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Advances in Space Research 41 (2008) 2131–2136
ADVANCES IN SPACE RESEARCH (a COSPAR publication)
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Thermovibrational instability of Rayleigh–Marangoni–Benard convection in two-layer fluid systems

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Received 31 October 2006; received in revised form 12 September 2007; accepted 14 September 2007

Abstract

The thermovibrational instability of Rayleigh–Marangoni–Benard convection in a two-layer system under the high-frequency vibration has been investigated by linear instability analysis in the present paper. General equations for the description of the convective flow and within this framework, the generalized Boussinesq approximation are formulated. These equations are dealt with using the averaging method. The theoretical analysis results show that the high-frequency thermovibrations can change the Marangoni–Benard convection instabilities as well as the oscillatory gaps of the Rayleigh–Marangoni–Benard convection in two-layer liquid systems. It is found that vertical high-frequency vibrations can delay convective instability of this system, and damp the convective flow down.

Keywords: Rayleigh–Marangoni–Benard convection; Thermovibration; Instability; Two-layer liquids

1. Introduction

Vibrations are known to be among the most effective ways of affecting the behavior of fluid systems in the sense of increasing or reducing the convective heat transfer. Most of the material in this paper is devoted to the case of high-frequency vibration, where it is necessary to suppose that the vibrations frequency is high enough for the period to be small with respect to all the reference hydrodynamic and thermal times. Besides, the amplitude of vibration must be finite. So we can use the averaging method.

We focus our attention on the case where a system filled with fluids heated from below is subjected to arbitrary high-frequency vibrations. The description of the thermovibrational flows in the limiting case of high-frequency and small amplitude of vibrations may be effectively obtained in the frame of the averaging method, which leads to the system of equations for averaged fields of velocities and temperatures. The averaging method is that proposed by Kapitsa when solving the problem of a pendulum with a vibrating pole. Simonenko and Zen'kovskaja (1966) were the first to use this method and proposed the time-averaged form of the Boussinesq equations. Subsequently, it was demonstrated that high-frequency vibrations are most relevant in modifying stability characteristics. Gershuni and Zhukhovitskii (1979) introduced the vibrational Rayleigh number, Ra_v, to present the intensity of the vibrational source. Using the time-averaged method, Gershuni and Zhukhovitskii studied the vibrational thermal convection under the weightlessness in a rectangular cavity, cylindrical enclosure and a heated cylinder, respectively, in an unconfined fluid. Due to the high-frequency assumption, many important phenomena, like the resonant state and the detailed variation of the heat transfer rate could not be investigated. A more detailed study was carried out by Fu and Shieh (1992) to investigate the effects of the vibration frequency and Rayleigh number on the thermal

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0273-1177/$34.00
doi:10.1016/j.asr.2007.09.016

convection. The vibration frequency varied from 1 to 1×10^4 and three different regimes of the Rayleigh number were discussed. According to the results, when the Rayleigh number is large enough ($Ra = 10^6$), gravitational thermal convection dominates, and the vibration motion does not markedly enhance the heat transfer rate. In contrast, in the lower Rayleigh number ($Ra = 10^4$) case, except in the quasi-static convection region, the vibration thermal convection is dominant, and the vibration enhances the heat transfer significantly.

Convection structure and instability in two-layer liquid system heated from below have been extensively studied in the past two decades (Johnson, 1975; Richter and McKenzie, 1981; Liu et al., 1998; Colinet et al., 2001). It has been known that the convection in two-layer system has many new features, which have no counterpart in a single-layer system (Zen'kovskaya and Shleikel, 2001; Cisse et al., 2004). The effects of thermovibrations on the stabilities have been studied about the classical Rayleigh–Benard problem (Cisse et al., 2004) and for Marangoni convection (surface-tension-driven convection) in one-layer system (Zen'kovskaya and Shleikel, 2001). All the results arrived at the conclusion that the vertical vibration can hinder the instability and horizontal vibration can enhance the instability. However the effects of high-frequency thermovibrations on two-layer system convection have not been studied so much. Only Birikh (2003) has done some work on thermocapillary convection in a two-layer system.

All the results were of great help for understanding the g-jitter effects during fluid and material science microgravity experiments. On available space platforms, a systematic characterization of the accelerations has shown that the microgravity environment is dynamic, depending on many sources, e.g., aerodynamic forces, on-board machinery, crew operation or servicing activities. It is recognized that the presence of g-disturbances may cause strong discrepancies, and so, the fluid science processes may be substantially changed by g-disturbances. To reduce the convective contributions of these g-jitters, it is convenient to consider acceleration fields as an expansion of harmonic oscillations, and to use a time-averaged method formulation.

2. Problem description and basic equations

We consider a two-layer system heated from below. Only in the vertical direction the depth is finite. The system is described in Fig. 1.

Our study will concern the effects of thermovibrations on the convective instability in Marangoni–Benard convection and Rayleigh–Marangoni–Benard convection in a two-layer system.

Here, we propose a theoretical model of two-layer convection with high-frequency thermovibrations as shown schematically in Fig. 1. In the system both the top wall and the bottom one are considered as rigid isothermal flats, perfectly conducting boundaries. The system is heated from the bottom. The walls are infinite in the horizontal direction. The fluids are bounded by the two flats and the fluid upper is Silicone oil (Si 10cst) and Flourinert (FC70) is the fluid bottom since this fluid pair has been more recently investigated theoretically (Liu, 2004; Liu et al., 2005; Zhou et al., 2004) and experimentally (Degen et al., 1998). The vector $\mathbf{n}$ ($\mathbf{n} = (\cos\alpha, \sin\alpha)$) is the unit vector of vibrations with α as the angle of vibrations and they are all changeable. Temperature difference $\Delta T = T_2 - T_1$ is imposed perpendicular to the interface of the fluids, where the subscripts 1 and 2 refer to the fluids upper and bottom, respectively. H_1, H_2 denote the depths of the fluids, respectively. According to the results about the deformable interface (Zen'kovskaya and Shleikel, 2001; Birikh, 2003) before here the interface is considered to be flat. The interfacial tension at the interface is assumed to be a linear function of temperature: $\sigma = \sigma_0 - \sigma_T(T - T_0)$, where T_0 is the reference temperature of the interface. The acceleration of vibrations can be defined as $\boldsymbol{a} = b\Omega^2 \cos \Omega t\boldsymbol{n}$, then the total acceleration will include both the static gravity acceleration and the vibrational part: $\mathbf{g} \to \mathbf{g} + (b\Omega^2 \cos\Omega t)\mathbf{n}$. Using the total depth of the system $H = H_1 + H_2$ as the non-dimensional scale for length, the non-dimensional depths $h_1 = H_1/H$, $h_2 = H_2/H$ can be got.

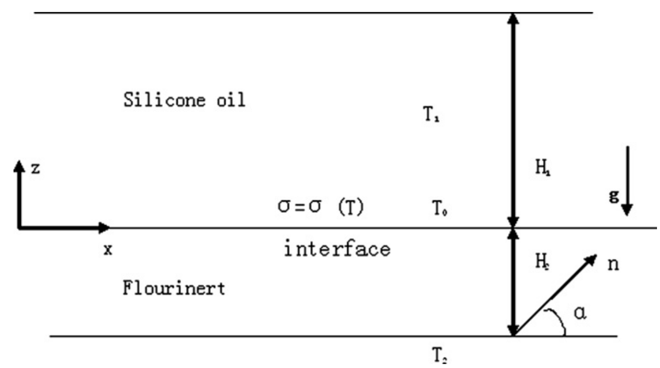

Fig. 1. Schematic diagram of the two-layer fluids.

In this non-inertial system, the equations describing natural convection differ from the standard Boussinesq equations only by the complete acceleration, which now includes both the static gravity acceleration and the vibrational part. Thus, the complete set of equations of natural convection under static gravity and vibration in the framework of Boussinesq approximation has the form as Eqs. (1)–(6)

$$\nabla \cdot \mathbf{v}_m = 0 \tag{1}$$

$$\frac{\partial \mathbf{v}_m}{\partial t} + (\mathbf{v}_m \cdot \nabla)\mathbf{v}_m = -\frac{1}{\rho_m}\nabla p_m + \nu_m \Delta \mathbf{v}_m + g\beta_m T_m \gamma + \beta_m T_m b\Omega^2 \cos \Omega t\mathbf{n} \tag{2}$$

$$\frac{\partial T_m}{\partial t} + \mathbf{v}_m \cdot (\nabla T_m) = \chi_m \Delta T_m \tag{3}$$

$$z = h_1 : u_1 = v_1 = 0, \quad T_1 = \theta_1 \tag{4}$$

$$z = -h_2 : u_2 = v_2 = 0, \quad T_2 = \theta_2 \tag{5}$$

$z = 0:$

$$u_1 = u_2, \quad v_1 = v_2 = 0, \quad T_1 = T_2, \quad \kappa_1 \partial T_1/\partial z = \kappa_2 \partial T_2/\partial z,$$
$$\mu_1 \partial u_1/\partial z - \mu_2 \partial u_2/\partial z = \partial\sigma/\partial x \tag{6}$$

Here the notations are as usual: $\mathbf{v}$ is the velocity, T is the temperature counted from relative zero, ρ is the constant density corresponding to $T = 0$, p is the pressure deviation from the hydrostatic one at the constant density ρ, γ is the unit vector directed vertically upward and v, χ, β are, respectively, the kinematical viscosity, heat conductivity and thermal expansion coefficients. Here $m = 1, 2$.

Then as usual ρ_2, κ_2, χ_2, v_2, β_2, μ_2 are chosen to denote the density, the thermal conductivity, the thermal diffusivity, the kinematical viscosity, the thermal expansion coefficient, the dynamical viscosity of the fluid below, respectively. At first the averaging method (Gershuni and Lyubimov, 1997) will be applied to deal with the governing equations Eq. (1) and boundary conditions Eq. (2). Then H^2/v_2, χ_2/H, ΔT, $\rho_2 v_2 \chi_2/H^2$ are adopted as the scaling factors for time, velocity, temperature and pressure, respectively, and the spatial normal perturbations proportional to $\exp[\lambda t + ikx]$ are employed into the linearization of all the governing equations. At last the complete set of dimensionless equations of convection under static gravity, thermocapillary and thermovibrations in the framework of Boussinesq approximation has the form described as Eqs. (7)–(16).

$$\lambda(D^2 - k^2)V_1 = \upsilon^*(D^2 - k^2)^2 V_1 + \mathrm{Ra_v}\beta^{*2}(-k^2 A_1 \times \cos^2\alpha\theta_1 + ikA_1\cos\alpha DW_1 + k^2 A_1 \times \sin\alpha W_1) - \mathrm{Ra}k^2\beta^*\theta_1 \tag{7}$$

$$\lambda \mathrm{Pr}\theta_1 - A_1 V_1 = \chi^*(D^2 - k^2)\theta_1 \tag{8}$$

$$(D^2 - k^2)W_1 = -(ik\cos\alpha D\theta_1 + k^2\sin\alpha\theta_1) \tag{9}$$

$$\lambda(D^2 - k^2)V_2 = (D^2 - k^2)^2 V_2 + \mathrm{Ra_v}(-k^2 A_2\cos^2\alpha\theta_2 + ikA_2\cos\alpha DW_2 + k^2 A_2\sin\alpha W_2) - \mathrm{Ra}k^2\theta_2 \tag{10}$$

$$\lambda \mathrm{Pr}\theta_2 - A_2 V_2 = (D^2 - k^2)\theta_2 \tag{11}$$

$$(D^2 - k^2)W_2 = -(ik\cos\alpha D\theta_2 + k^2\sin\alpha\theta_2) \tag{12}$$

$$z = h_1 : V_1 = \mathrm{d}V_1/\mathrm{d}z = 0, \quad \theta_1 = 0, \quad W_1 = 0 \tag{13}$$

$$z = -h_2 : V_2 = \mathrm{d}V_2/\mathrm{d}z = 0, \quad \theta_2 = 0, \quad W_2 = 0 \tag{14}$$

$z = 0$:

$$W_1 = W_2 = 0, \quad V_1 = V_2 = 0, \quad \theta_1 = \theta_2, \quad \mathrm{d}V_1/\mathrm{d}z = \mathrm{d}V_2/\mathrm{d}z \tag{15}$$

$$\kappa^*\mathrm{d}\theta_1/\mathrm{d}z = \mathrm{d}\theta_2/\mathrm{d}z, \quad \mu^*\mathrm{d}^2V_1/\mathrm{d}z^2 - \mathrm{d}^2V_2/\mathrm{d}z^2 = k^2\mathrm{Ma}\theta_2 \tag{16}$$

where D is the dimensionless differential operator $D = \mathrm{d}/\mathrm{d}z$. k, λ, α denote the dimensionless wavenumber, the time growth rate, and the angle of vibration, respectively. And ρ^*, κ^*, χ^*, v^*, β^*, μ^* are the ratios between the properties of the upper and the lower layers. Ma is the Marangoni number defined as $\mathrm{Ma} = \sigma_T \Delta T H/\mu_2\chi_2$, and Ra is the Rayleigh number defined as $\mathrm{Ra} = g\beta_2 H^3 \Delta T/v_2\chi_2$. $\mathrm{Ra_v}$, which is defined as $\mathrm{Ra_v} = (\beta_2 b\Omega\Delta TH)^2/2v_2\chi_2$, is the vibrational Rayleigh number. Here $\rho^* = 0.4844$, $\kappa^* = 1.917$, $\chi^* = 2.7616$, $\mu^* = 0.3443$, $\upsilon^* = 0.7143$, $\beta^* = 1.10$, $\mathrm{Pr} = v_2/\chi_2 = 406.0$. We select the relation between the Ra and Ma numbers, $\mathrm{Bond} = \mathrm{Ra}/\mathrm{Ma} = g\beta_2\rho_2 H^2/\sigma_T$.

3. Numerical results

The linear governing Eqs. (7)–(12) together with the boundary conditions Eqs. (13)–(16) are discretized using the spectral numerical method (Tau-Chebychev) (Steven and Qrszag, 1971) and then resolved as the general eigenvalue problem. The complex time growth rate λ is computed in complex double precision.

As it is known in a system there are two convection models, when there is not gravity or the gravity is micro the convection is Marangoni–Benard convection driven by thermocapillary force and the other convection model is the Rayleigh–Marangoni–Benard convection driven by gravity and thermocapillary. Then the results in our presentation will be studied in two parts. The first one is the effects on Marangoni–Benard convection, and then it is about the Rayleigh–Marangoni–Benard convection. Many of our stability results are concerned with the vibrational parameter $\mathrm{Ra_v}$, which depends on the vibrational effects and the temperature difference.

3.1. The effects of thermovibration on the Marangoni–Benard convection

Here two cases will be selected to discuss. As we know at different depth ratio $H_r (H_r = H_1/H_2)$, the neutral curves of Ma–k are different (Colinet et al., 2001).

When the depth ratio $H_r = 0.667$, the results are shown as Figs. 2 and 3. In Fig. 2, it is about the Ma numbers at different wavenumbers under vertical vibrations or hori-

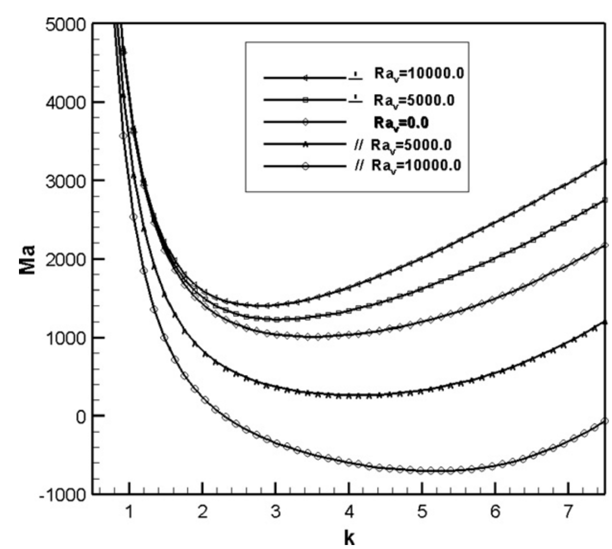

Fig. 2. $H_r = 0.667$. At vertical and horizontal vibrations, the neural curves of Ma–k.

zontal vibrations. At the same angle but different Ra_v, the corresponding Marangoni numbers are different. Each curve has a smallest Marangoni number we call it critical Marangoni number Ma_c. In Fig. 3, it is about the curves of Ma_c–Ra_v, which give the critical Marangoni numbers under different vibrations.

In all the figures, the symbol // means the vibration is horizontal, and ⊥ means the vibration is vertical. And they have the same meaning in the flowing.

The results show that in the Marangoni–Benard convection, when the vibration is horizontal, the system is more apt to lose stability, and when the vibration is vertical the instability of the system is hindered. From Fig. 3, we can say that the critical Marangoni numbers Ma_c, which is the smallest Ma at this depth ratio under all the wavenumbers, change as the same pattern. At the same vibration but different vibrational angles the critical Marangoni numbers are different. The critical Marangoni numbers Ma_c change very little when the vibrations are in the direction of angle 60°.

Then it is the case when the depth ratio H_r is 3.0. The results are shown in Figs. 4 and 5.

The results in Figs. 4 and 5 show that when the wavenumbers are small there are oscillatory regions and the larger the vibrations in horizontal the wider the regions, while the larger the vibrations in vertical the sharper the regions. In the oscillatory regions the image of the eigenvalue λ_i is larger than zero. From Figs. 4 and 5, all the results show that, at small and large k regions except the intermediate regions from the small k to the large k, vertical vibration can enhance the stability of the system and the vibrations in horizontal enable the system easier to lose stable.

From the results above the conclusion will be presented as that in the Marangoni convection the vibrations in vertical can hinder the instability of the system and the system with horizontal vibrations will lose stability in small wavenumbers compared with the system without vibrations. In all the vibrations in arbitrary angles, the vibrations in the

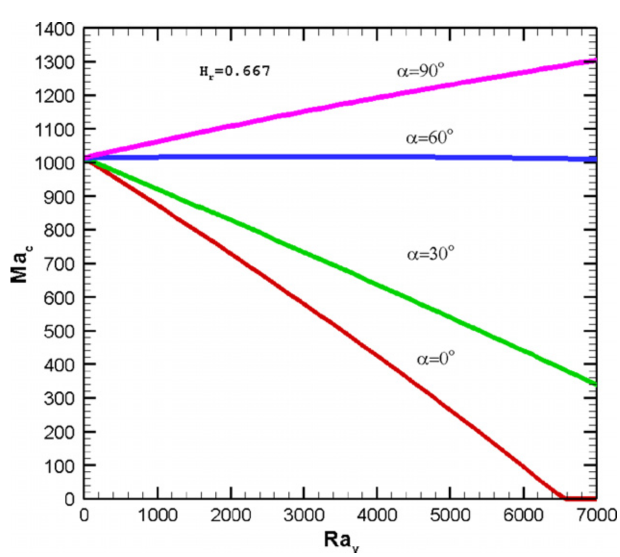

Fig. 3. $H_r = 0.667$. The neutral curves of Ma_c–Ra_v.

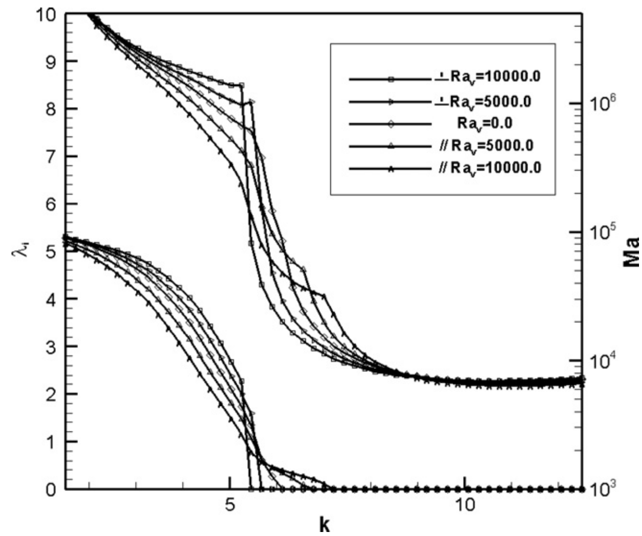

Fig. 4. $H_r = 3.0$. The curves of Ma–k and λ_i–k.

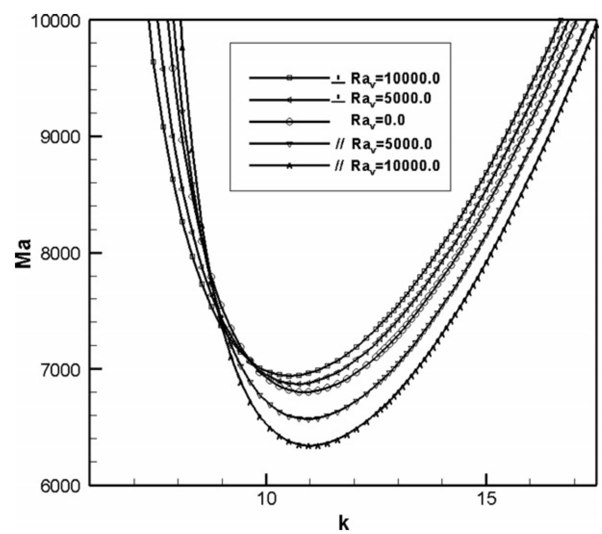

Fig. 5. $H_r = 3.0$. The curves of Ma–k and λ_i–k at large k.

direction of 60° have smallest effects on the instability of the system. In this case, thermovibrations have two effects of vertical vibrations and horizontal vibrations and the two effects in this direction are equal and so the vibrations' effects are very weak.

3.2. The effects of thermovibration on the Rayleigh–Marangoni–Benard convection

Previously a lot of results (Nepomnyashchy et al., 2006; Liu, 2004; Liu et al., 2005; Zhou et al., 2004; Colinet and Legros, 1994; Simanovskii and Nepomnyashchy, Convective Instabilities in Systems with Interface, 1993; Gershuni and Zhukhovitsky, 1982) have been got about the convective instability in a two-layer system, especially about the Hopf bifurcation occurring in the two-layer Rayleigh–Benard and Rayleigh–Marangoni–Benard convective instability. In order to understand the effects of g-jitter on this problem, we study hereafter the effects of vibrations on different depth ratios in the Rayleigh–Marangoni–Benard convection, in comparison with previous works which concerned the Hopf bifurcation with the effects of buoyancy or thermocapillary. The results will be discussed in three parts when the depth of the system H is 4 mm when the Bond number is 6.82.

As it is known the neutral curves in Rayleigh–Marangoni–Benard convection can be described in Fig. 6. The results in the figure show that the neutral curve is divided into three parts and there is an oscillatory gap. So three depth ratios will be selected to discuss, $H_r = 1.5$, $H_r = 2.5$ and $H_r = 4.0$, respectively. The case when the depth ratio $H_r = 2.5$ is in the oscillatory gap.

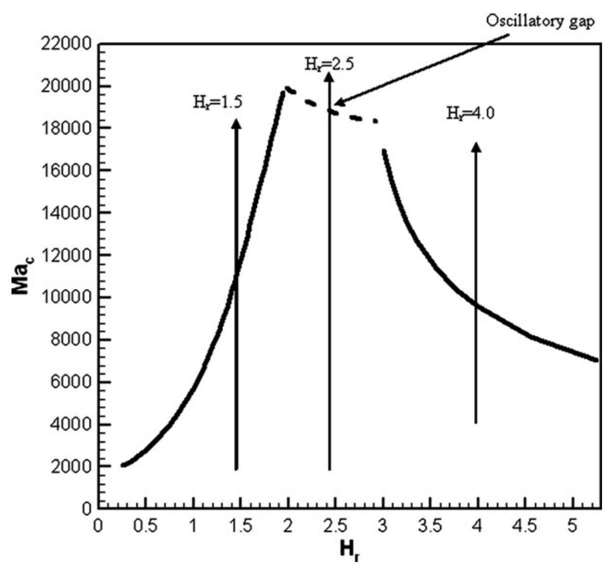

Fig. 6. Neutral curves of Ma_c–H_r in R–M–B.

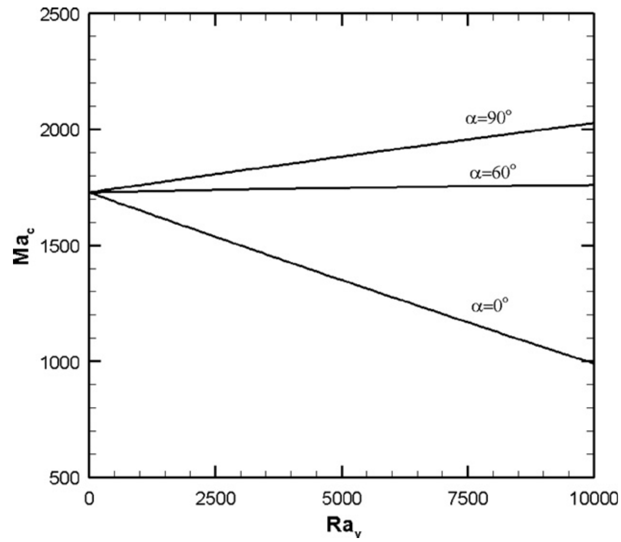

Fig. 8. $H_r = 1.5$. The curves of Ma_c–Ra_v.

At first we will discuss the case when the depth ratio H_r is 1.5 which is a case before the oscillatory gap. In Figs. 7 and 8, the results show that there are oscillatory regions when the wavenumbers are small. The vibrations also have effects on the oscillatory regions. Here we can see in the Rayleigh–Marangoni–Benard convection the vibrations in vertical direction also enhance the stability of the system and the horizontal vibrations can decrease the stability and the vibrations in 60° also have the least effects.

Then the case when the depth ratio $H_r = 2.5$ is discussed, which shown in Fig. 9 and it is in the oscillatory gap. The last part of the results is about the case after the oscillatory gap when the depth ratio $H_r = 4.0$, which shown in Fig. 10. Results in Figs. 9 and 10 show that the

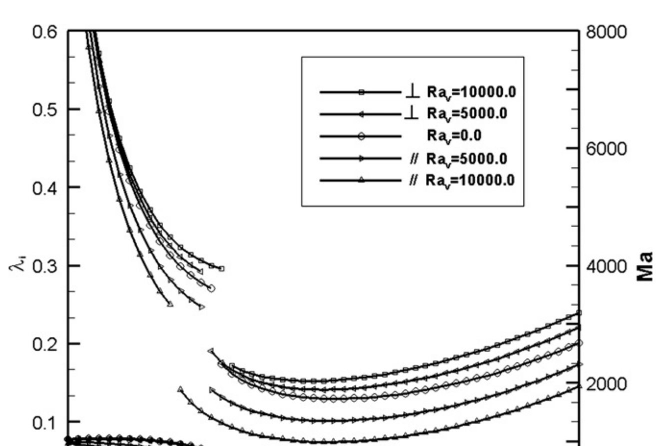

Fig. 7. $H_r = 1.5$. The curves of Ma_c–k, λ_i–k.

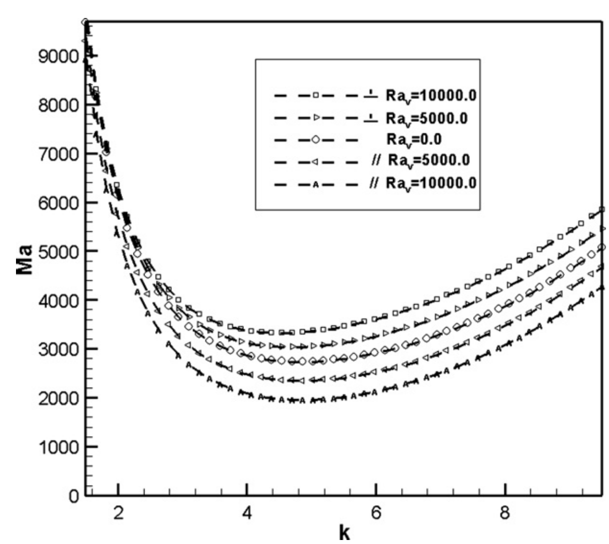

Fig. 9. $H_r = 2.5$. The curves of Ma–k, the dashed lines means it is oscillatory instability.

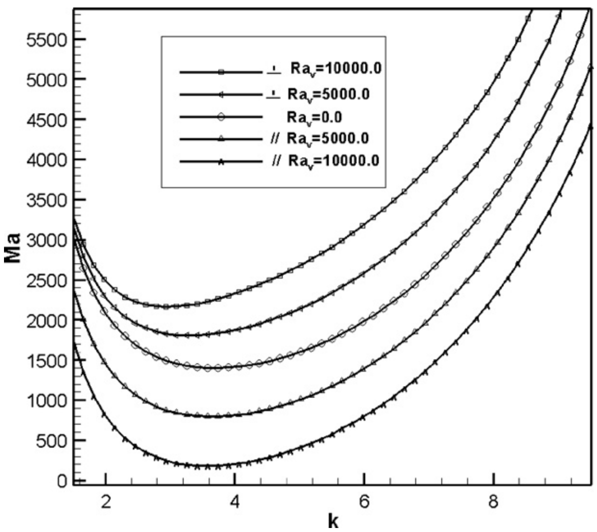

Fig. 10. $H_r = 4.0$. The curves of Ma–k.

vertical vibrations can hinder the instability in the Rayleigh–Marangoni–Benard convection, and the horizontal vibrations have the opposite effects.

All the results above show that thermovibrations have complex effects on the R–M–B convection and the effects depending on the angle of the vibrations mainly. The effects of thermovibrations on the Hopf bifurcations will be discussed elsewhere.

4. Conclusions and discussion

The thermalvibrational instabilities of the Marangoni–Benard convection and Rayleigh–Marangoni–Benard convection in a two-layer liquid system have been studied theoretically in the present paper. The linear instability analysis results show that thermovibration have the similar effects on the Marangoni–Benard convection and the Rayleigh–Marangoni–Benard convection. The vibrations with the direction vertical to the interface can enhance the stability of the two-layers system and damp the convective flow down. Horizontal vibrations enhance the instability of the system and make the system apt to lose stability. When the angle of vibrations is 60°, the thermovibrations have almost no evident effects on the instability of the system. It is noted that thermovibrations can also change the oscillatory regions, especially when the wavenumber of the system is small. In summary, former work on the thermovibrations mainly focused on single layer, and here we presented the effects of thermovibrations on the instability of two-layer system convection.

Acknowledgements

The work presented in the paper was performed with the financial support of the National Natural Science Foundation of China (10772185, 10372105) and the joint Russian–Chinese project (RFBR-04-01-39021, NSFC 103-72-105).

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