Transp Porous Med (2008) 75:401–412
DOI 10.1007/s11242-008-9227-7

Designing of Partial Similarity Models and Evaluation Method in Polymer Flooding Experiment

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Received: 2 January 2008 / Accepted: 26 March 2008 / Published online: 9 April 2008

Abstract Based on the scaling criteria of polymer flooding reservoir obtained in our previous work in which the gravity and capillary forces, compressibility, non-Newtonian behavior, absorption, dispersion, and diffusion are considered, eight partial similarity models are designed. A new numerical approach of sensitivity analysis is suggested to quantify the dominance degree of relaxed dimensionless parameters for partial similarity model. The sensitivity factor quantifying the dominance degree of relaxed dimensionless parameter is defined. By solving the dimensionless governing equations including all dimensionless parameters, the sensitivity factor of each relaxed dimensionless parameter is calculated for each partial similarity model; thus, the dominance degree of the relaxed one is quantitatively determined. Based on the sensitivity analysis, the effect coefficient of partial similarity model is defined as the summation of product of sensitivity factor of relaxed dimensionless parameter and its relative relaxation quantity. The effect coefficient is used as a criterion to evaluate each partial similarity model. Then the partial similarity model with the smallest effect coefficient can be singled out to approximate to the prototype. Results show that the precision of partial similarity model is not only determined by the number of satisfied dimensionless parameters but also the relative relaxation quantity of the relaxed ones.

Keywords Polymer flooding · Partial similarity model · Dimensionless parameter · Sensitivity analysis

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1 Introduction

Polymer flooding, being widely carried out in China, is a kind of effective tertiary recovery. The addition of polymer to water flooding increases the viscosity of the aqueous phase to adapt the oil viscous force. The polymer solution can also reduce the permeability of aqueous phase. These two factors result in improving the mobility ratio between aqueous and oleic phases, increasing areal and volumetric sweep efficiency. The field and laboratory experiments have proved that polymer flooding can enhance oil recovery by 10–20% (Lu et al. 1996).

Polymer flooding experiment, based on scaling criteria, plays an important role in revealing the mechanism of driving process and optimizing the development programs in a short term at a low cost. Though some experiments (Delamaide et al. 1994; Sorbie et al. 1985; Wang et al. 1993) have been carried out, few literatures have reported the experiments guided by the principle of similarity or scaling law. The reason is the complex flooding process in which the convection, diffusion, dispersion, and adsorption are involved. There are many parameters involved in the process of polymer flooding flows. Therefore, it is very difficult or sometimes even impossible to keep all the similarity parameters identical in the laboratory experiment. Pozzi and Blackwell (1963) pointed out that precise scaling of transverse dispersion coupled with the requirement of geometric similarity would impractically require a large model and a very long time interval for experiments. In some cases the geometry similarity and the gravity force have to be relaxed to satisfy the requirement of the miscible displacement (Shen 2000). Islam and Farouq Ali (1989, 1990) got the scaling criteria for polymer, surfactant-enhanced alkaline/polymer multiple flooding flows. They reported that in one physical simulation it was impossible to realize the modeling of pressure, gravity and capillary forces, dispersion, and geometry similarity. They designed some partial similarity models to deal with this kind of problem.

These partial similarity models by relaxing some dimensionless parameters are effective to reveal certain mechanism. However, some problems may appear when extending these experimental results to oil field. Presently, few literatures have reported the method to quantitatively evaluate the difference between the partial similarity model and full similarity model. The method of quantitative comparison among different partial similarity models is not yet found from present literatures. In this paper a method is suggested based on our previous work on the scaling criteria and sensitivity analysis of dimensionless parameter of polymer flooding experiment.

2 The Evaluation Method of Partial Similarity Model

2.1 Sensitivity Analysis of Relaxed Dimensionless Parameter

To quantify the difference among partial similarity models, the sensitivity of relaxed dimensionless parameter is firstly analyzed by extending our previous proposal method (Bai et al. 2005). The sensitivity factor S_i representing the dominance degree of the relaxed dimensionless parameter π_i is defined as follows

$$S_i = \frac{\partial[f(\pi_1, \pi_2, \ldots, \pi_N)/f_{pm}]}{\partial(\pi_i/\pi_{ipm})}, \quad (i = 1, 2, \ldots, M), \tag{1}$$

where π_i is the ith relaxed dimensionless parameter, N the number of all the dimensionless parameters, and M the number of relaxed dimensionless parameters in partial similarity model. The subscript pm denotes the partial similarity model. S_i means the relative variation ration of the target function with respect to that of a relaxed dimensionless parameter. Here $f(\pi_1, \pi_2, \ldots, \pi_N)$ being the function of all dimensionless parameters denotes a target function concerned in polymer flooding experiment. It can be pressure, saturation, oil recovery, etc. We are mainly concerned with the oil recovery in polymer flooding; so the target function can be expressed as

$$f(\pi_1, \pi_2, \ldots, \pi_N) = \int_0^{T_\mathrm{D}} \eta(\pi_1, \pi_2, \ldots, \pi_N, t_\mathrm{D}) \mathrm{d}t_\mathrm{D}, \tag{2}$$

in which $\eta(\pi_1, \pi_2, \ldots, \pi_N, t_\mathrm{D})$ is the function of all dimensionless parameters and dimensionless time t_D represents the oil recovery.

In numerical approach, the sensitivity factor is given by

$$S_i = \Delta f / w_i, \tag{3}$$

in which

$$\Delta f = \frac{\int_0^{T_\mathrm{D}} \left| \eta_{pm}(\pi_1, \pi_2 \ldots, \pi_i, \ldots, \pi_N, t_\mathrm{D}) - \eta_{pm}(\pi_1, \pi_2, \ldots, \pi_{ipm}, \ldots, \pi_N, t_\mathrm{D}) \right| \mathrm{d}t_\mathrm{D}}{\int_0^{T_\mathrm{D}} \eta_{pm}(\pi_1, \pi_2, \ldots, \pi_{ipm}, \ldots, \pi_N, t_\mathrm{D}) \mathrm{d}t_\mathrm{D}}, \tag{4}$$

$$w_i = \left| \frac{\pi_i - \pi_{ipm}}{\pi_{ipm}} \right|. \tag{5}$$

Let us set the deviation coefficient w_i of each relaxed dimensionless parameter to be 1%, respectively. Obviously, we can exhibit the dominance degree of each dimensionless parameter by the value of the sensitivity factor.

2.2 The evaluation method of partial similarity model

To quantitatively evaluate the different partial similarity model derived from the same prototype, the effect coefficient α is defined as follows

$$\alpha = \sum_i S_i R_i, \tag{6}$$

in which

$$R_i = \left| \frac{\pi_i - \pi_{ipm}}{\pi_{ipm}} \right|, \tag{7}$$

denoting the relative relaxation quantity of π_i with respect to S_i. The effect coefficient is the summation of product of sensitivity factor of relaxed dimensionless parameter and its relative relaxation quantity. The larger the effect coefficient is, the more the difference between the partial similarity model and full similarity model.

3 Scaling Criteria of Polymer Flooding

The process of polymer flooding is actually the flow of the oleic and the aqueous phases. The oleic phase contains only oil and the aqueous phase contains water and polymer. In addition, the following assumptions are made: (1) Isothermal flow occurs in a homogeneous and isotropic medium; (2) Darcy's and Fick's laws are valid; (3) The polymer only reduces the permeability of aqueous phase and has no effect on permeability of oleic phase; (4) Aqueous and oleic phases and the rock are slightly compressible. Then, the scaling criteria considering the effect of gravity force, capillary force, compressibility, non-Newtonian characteristics of polymer solution, adsorption, and dispersion can be derived as follows (Bai et al. 2007)

$$\pi_1 = \frac{K_{cwo}}{K_{row}}, \quad \pi_2 = \frac{K_o}{K_{cwo}}, \quad \pi_3 = \frac{K_w}{K_{row}}, \quad \pi_4 = \frac{y_R}{x_R}, \quad \pi_5 = \frac{x_R}{z_R}, \quad \pi_6 = \frac{x_p}{x_R},$$
$$\pi_7 = \frac{y_p}{y_R}, \quad \pi_8 = \frac{r_{eo}}{x_R}, \quad \pi_9 = \frac{r_o}{x_R}, \quad \pi_{10} = \frac{s_{cw}}{\Delta s}, \quad \pi_{11} = \frac{s_{ro}}{\Delta s}, \quad \pi_{12} = \frac{s_{oi} - s_{ro}}{\Delta s},$$
$$\pi_{13} = \frac{\sigma\sqrt{\phi_0/K}\cos\theta K_{row}h}{q_I\mu_w}, \quad \pi_{14} = \frac{\mu_o}{\mu_w}, \quad \pi_{15} = \frac{\rho_{o0}}{\rho_{a0}}, \quad \pi_{16} = \frac{K_{row}h}{q_I\mu_w}\rho_{a0}gz_R,$$
$$\pi_{17} = \frac{C_o q_I \mu_w}{K_{row}h}, \quad \pi_{18} = \frac{C_a q_I \mu_w}{K_{row}h}, \quad \pi_{19} = \frac{C_\phi q_I \mu_w}{K_{row}h}, \quad \pi_{20} = \frac{p_{a0}K_{row}h}{q_I\mu_w},$$
$$\pi_{21} = \frac{p_{o0}K_{row}h}{q_I\mu_w}, \quad \pi_{22} = \frac{p_{wf}K_{row}h}{q_I\mu_w}, \quad \pi_{23} = \frac{p_{oi}K_{row}h}{q_I\mu_w}, \quad \pi_{24} = J(\bar{s}_w),$$
$$\pi_{25} = R_{k\,\max}, \quad \pi_{26} = \frac{\rho_r b\hat{C}_{ad\,\max}}{\phi_0\Delta s}, \quad \pi_{27} = \frac{\rho_r b\hat{C}_{ad\,\max}}{\Delta s}, \quad \pi_{28} = bC_{awp},$$
$$\pi_{29} = \frac{D_0\Delta sh}{q_I F}, \quad \pi_{30} = \frac{\delta d_p\phi_0\Delta s}{R_{k\,\max}x_R}, \quad \pi_{31} = n, \quad \pi_{32} = \frac{q_I}{hx_R\gamma_{\min}\sqrt{k_{row}}}, \quad \pi_{33} = c'\phi_0\Delta s,$$
$$\pi_{34} = A_{p1}C_{awp}, \quad \pi_{35} = A_{p2}C_{awp}^2, \quad \pi_{36} = A_{p3}C_{awp}^3, \quad \pi_{37} = \frac{C_{iap}}{C_{awp}}.$$

From the physical point of view, π_1, π_2, and π_3 denote the similarity of relative permeability of oil and water phases, π_4, π_5, π_6, π_7, π_8, and π_9 the similarity of geometry, well position, and well radius, respectively; π_{10} and π_{11} the ratios of the irreducible water saturation and residual oil saturation to the mobile oil saturation, respectively; π_{12} the reduced initial water saturation; π_{13} the ratio of the capillary force to the reservoir pressure difference induced by the injection rate q_I; π_{14} and π_{15} the ratios of the viscosity and density of water to oil, respectively; π_{16} the ratio of the gravitational force to the driving force; π_{17}, π_{18}, and π_{19} the relative volume variation ratio of oil, water, and rock caused by the reservoir pressure difference induced by the injection rate q_I, respectively; π_{20}, π_{21}, π_{22}, and π_{23} the respective ratios of the reference pressure of water and oil, the pressure of well bottom, and the initial pressure to the reservoir pressure difference; π_{24} the capillary force function; π_{25} the maximum permeability reduction factor; π_{26}, π_{27}, and π_{28} the rock characteristics related to adsorption behavior; π_{29} the area ratio between the molecular diffusion and the sweep area; π_{30} the ratio of the mean particle diameter in porous media to the reservoir dimension; π_{31} the shear rate exponent; π_{32} the ratio of the shear rate to the minimum shear rate within the pseudoplastic polymer solution; π_{33} the tortuosity attributed to rock characteristics; π_{34}, π_{35}, and π_{36} the dimensionless parameters in the viscosity expression of polymer solution, and π_{37} the dimensionless initial concentration of polymer solution.

Where p, μ, ρ, K, and s mean the pressure, the viscosity, the density, the effective permeability, and saturation; subscripts a and o indicate aqueous and oleic phases, respectively; x_R, y_R, and z_R the reference lengths in three dimensional directions, respectively; s_{ro} and s_{cw} the residual oil saturation and the irreducible water saturation, respectively; K_{cwo} the effective permeability of oil phase under the condition of the irreducible water saturation; K_{row} the effective permeability of water phase under the condition of the residual oil saturation; p_{wf}, p_c the bottom pressure of the production well and the capillary force, respectively; ϕ the porosity of rock; g the gravitational acceleration; x_p and y_p the coordinates of the production well; q_I the injection rate; r_o the well radius; C_{iap} the initial polymer concentration in aqueous phase; $\hat{C}_{ad\max}$ the maximum amount of polymer adsorbed per unit mass of rock; C_{awp} the polymer concentration in production well; ρ_r the density of rock; σ, θ the interfacial tension and the contact angle between aqueous and oleic phases; $J(s_a)$ the capillary force function; C_o, C_a, and C_ϕ the compressibility of oil, water, and rock, respectively; $R_{k\max}$ the maximum permeability reduction factor; $\gamma_{\min}$ the minimum shear rate within the range of pseudoplastic fluid; n the shear rate exponent; m the salinity exponent; A_{p1}, A_{p2}, and A_{p3} the constants; c' the tortuosity; D_0 molecular diffusion coefficient; F the formation electrical resistivity factor; d_p the mean particle diameter in porous media; δ the inhomogeneity of the formation.

4 Designing of Partial Similarity Model

Theoretically, the physical model should fully obey all the dimensionless parameters to obtain similar experimental results with the field prototype. However, it hardly satisfies all the dimensionless parameters in physical simulation of polymer flooding. One way of tackling this problem is to relax some dimensionless parameters to satisfy others, namely the partial similarity model is used to approximate the full similarity model. Based on the different experimental goals and conditions, eight different partial similarity models from cases1 to 8 are designed as follows.

Case1: The model and the prototype are the same porous media, the same fluids, the same pressure drops, and relaxed geometry similarity. The satisfied dimensionless parameters in model are as follows

$$\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{10}, \pi_{11}, \pi_{12}, \pi_{14}, \pi_{15}, \pi_{16}, \pi_{20}, \pi_{21}, \pi_{22}, \pi_{23}, \pi_{24},$$
$$\pi_{25}, \pi_{26}, \pi_{27}, \pi_{28}, \pi_{31}, \pi_{32}, \pi_{33}, \pi_{34}, \pi_{35}, \pi_{36}, \pi_{37}.$$

This approach has the advantage that all properties that depend on porous media can be scaled more accurately due to the same porous media being used in the partial similarity model as that in prototype. The irreducible water saturation, the residual oil saturation, and the relative permeability that associate with porous media can be properly scaled. The gravity force and the pressure distribution can be properly scaled while relaxing geometry similarity requirement. The non-Newtonian behavior, the effect of shear rate on non-Newtonian, the permeability reduction, and the adsorption can be properly scaled due to the same fluids between partial similarity model and prototype. The disadvantage of this approach is that properties which depend on pressure cannot be accurately scaled due to the different pressure between the model and prototype. The compressibility of water, oil, and porosity, the capillary force, and the dispersion cannot be properly scaled. Giving $x_R = l$, $y_R = w$, and $z_R = h$. Let $r(\xi) = \xi_{prototype}/\xi_{\text{model}}$ denoting the variable ratio of prototype to that of model. If $r(l) = a$, then $r(w) = r(h) = a, r(x_p) = r(y_p) = a, r(p_{a0}) = r(p_{o0}) = a, r(p_{wf}) = a, r(p_{oi}) = a, r(t) = a$, and $r(q_I) = a^2$. The other variable values in model are the same as those of prototype.

Case2: The model and the prototype are the same porous media, the same fluids, geometry similarity, and the same pressure drops. The satisfied dimensionless parameters in the model can be as follows

$$\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{10}, \pi_{11}, \pi_{12}, \pi_{13}, \pi_{14}, \pi_{15}, \pi_{17}, \pi_{18}, \pi_{19}, \pi_{20}, \pi_{21},$$
$$\pi_{22}, \pi_{23}, \pi_{24}, \pi_{25}, \pi_{26}, \pi_{27}, \pi_{28}, \pi_{29}, \pi_{31}, \pi_{33}, \pi_{34}, \pi_{35}, \pi_{36}, \pi_{37}.$$

In this approach, the relative permeability, the irreducible water saturation, and the residual oil saturation can be properly scaled due to the same porous media between the model and prototype. The adsorption behavior and the dispersion can be properly scaled too. Such attribution as compressibility that depends on pressure can be scaled because of the same pressure used in model. The effect of viscosity, the capillary force, and the partial non-Newtonian characteristics on flow can be properly scaled. However, the effect of shear rate on non-Newtonian characteristics and the gravity force cannot be scaled. Let $r(l) = a$, then $r(w) = r(h) = a, r(q_I) = a, r(x_p) = r(y_p) = a$, and $r(t) = a^2$. The other variable values in model are the same as that of prototype.

Case3: The model and the prototype are the same porous media, the same fluids, relaxed geometry similarity, and different pressure drops. The satisfied dimensionless parameters in the model can be derived as follows

$$\pi_1, \pi_2, \pi_3, \pi_4, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{10}, \pi_{11}, \pi_{12}, \pi_{14}, \pi_{15}, \pi_{16}, \pi_{20}, \pi_{21}, \pi_{22}, \pi_{23}, \pi_{24}, \pi_{25}, \\ \pi_{26}, \pi_{27}, \pi_{28}, \pi_{31}, \pi_{33}, \pi_{34}, \pi_{35}, \pi_{36}, \pi_{37}.$$

This approach can realize the scaling of permeability, the irreducible water saturation, the residual oil saturation, and adsorption due to the same porous media. The gravity force can be properly scaled while relaxing the geometry similarity. The effect of viscosity and partial non-Newtonian behavior can be scaled. However, the effect of shear rate on non-Newtonian behavior, the vertical characteristics of flow, the properties depending on pressure, and the dispersion cannot be properly scaled. If the vertical scale is so small compared with the horizontal scale in prototype, the geometry similarity should be relaxed in the model. If $r(h) = a/n$, i.e. the thickness of partial similarity model is n times as large as that of theoretical model. Let $r(l) = a$, then $r(w) = a, r(q_I) = (a/n)^2, r(x_p) = r(y_p) = a, r(p_{a0}) = r(p_{o0}) = a/n, r(p_{wf}) = a/n, r(p_{oi}) = a/n$, and $r(t) = na$. The other variable values in model are the same as that of prototype.

Case4: The model and the prototype are the same porous media, the same fluids, the same pressure drops, and the relaxed geometry similarity. The satisfied dimensionless parameters in the model can be as follows

$$\pi_1, \pi_2, \pi_3, \pi_4, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{10}, \pi_{11}, \pi_{12}, \pi_{13}, \pi_{14}, \pi_{15}, \pi_{17}, \pi_{18}, \pi_{19}, \pi_{20}, \pi_{21}, \pi_{22}, \\ \pi_{23}, \pi_{24}, \pi_{25}, \pi_{26}, \pi_{27}, \pi_{28}, \pi_{29}, \pi_{31}, \pi_{33}, \pi_{34}, \pi_{35}, \pi_{36}, \pi_{37}.$$

In this model the permeability, the irreducible water saturation, the residual oil saturation, and the adsorption can be properly scaled with the same porous media. The effect of viscosity, partial non-Newtonian behavior, and attributions depending on pressure can be scaled. The capillary force and dispersion can be scaled due to the relaxed geometry similarity and different pressure drops. The vertical flow characteristics, the gravity force, and the effect of shear rate cannot be scaled properly. Let $r(h) = a/n$ and $r(l) = a$, then $r(w) = a, r(x_p) = r(y_p) = a, r(q_I) = a/n$, and $r(t) = a^2$. The other variable values in model are the same as that of prototype.

Case5: The model and the prototype are the different porous media, the same fluids, geometry similarity, and different pressure drops. In some cases, the porous media used in model is different from that of prototype. The satisfied dimensionless parameters can be as follows

$$\pi_4, \pi_5, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{12}, \pi_{14}, \pi_{15}, \pi_{16}, \pi_{20}, \pi_{21}, \pi_{22}, \pi_{23}, \pi_{31}, \pi_{34}, \pi_{35}, \pi_{36}, \pi_{37}.$$

This approach cannot realize the accurate scaling of permeability, the irreducible water saturation, the residual oil saturation, the adsorption, and the effect of permeability reduction due to the different porous media considered. However, the difference of these characteristics can be reduced to be as small as possible by approximating the model porous media to that of prototype. The properties depending on pressure cannot be scaled properly due to the different pressure drops. The dispersion, the capillary, and the effect of shear rate cannot be scaled properly too. The gravity force can be properly scaled by relaxing the pressure drop. Let $r(l) = a$, then $r(w) = a, r(h) = a, r(x_p) = r(y_p) = a, r(p_{a0}) = r(p_{o0}) = a, r(p_{wf}) = a$, $r(p_{oi}) = a$, $r(k_{row}/q_I) = a^{-2}$, $r(k_{row}) = a^{-1/2}$, $r(q_I) = a^{3/2}$, and $r(t) = a^{3/2}\Delta s$. The other variable values in model are the same as that of prototype.

Case6: The model and the prototype are different porous media, the same fluids, geometry similarity, and the same pressure drops. The satisfied dimensionless parameters are as follows

$$\pi_4,\ \pi_5, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{12}, \pi_{14}, \pi_{15}, \pi_{17}, \pi_{18}, \pi_{20}, \pi_{21}, \pi_{22}, \pi_{23}, \pi_{31}, \pi_{34}, \pi_{35}, \pi_{36}, \pi_{37}.$$

In this partial similarity model, the effect of viscosity, the distribution of pressure, partial non-Newtonian behavior, and the properties depending on pressure can be scaled properly. The permeability, the irreducible water saturation, the residual oil saturation, adsorption, dispersion, and the permeability reduction cannot be scaled properly due to the different porous media used in this model. Let $r(l) = a$, then $r(w) = a, r(h) = a, r(x_p) = r(y_p) = a$, $r(k_{row}/q_I) = a^{-1}$, $r(k_{row}) = a^{-1/2}$, $r(q_I) = a^{1/2}$, and $r(t) = a^{5/2}\Delta s$. The other variable values in model are the same as those of prototype.

Case7: The model and the prototype are different porous media, the same fluids, relaxed geometry similarity, and the different pressure drops. The satisfied dimensionless parameters are as follows

$$\pi_4, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{12}, \pi_{14}, \pi_{15}, \pi_{16}, \pi_{20}, \pi_{21}, \pi_{22}, \pi_{23}, \pi_{31}, \pi_{34}, \pi_{35}, \pi_{36}, \pi_{37}.$$

In this model, the effect of viscosity, the gravity force, the pressure distribution, and partial non-Newtonian behavior can be properly scaled while relaxing the geometry similarity and different pressure. Due to the different porous media considered in the model, the permeability, the irreducible water saturation, the residual oil saturation, capillary force, compressibility, dispersion, adsorption, permeability reduction, and the effect of shear rate cannot be scaled accurately. Let $r(l)=a$, then $r(w)=a$, $r(h)=a/n$, $r(x_p) = r(y_p)=a$, $r(p_{a0})=r(p_{o0}) = a/n$, $r(p_{wf}) = a/n$, $r(p_{oi}) = a/n$, $r(k_{row}/q_I) = n^2a^{-2}$, $r(k_{row}) = a^{-1/2}$, $r(q_I) = n^{-2}a^{3/2}$, and $r(t) = na^{3/2}\Delta s$. The other variable values in model are the same as that of prototype.

Case8: The model and prototype are different porous media, the same fluids, relaxed geometry similarity, and the same pressure drops. The satisfied dimensionless parameters can be as follows

$$\pi_4, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{12}, \pi_{14}, \pi_{15}, \pi_{17}, \pi_{18}, \pi_{20}, \pi_{21}, \pi_{22}, \pi_{23}, \pi_{31}, \pi_{34}, \pi_{35}, \pi_{36}, \pi_{37}.$$

This model can properly scale the effect of viscosity, the pressure distribution, compressibility, and partial non-Newtonian behavior by relaxing geometry similarity. Due to the different porous media used in the model, the permeability and permeability reduction, the irreducible water saturation, the residual oil saturation, capillary force, dispersion, adsorption, and the effect of shear rate cannot be scaled accurately. Let $r(l) = a$, $r(h) = a/n$, then $r(w) = a$, $r(x_p) = r(y_p) = a$, $r(k_{row}/q_I) = na^{-1}$, $r(k_{row}) = a^{-1/2}$, $r(q_I) = n^{-1}a^{1/2}$, and $r(t) = a^{5/2}\Delta s$. The other variable values in model are the same as those of prototype.

5 Dimensionless Governing Equations and Solution Procedure

The dimensionless governing equations including all dimensionless parameters are derived and resolved to obtain the sensitivity factor.

The dimensionless equation of mass balance of oleic phase reads

$$\begin{aligned}&\pi_4\pi_1\frac{\partial}{\partial x_D}\left(\rho_{oD}\frac{\pi_2}{\pi_{14}}\frac{\partial p_{oD}}{\partial x_D}\right)+\frac{\pi_1}{\pi_4}\frac{\partial}{\partial y_D}\left(\rho_{oD}\frac{\pi_2}{\pi_{14}}\frac{\partial p_{oD}}{\partial y_D}\right)\\&+\pi_4\pi_5^2\pi_1\frac{\partial}{\partial z_D}\left(\rho_{oD}\frac{\pi_2}{\pi_{14}}\frac{\partial p_{oD}}{\partial z_D}\right)+\pi_{16}\pi_1\pi_4\pi_5^2\frac{\partial}{\partial z_D}\left(\rho_{oD}^2\pi_{15}\frac{\pi_2}{\pi_{14}}\right)\\&+\pi_1\rho_{oD}\frac{\pi\pi_2\left(\pi_{22}-p_{oD}\right)}{\pi_{14}2\ln\pi_8/\pi_9}\delta\left(x_D-\pi_6\right)\delta(y_D-\pi_7)=\frac{\partial(\rho_{oD}\phi_D\bar{s}_o)}{\partial t_{\mathrm{D}}}+\pi_{11}\frac{\partial(\rho_{oD}\phi_D)}{\partial t_{\mathrm{D}}}.\end{aligned}\tag{8}$$

The dimensionless equation of mass balance of aqueous phase is written as

$$\begin{aligned}&\frac{\pi_4}{\pi_{25}}\frac{\partial}{\partial x_D}\left(\frac{\rho_{aD}\pi_3}{R_{kD}\mu_{aD}}\frac{\partial p_{aD}}{\partial x_D}\right)+\frac{1}{\pi_4\pi_{25}}\frac{\partial}{\partial y_D}\left(\frac{\rho_{aD}\pi_3}{R_{kD}\mu_{aD}}\frac{\partial p_{aD}}{\partial y_D}\right)+\frac{\pi_4\pi_5^2}{\pi_{25}}\frac{\partial}{\partial z_D}\\&\times\left(\frac{\rho_{aD}\pi_3}{R_{kD}\mu_{aD}}\frac{\partial p_{aD}}{\partial z_D}\right)+\frac{\pi_{16}\pi_4\pi_5^2}{\pi_{25}}\frac{\partial}{\partial z_D}\left(\rho_{aD}^2\frac{\pi_3}{R_{kD}\mu_{aD}}\right)+\rho_{aD}\left[\frac{1}{4}\delta(x_D)\delta(y_D)+\frac{1}{\pi_{25}}\right.\\&\left.\times\frac{\pi\pi_3(\pi_{22}-p_{aD})}{R_{kD}\mu_{aD}2\ln\pi_8/\pi_9}\delta(x_D-\pi_6)\delta(y_D-\pi_7)\right]=\frac{\partial(\rho_{aD}\phi_D\bar{s}_a)}{\partial t_{\mathrm{D}}}+\pi_{10}\frac{\partial(\rho_{aD}\phi_D)}{\partial t_{\mathrm{D}}}.\end{aligned}\tag{9}$$

The dimensionless equation of mass balance of polymer component is given by

$$\begin{aligned}&\frac{\partial}{\partial x_D}\left((\bar{s}_aD_{xD1}+D_{xD2})\frac{\partial}{\partial x_D}(C_{apD}\rho_{aD})\right)+\frac{\partial}{\partial y_D}\left((\bar{s}_aD_{yD1}+D_{yD2})\frac{\partial}{\partial y_D}(C_{apD}\rho_{aD})\right)\\&+\frac{\partial}{\partial z_D}\left((\bar{s}_aD_{zD1}+D_{zD2})\frac{\partial}{\partial z_D}(C_{apD}\rho_{aD})\right)+\frac{\pi_4}{\pi_{25}}\frac{\partial}{\partial x_D}\left(\frac{\rho_{aD}C_{apD}\pi_3}{R_{kD}\mu_{aD}}\frac{\partial p_{aD}}{\partial x_D}\right)\\&+\frac{1}{\pi_4\pi_{25}}\frac{\partial}{\partial y_D}\left(\frac{\rho_{aD}C_{apD}\pi_3}{R_{kD}\mu_{aD}}\frac{\partial p_{aD}}{\partial y_D}\right)+\frac{\pi_4\pi_5^2}{\pi_{25}}\\&\times\frac{\partial}{\partial z_D}\left(\frac{\rho_{aD}C_{apD}\pi_3}{R_{kD}\mu_{aD}}\frac{\partial p_{aD}}{\partial z_D}+\pi_{16}\rho_{aD}^2\frac{C_{apD}\pi_3}{R_{kD}\mu_{aD}}\right)\\&+\rho_{aD}\left[\frac{1}{4}\delta(x_D)\delta(y_D)+\frac{1}{\pi_{25}}\frac{\pi C_{apD}\pi_3(\pi_{22}-p_{aD})}{2R_{kD}\mu_{aD}\ln\pi_8/\pi_9}\delta(x_D-\pi_6)\delta(y_D-\pi_7)\right]\\&=\frac{\partial(\rho_{aD}\phi_D\bar{s}_aC_{apD})}{\partial t_{\mathrm{D}}}+\pi_{10}\frac{\partial(\rho_{aD}\phi_DC_{apD})}{\partial t_{\mathrm{D}}}+\frac{\pi_{26}}{(1+\pi_{28}C_{apD})^2}\frac{\partial(C_{apD}\rho_{aD})}{\partial t_{\mathrm{D}}}\\&-\frac{\pi_{27}}{(1+\pi_{28}C_{apD})^2}\frac{\partial(C_{apD}\phi_D\rho_{aD})}{\partial t_{\mathrm{D}}}.\end{aligned}\tag{10}$$

Here

$$\begin{aligned}&\rho_{aD}=1+\pi_{18}(p_{aD}-\pi_{21})+\pi_{16}C_{aD}z_D,\ \rho_{oD}=1+\pi_{17}(p_{oD}-\pi_{21})+\pi_{16}\pi_{17}\pi_{15}z_D,\\&\phi_D=1+\pi_{19}\left(\frac{p_{aD}+p_{oD}}{2}-\frac{\pi_{20}+\pi_{21}}{2}\right)+\frac{\pi_{16}\pi_{15}\pi_{19}}{2}z_D+\frac{\pi_{16}\pi_{19}}{2}z_D,\end{aligned}$$

$$D_{xD1} = \pi_{29}\pi_4 - \pi_{30}\pi_4\pi_3 \frac{0.5\phi_D}{R_{kD}\mu_{aD}} \frac{\partial p_{aD}}{\partial x_D}, \quad D_{yD1} = \frac{\pi_{29}}{\pi_4} - \frac{\pi_{30}\pi_3}{\pi_4^2} \frac{0.5\phi_D}{R_{kD}\mu_{aD}} \frac{\partial p_{aD}}{\partial y_D},$$
$$D_{zD1} = \pi_{29}\pi_4\pi_5^2 - \pi_{30}\pi_4\pi_5^3\pi_3 \frac{0.5\phi_D}{R_{kD}\mu_{aD}} \frac{\partial p_{aD}}{\partial z_D} - \pi_{16}\pi_{30}\pi_4\pi_5^3\pi_3 \frac{0.5\rho_{aD}\phi_D}{R_{kD}\mu_{aD}},$$
$$D_{xD2} = \pi_{10}D_{xD1}, \quad D_{yD2} = \pi_{10}D_{yD1}, \quad D_{zD2} = \pi_{10}D_{zD1}.$$

The dimensionless saturation relation is

$$\bar{s}_o + \bar{s}_a = 1. \tag{11}$$

The dimensionless capillary force can be expressed as

$$p_{cD} = (p_{oD} - p_{wD}) = \pi_{13}\sqrt{\phi_D}\pi_{24}. \tag{12}$$

The dimensionless initial conditions are

$$p_{oD}(x_D, y_D, z_D, 0) = \pi_{23}, \quad \bar{s}_o(x_D, y_D, z_D, 0) = \pi_{12}, \quad C_{apD}(x_D, y_D, z_D, 0) = \pi_{37}, \tag{13}$$

and the dimensionless boundary conditions are specified as

$$\frac{\partial p_{lD}}{\partial x_D} = 0, \quad \frac{\partial p_{lD}}{\partial y_D} = 0, \quad \frac{\partial p_{lD}}{\partial z_D} + \pi_{16}\rho_{aD} = 0. \tag{14}$$

The dimensionless viscosity equation of polymer solution is given as follows

$$\mu_{aD} = \left(1 + \left(\frac{\gamma}{\gamma_{\min}}\right)^{\pi_{31}-1} \left(\pi_{34}C_{apD} + \pi_{35}C_{apD}^2 + \pi_{36}C_{apD}^3\right)\right). \tag{15}$$

Here

$$\frac{\gamma}{\gamma_{\min}} = \left(\frac{3\pi_{31}+1}{4\pi_{31}}\right)^{\frac{\pi_{31}}{\pi_{31}-1}} \frac{\pi_3}{\pi_{25}R_{kD}\mu_{aD}}$$
$$\times \sqrt{\frac{\pi_{32}^2\left(\frac{\partial p_{aD}}{\partial x_D}\right)^2 + \left(\frac{\pi_{32}}{\pi_4}\right)^2\left(\frac{\partial p_{aD}}{\partial y_D}\right)^2 + (\pi_{32}\pi_5)^2\left(\frac{\partial p_{aD}}{\partial z_D}\right)^2 + (\pi_{32}\pi_5\pi_{16})^2\rho_{aD}^2}{0.5\pi_{33}\pi_3\phi_D(\bar{s}_a + \pi_{10})/(\pi_{25}R_{kD})}}$$
$$\text{in which } R_{kD} = \frac{1}{\pi_{25}} + \left(1 - \frac{1}{\pi_{25}}\right)\frac{\pi_{28}C_{apD}}{1 + \pi_{28}C_{apD}}. \tag{16}$$

Subscript D denotes the dimensionless. The dimensionless governing equations are multivariable coupled partial differential equation groups with nonlinearity. Therefore, the numerical method is used to solve them. The governing equations are discretized using finite difference scheme with the convection term discretized using upwind scheme and the diffusion term using second-order central difference scheme. The discretized equations are solved by the conventional implicit pressure–explicit saturation (Bai et al. 2005; Khalid and Antonin 1979) and implicit concentration method (Cui and Luan 1997). Both the grid scale and the time scale are dimensionless scales in numerical scheme to corresponding to dimensionless governing equations. With an amount of pilot computations, we find that the mesh scales in x, y, and z dimensional directions equal to 0.05, 0.05, and 0.2, respectively, and time scale equal to 2.59E-4 can satisfy our required precision.

Table 1 Main physical variables for polymer flooding

Physical variable	Value	Physical variable	Value
Length l (m)	140	Density of rock ρ_r (kg/m^3)	2.5E+3
Width w (m)	140	Bottom pressure of the production well p_{wf} (Pa)	10E+6
Thickness h (m)	10	Injection rate q_I (m^3/s)	8E-3
Density of oleic phase under a given condition ρ_{o0}(kg/m^3)	800	Maximum permeability reduction factor $R_{k\max}$	15
Shear rate exponent n	0.68	Well radius r_o(m)	0.1
Initial oleic pressure p_{oi} (Pa)	12E+6	Compressibility of aqueous phase C_a(Pa^{-1})	5E-10
Density of oleic phase under a given condition ρ_{w0} (kg/m^3)	1000	Maximum adsorption amount per unit mass of rock $\hat{C}_{ad\max}$(kg/kg)	3E-6
Compressibility of oleic phase C_o(Pa^{-1})	8E-10	Parameter in viscosity equationA_{p1}	1.02
Langmuir adsorption constant b	0.12	Parameter in viscosity equation A_{p2}	0.18
Compressibility of rock C_ϕ(Pa^{-1})	6E-10	Parameter in viscosity equation A_{p3}	0.63
Porosity under a given condition ϕ_0	0.25	Molecular diffusion coefficient D_0(m^2/s)	1.346E-9
Viscosity of water μ_w (Pa s)	1E-3	The formation electrical resistivity factor F	5.88
Viscosity of oil μ_o (Pa s)	5E-3	Mean particle diameter d_p(m)	8.57E-4

6 Evaluation of Partial Similarity Model

For the reservoir prototype defined by the parameters listed in Table 1, the only theoretical full similarity model can be designed according to the derived scaling criteria. The dimensionless parameters in the full similarity model are calculated as listed in Table 2 except for π_2,π_3, and π_{24}. π_2 and π_3 are normalized permeability of oil and water phases, and $\pi_{24}$the capillary force function. However, it hardly satisfies all the dimensionless parameters in physical model. Therefore, some dimensionless parameters have to be relaxed. According to the above designed method of partial similarity model, the parameters from cases1 to 8 can be determined. In each partial similarity model, the sensitivity factors of the relaxed dimensionless parameters can be calculated. The main procedures to obtain the sensitivity factor are as follows. Firstly, the calculated dimensionless parameters relevant to one partial similarity model are substituted into the governing equations as shown from Eq. 8–16. Then, the governing equations are solved and the variation of oil recovery with dimensionless time can be determined. Finally, the sensitivity factor in the partial similarity model can be calculated according to Eq. 1. Taking case1 as an example, we obtain the relaxed dimensionless parameters and their values as follows, $\pi_{13} = 1.156$, $\pi_{17} = 8.649$E-6, $\pi_{18} = 5.405$E-6, $\pi_{19} = 6.487$E-6, $\pi_{29} = 3.893$E-5, $\pi_{30} = 6.539$E-4. Therefore, the sensitivity factors of these relaxed dimensionless parameters in case1 can be calculated as shown in Table 3. We can see that π_{13} with the largest sensitive factor is the most sensitive among the relaxed dimensionless parameters, which means that it should be relaxed as small as possible when it is contradictory with other relaxed ones to reduce the difference between model and prototype.

Table 2 The dimensionless parameter values

Dimensionless parameter	Value	Dimensionless parameter	Value	Dimensionless parameter	Value
π_1	1.339	π_{17}	11.189E-4	π_{29}	1.456E-7
π_4	1	π_{18}	6.933E-4	π_{30}	2.428E-6
π_5	14	π_{19}	8.392E-4	π_{31}	0.68
π_9	7.143E-4	π_{20}	8.58	π_{32}	2.519
π_{10}	0.287	π_{21}	8.58	π_{33}	0.278
π_{11}	0.693	π_{22}	7.15	π_{34}	1.02
π_{12}	1	π_{23}	8.58	π_{35}	0.18
π_{13}	8.938E-3	π_{25}	15	π_{36}	0.63
π_{14}	5	π_{26}	7.128E-3	π_{37}	0
π_{15}	0.8	π_{27}	0.12		
π_{16}	7.008E-2	π_{28}	0.12		

Table 3 Sensitivity factors of relaxed dimensionless parameters

π_i	π_{13}	π_{17}	π_{18}	π_{19}	π_{29}	π_{30}
S_i	**1.558E-2**	3.269E-5	4.358E-5	4.358E-5	5.448E-5	2.942E-4

Note: Bold indicates the dominance of the number because it is much larger than others

Table 4 The effect coefficients of partial similarity models

Case	Case1	Case2	Case3	Case4
α	1.31378	11.10954	0.74019	7.81891
Case	Case5	Case6	Case7	Case8
α	1.25826	23.39068	0.81174	20.10005

With the same analysis, we find that π_{32} is the most sensitive in cases2 and 4. In case3 π_5 and π_{32} are the most sensitive, and π_{13} is more sensitive. In cases 5 to 8 the most sensitive dimensionless parameters are all π_1,π_2, and π_3, which means that when the porous media used in model is different from that in prototype we should try our best to scale the porous media parameters. The more sensitive dimensionless parameters are π_{28}, π_{25}, and π_{32} in case5; π_{25} in case6; π_5, π_{25}, and π_{32} in case7; π_{25} in case8; respectively.

Based on the sensitivity analysis, the effect coefficient of each partial similarity model as shown in Table 4 can be calculated according to Eq. 6. We can see that cases3 and 7 with the smaller effect coefficient are more approximate to prototype. Though case2 satisfies the dimensionless parameters with the most number, its effect coefficient is not smallest. Though the number of relaxed dimensionless parameters in case7 is not the smallest, it is with smaller effect coefficient. It should be noted that, the effect coefficients of cases5 and 7 are not as much as we suppose due to the difference of 1% of porous media between model and prototype is assigned. We can reasonably conclude that the effect coefficient should increase with the difference between model and prototype. From the above analysis we can draw the conclusion that the precision of partial similarity model is not only determined by the number of satisfied dimensionless parameters but also the relative relaxation quantity of relaxed dimensionless parameters. Case2 is the best proof in which the number of satisfied dimensionless parameter is the most but the effect coefficient is not the smallest due to the larger relative relaxation quantity of dimensionless parameter.

7 Conclusions

A new numerical approach of sensitivity analysis is suggested in this paper to quantify the sensitivity of relaxed dimensionless parameter for partial similarity model in polymer flooding experiment. The sensitivity factor quantifying the dominance degree of all relaxed dimensionless parameters is defined. Based on this sensitivity analysis, the effect coefficient of partial similarity model is defined as the summation of product of sensitivity factor of relaxed dimensionless parameter and its relative relaxation quantity. By comparing the effect coefficient, the partial similarity model that is most approximate to prototype can be singled out.

Eight partial similarity models are designed based on the derived scaling criteria of polymer flooding experiment. The sensitivity factor of each relaxed dimensionless parameter is calculated for each partial similarity model; thus, the dominance degree of relaxed one is quantitatively determined. The effect coefficient is calculated and used as a criterion to evaluate the deviation of each partial similarity model from the prototype. Results show that the precision of partial similarity model is not only determined by the number of satisfied dimensionless parameters but also the relative relaxation quantity of relaxed dimensionless parameters.

Acknowledgements The writers greatly appreciate the financial support of the National Basic Research Program of China (Grant No. 2005CB221300) and the Innovative Project of Chinese Academy of Sciences (Grant No. KJCX-SW-L08). The writers also greatly appreciate Professor Li Jiachun, academician of Chinese Academy of Sciences, for his help.

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