1016/j.actamat.2003.09.036