A NEW METHOD TO ANALYSE THE PEIERLS-NABARRO MODEL OF AN EDGE DISLOCATION IN A RECTANGULAR PLATE

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ABSTRACT

A new method is presented here to analyse the Peierls-Nabarro model of an edge dislocation in a rectangular plate. The analysis is based on the superposition scheme and series expansions of complex potentials. The stress field and dislocation density field on the slip plane can be expressed as the first and the second Chebyshev polynomial series respectively. Two sets of governing equations are obtained on the slip plane and outer boundary of the rectangular plate respectively. Three numerical methods are used to solve the governing equations.

Key Words: Peierls-Nabarro model; edge dislocation; finite body

INTRODUCTION

The original concept of the Peierls-Nabarro(P-N) model [1-2] is based on the balance of stress in the slip plane. The solution of the Peierls-Nabarro's integrodifferential equation determines the core structure of an edge dislocation. But the original solution of P-N model can not directly apply to the problem of an edge dislocation in a finite rectangular plate. A general method to analyse an edge dislocation with core structure of P-N model in a rectangular plate is presented in this paper. The basic procedure is developed by Wang and co-workers [3-4]. Based on the superposition scheme and series expansions of complex potentials, the stress field and dislocation density field on the slip plane can be expressed as the first and the second Chebyshev polynomial series respectively. In order to meet the boundary condition on the outer boundary, two analytical complex functions are introduced. Two sets of governing equations are obtained on the slip plane and the outer boundary of the rectangular respectively. Three numerical methods including the boundary collocation, least square and New-Raphson method are used to solve the governing equation. In order to compare with the original results of P-N model, the case of an edge dislocation in an infinite body is also analysed.

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Figure 1. Rectangular plate with an edge dislocation.

PROCEDURES, RESULTS AND DISCUSSION

Suppose an edge dislocation in a simple cubic crystal. τ_{xy} and δ_x denote the shear stress and the relative displacement of two atomic plane adjacent to the slip plane, respectively. According to Peierls [1] and Nabarro [2], assume that the relation between τ_{xy} and δ_x is the Frenkel sinusoidal function. We have

$$\tau_{xy} = \tau_{max} \sin\left(2\pi \frac{\delta_x}{b}\right) \qquad (1)$$

Consider a rectangular plate with an edge dislocation that is subjected no traction on the outer boundary, as shown in Figure 1. Here $\Phi(z)$ and $\Omega(z)$ are used as the two complex potentials. Stress can be derived from [5]

$$\sigma_x + \sigma_y = 2\left[\Phi(z) + \overline{\Phi(z)}\right]$$
$$\sigma_y - i\tau_{xy} = \Phi(z) + \Omega(\bar{z}) + (z - \bar{z})\overline{\Phi'(z)} \qquad (2)$$

For a slip plane lying on the real axis between $s = -L$ and $s = L$, the complex potentials $\Phi(z)$ and $\Omega(z)$ are given by the following formula:

$$\Phi(z) = \Omega(z) = \frac{\mu}{\pi i(\kappa + 1)} \int_{-L}^{L} \frac{b(s)}{z - s} ds \qquad (3)$$

here $b(s)$ is the dislocation density at point $z = s$ on the slip plane. For the plane strain problem $\kappa = 3 - 4\nu$, and plain stress problem $\kappa = (3 - \nu)/(1 + \nu)$. μ is the shear modulus, ν is Poisson's ratio.

The dislocation density can be expressed as the following series [4]:

$$\frac{\mu}{i(\kappa + 1)} b(s) = -\frac{\mu}{i(\kappa + 1)} \frac{d\delta(s)}{ds} = \sum_{m=0}^{\infty} \alpha_m \frac{T_m(\xi)}{\sqrt{1 - \xi^2}} \qquad (4)$$

Then substituting Eq.(4) into Eq.(3), we obtained

$$\Phi(z) = \Omega(z) = \sum_{m=0}^{\infty} \alpha_m \left(\frac{z}{L} - \sqrt{\left(\frac{z}{L}\right)^2 - 1} \right)^m \Bigg/ \sqrt{\left(\frac{z}{L}\right)^2 - 1} \tag{5}$$

Substitution Eq.(5) into Eq.(2), the stress field at any point due to the dislocation can be expressed as a series. Especially, the stress field on the slip plane can be expressed as

$$\sigma_y - i\tau_{xy} = -2\sum_{m=0}^{\infty} \alpha_m U_{m-1}(s/L) \tag{6}$$

In order to meet the traction free condition on the outer boundary, two additional complex potentials $\Phi_0(z)$ and $\Omega_0(z)$ are introduced here, we set

$$\Phi_0 = \sum_{n=1}^{\infty} n\beta_n z^{n-1}, \qquad \Omega_0 = \sum_{n=1}^{\infty} n\gamma_n z^{n-1} \tag{7}$$

According to the superposition scheme, the stress on the slip plane can be written as follows

$$\sigma_y - i\tau_{xy} = -2\sum_{m=0}^{\infty} \alpha_m U_{m-1}(s/L) + \sum_{n=1}^{\infty} n(\beta_n + \gamma_n) s^{n-1} \tag{8}$$

The traction free condition on the outer boundary can be written in terms of resultant forces from A to A^*, as shown in Figure 1. Point A is assumed to be fixed at all time, point A^* is permitted to move along the outer boundary. The boundary condition can be expressed as:

$$X(z) + iY(z) = -i\left[\phi_0(z) + \omega_0(\bar{z}) + (z - \bar{z})\overline{\Phi_0(z)} \right]_A^{A^*}$$
$$-i\left[\phi(z) + \omega(\bar{z}) + (z - \bar{z})\overline{\Phi(z)} \right]_A^{A^*} = 0 \tag{9}$$

From Eq.(4), we obtain

$$\delta(s) = -\int_0^s \frac{i(\kappa+1)}{\mu} \sum_{m=0}^{\infty} \alpha_m \frac{T_m(s/L)}{\sqrt{1 - (s/L)^2}} ds$$
$$= \frac{b}{2} + \frac{iL(\kappa+1)}{\mu} \left[\sum_{m=2,4}^{\infty} \frac{\alpha_m}{m} \sin m\theta + \alpha_0 \left(\theta - \frac{\pi}{2} \right) \right] \tag{10}$$

Equation (1) and Equations (8)-(10) are the governing equations for determining the unknown coefficients $\{\alpha_m\}$, $\{\beta_m\}$ and $\{\gamma_m\}$.

It is difficult to solve the governing equations analytically. The governing equations can be reduced to a set of nonlinear algebraic equations on the slip plane and a set of linear algebraic equations along the outer boundary of the rectangular plate. The boundary collocation method and the least square method are used to solve the linear algebraic equations while the Newton-Raphson method is used to solve the nonlinear algebraic equations. The collocation points on the outer boundary are taken as shown in Figure 1. The collocation points on the slip plane are given by the following expression

$$x_i = L\cos\left(\frac{i\pi}{M+1}\right), \qquad i = 1, 2, ..., M$$

In this paper, the plain strain condition is considered and calculation is carried out with materials parameter $\nu = 0.3$, $\tau_{max}/\mu = 0.159$. Figure 2a and Figure 2b show the shear stress and slip displacement for the infinite body. The comparison of the present results with the original results of [1-2] is also shown in Figure 2a and Figure 2b. As shown in Figure 2, the present results agree completely with the original results given by Peierls [1] and Nabarro [2]. For rectangular plate, we take $H/L = 1$, the case of $L/b = 5$, $L/b = 8$, $L/b = 10$ and $L/b = 20$ are calculated and the results are shown in Figure 3a and Figure 3b. The calculation results show that the present method is a direct and efficient approach to analyse the edge dislocation in the finite body.
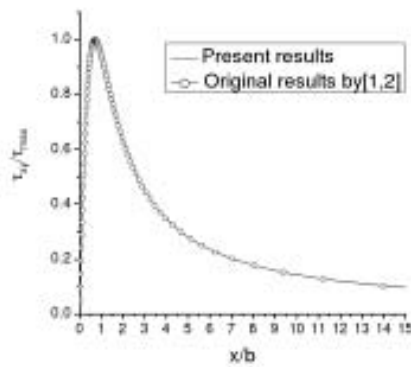

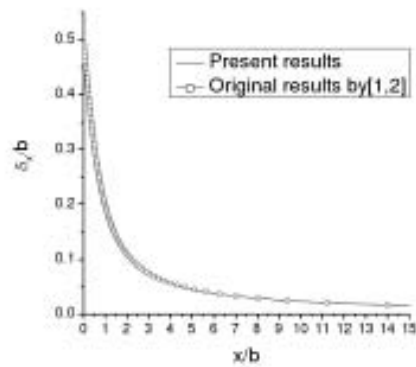Figure 2a. Shear stresses on the slip plane.
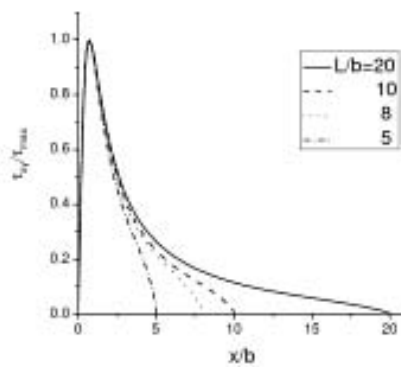
Figure 2b. Shear displacements on the slip plane.

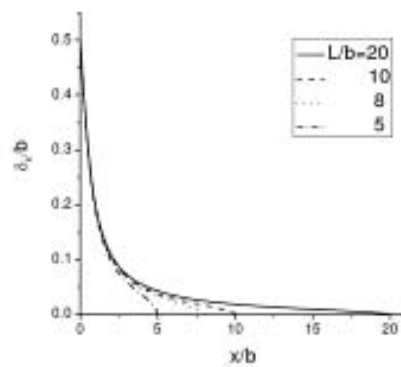

Figure 3a. Shear stresses of rectangular plate.

Figure 3b. Shear displacements of rectangular plate.

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