China Ocean Engineering, Vol.23, No.1, pp.119 – 132

Physical Modeling and Parametric Study on Two-Degree-of-Freedom VIV of A Cylinder near Rigid Wall*

YANG Bing (杨　兵)[1], GAO Fu-ping (高福平),
LI Dong-hui (李东晖) and WU Ying-xiang (吴应湘)
Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China

(Received 25 May 2008; accepted 18 August 2008)

ABSTRACT

Unlike most previous studies on the transverse vortex-induced vibration(VIV) of a cylinder mainly under the wall-free condition (Williamson & Govardhan, 2004), this paper experimentally investigates the vortex-induced vibration of a cylinder with two degrees of freedom near a rigid wall exposed to steady flow. The amplitude and frequency responses of the cylinder are discussed. The lee wake flow patterns of the cylinder undergoing VIV were visualized by employing the hydrogen bubble technique. The effects of the gap-to-diameter ratio (e_0/D) and the mass ratio on the vibration amplitude and frequency are analyzed. Comparisons of VIV response of the cylinder are made between one degree (only transverse) and two degrees of freedom (streamwise and transverse) and those between the present study and previous ones. The experimental observation indicates that there are two types of streamwise vibration, i.e. the first streamwise vibration (FSV) with small amplitude and the second streamwise vibration (SSV) which coexists with transverse vibration. The vortex shedding pattern for the FSV is approximately symmetric and that for the SSV is alternate. The first streamwise vibration tends to disappear with the decrease of e_0/D. For the case of large gap-to-diameter ratios (e.g. $e_0/D = 0.54 \sim 1.58$), the maximum amplitudes of the second streamwise vibration and transverse one increase with the increasing gap-to-diameter ratio. But for the case of small gap-to-diameter ratios (e.g. $e_0/D = 0.16$, 0.23), the vibration amplitude of the cylinder increases slowly at the initial stage (i.e. at small reduced velocity V_r), and across the maximum amplitude it decreases quickly at the last stage (i.e. at large V_r). Within the range of the examined small mass ratio ($m^* < 4$), both streamwise and transverse vibration amplitude of the cylinder decrease with the increase of mass ratio for the fixed value of V_r. The vibration range (in terms of V_r) tends to widen with the decrease of the mass ratio. In the second streamwise vibration region, the vibration frequency of the cylinder with a small mass ratio (e.g. $m_x^* = 1.44$) undergoes a jump at a certain V_r. The maximum amplitudes of the transverse vibration for two-degree-of-freedom case is larger than that for one-degree-of-freedom case, but the transverse vibration frequency of the cylinder with two degrees of freedom is lower than that with one degree of freedom (transverse).

Key words: *vortex-induced vibration*; *cylinder near a wall*; *two degrees of freedom*; *steady flow*; *gap-to-diameter ratio*; *mass ratio*

1. Introduction

The vortex-induced vibration (VIV) of structures has received much attention mainly owing to its practical significance in many branches of engineering. Several reviewers have discussed this problem

---

* This work was financially supported by the Eleventh Five-Year Plan of Chinese Academy of Sciences (Grant No. KJCX2-YW-L02) and the National Natural Science Foundation of China (Grant No. 50509022)

1 Corresponding author. E-mail: byang@imech.ac.cn

(Sarpkaya, 1979; Griffin and Ramberg, 1982; Bearman, 1984; Parkinson, 1989; Sumer and Fredsoe, 1997; Williamson and Govardhan, 2004). Most of the existing studies focused on the transverse dynamic responses of a cylinder undergoing vortex-induced vibration with one degree of freedom. Feng (1968) studied the vortex-induced vibration of a cylinder with a single degree of freedom in the transverse flow direction, and demonstrated that the vortex shedding frequency is not controlled by Strouhal law but locked into the vibration frequency of the cylinder when the vortex-shedding frequency at the wake region of the cylinder is close to the natural frequency of the cylinder. Anand (1985) investigated the response of a cylinder exposed to water flow and found that the vibration frequency of the cylinder placed in the water increases monotonously with the reduced velocity in the lock-in range. The branches of amplitude response have been investigated in detail by Brika and Laneville (1993), Khalak and Williamson (1996; 1997a, 1997b; 1999), and Govardhan and Williamson (2000).

Until now, only few research has involved the vortex-induced vibration of the cylinder with two degrees of freedom (Williamson and Govardhan, 2004). Moe and Wu (1990) found that the position of the maximum response amplitude of the cylinder with two degrees of freedom shifts to a higher value of reduced velocity and the maximum amplitude also reaches a little higher value under the condition of different mass ratios and natural frequencies in the streamwise and transverse direction compared with those of transverse motion. Sarpkaya (1995) also drew a conclusion similar to that reported by Moe and Wu (1990). Jauvtis and Williamson (2003) studied the response of an elastically mounted cylinder with two degrees of freedom at a low mass-damping parameter. They found that the freedom to oscillate in-line with the flow affects the transverse vibration very slightly for $m^* \geq 5$, m^* being the ratio between oscillating structure mass and displaced fluid mass. The experimental results obtained by Williamson and Jauvtis (2004) indicate that there is much difference between the response of a cylinder with one degree of freedom and that with two degrees of freedom for the case of $m^* < 6$. In fact, under some circumstances the cylinder will be close to a wall (e.g. the pipelines are installed on the seabed). However, VIV response of a cylinder with two degrees of freedom near a wall has not been investigated extensively. For this, Tsahalis (1984) studied the streamwise and transverse vibration responses of a flexible cylinder in steady currents, and found that the proximity to the plane boundary has a pronounced effect not only on the transverse amplitude response but also on the streamwise one. On the basis of the experiments on an elastically-mounted rigid cylinder, Yang *et al*. (2008) mainly discussed the correlation between the vortex shedding frequency and the vibration frequency of the cylinder. Until now, the VIVs of a near-wall cylinder with two degrees of freedom have not been well understood. For example, the flow characteristics around the cylinder close to a wall, the effect of the cylinder's proximity to the wall upon the vibration amplitude and vibration frequency of the cylinder for the case of a small value of e_0/D (especially at $e_0/D < 1$), the influence of the mass parameter upon the dynamic response of the near-wall cylinder with two degrees of freedom, and so on, need to be further investigated.

In the present study, the two-degree-of-freedom VIV of a cylinder near a rigid wall is physically modeled with a newly designed hydro-elastic apparatus. By employing the hydrogen bubble technique, the lee wake flow patterns of the cylinder undergoing VIV are visualized. Based on the similarity analyses, a parametric study is conducted to investigate the effects of the gap-to-diameter ratio and the mass ratio on two-degree-of-freedom VIV responses of the near-wall cylinder. Moreover, a comparison is made between the present results and the previous ones.

2. Experimental Details

A special hydro-elastic apparatus in the conjunction with a flume was used for the present experiments (Fig. 1). The flume, 0.5 m in width, 0.6 m in height and 19 m in length, can produce steady currents with the velocity up to 0.6 m/s. The water depth was kept at 0.4 m. The test cylinders with a diameter of 0.032 m and 0.050 m were elastically supported by vertical and horizontal springs, as depicted in Fig. 1. With a length of 0.47 m, the cylinder has a smooth surface. The details describing the apparatus can be found in the work by Yang *et al*. (2006). The laser displacement transducers were used to measure the vibration displacements of the cylinder. The one with a dynamic resolution of 0.25 mm was used for measurement of the vertical displacement and the one with 0.025 mm for the horizontal displacement. The natural frequency of the cylinder (f_n) was obtained by spectrum analyses of free-decay tests in still water, and the structural damping of the cylinder was measured with the method of free-decay tests, i.e. the cylinder was given a prescribed displacement and then released in still water. The structural damping factor (ζ) can be estimated with $\zeta = \ln(A_i/A_{i+n})/(2\pi n)$, where A_i is the initial amplitude of cylinder vibrations, and A_{i+n} is the amplitude after n cycles (Blevins, 1977). In the present study, the value of n is specified as 5. A micro-propeller current meter was adopted to measure the flow velocity.

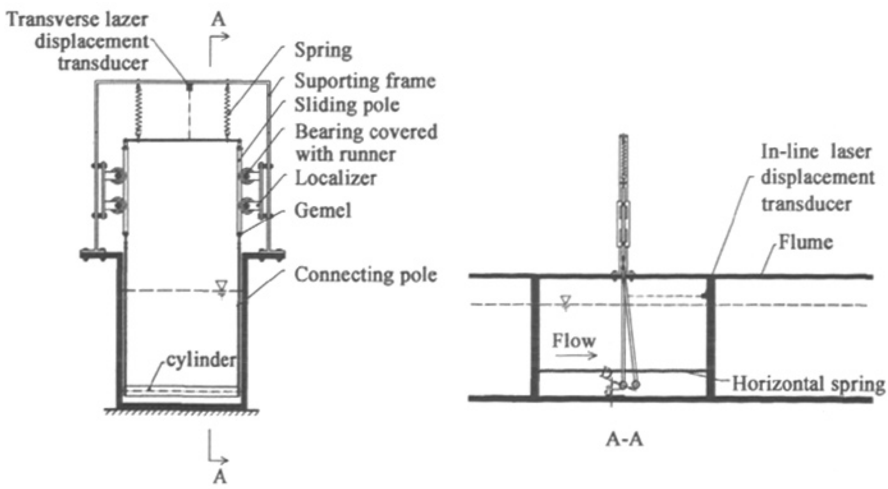


Fig. 1. Schematic diagram of the experimental apparatus.

The hot film current meter was employed to measure the pulse velocity in the wake region. In every group of experiments, the flow velocity is added gradually when other parameters (e.g. the diameter of the cylinder, the natural frequency of the cylinder) are fixed. Thus, at each flow velocity, the vibration amplitude, vibration frequency of the cylinder and the flow field around the cylinder can be obtained.

3. Dimensional Analysis

When a near-wall cylinder with two degrees of freedom is exposed to steady flow, there exist dynamic interaction between cylinder, flow and wall. The physical quantities influencing the dynamic responses of the cylinder under the action of steady flow are listed in Table 1.

Table 1 Physical quantities related to dynamic interactions between cylinder, flow and wall

	Physical quantity	Symbol	Dimension
Relative to flow	Mass density of fluid	ρ	ML^{-3}
	Dynamic viscosity of fluid	μ	$ML^{-1}T^{-1}$
	Undisturbed incoming flow velocity	U	LT^{-1}
Relative to the cylinder	Diameter of the cylinder	D	L
	Relative roughness of the cylinder's surface	κ	1
	Mass of the cylinder per meter	m	M
	Natural frequency of the cylinder in still water	f_n	T^{-1}
	Structural damping factor of the cylinder	ζ	1
	Initial gap between cylinder and plane boundary	e_0	L

Thus, the vibration amplitude and frequency of the cylinder undergoing vortex-induced vibration can be written as:

$$\begin{Bmatrix} A \\ f \end{Bmatrix} = \phi(\rho, \mu, U, D, \kappa, m, f_n, \zeta, e_0), \tag{1}$$

where A is the vibration amplitude of the cylinder and f is the vibration frequency. According to PI theorem, the nine physical quantities in Table 1 can be combined as six dimensionless parameters, then Eq. (1) can be rewritten as:

$$\begin{Bmatrix} A/D \\ f/f_n \end{Bmatrix} = \phi(Re, V_r, \kappa, m^*, K_s, e_0/D), \tag{2}$$

in which Re is the Reynolds number, V_r is the reduced velocity, m^* is the mass ratio, and K_s is the stability parameter. They are defined respectively as:

$$Re = \frac{\rho UD}{\mu}, \quad V_r = \frac{U}{f_n D}, \quad m^* = \frac{4m}{\pi\rho D^2} \quad \text{and} \quad K_s = \frac{4(m+m_a)\zeta}{\pi\rho D^2}, \tag{3}$$

where m_a is the added mass and $m_a = C_A m_d$; C_A is the coefficient of the added mass, C_A is set to be 1.0 and $m_d = (\pi\rho D^2)/4$ in the present study.

4. Results and Discussions

4.1 Typical Phenomenon of VIV Responses of the Near-Wall Cylinder

Fig. 2 presents the typical amplitude and frequency response of the cylinder with two degrees of freedom. The experimental parameters are set as follows: $D = 0.05$ m, $e_0/D = 1.02$, $m_x^* = 1.87$, $\zeta_x = 0.0392$, $K_{sx} = 0.1125$, $m_y^* = 2.11$, $\zeta_y = 0.0681$, $K_{sy} = 0.2118$, and $f_{nx}/f_{ny} = 0.75$. The subscripts x and y indicate streamwise and transverse direction, respectively. It can be seen from the figure that, when the reduced velocity V_r is in the range of $1.5 < V_r < 2.1$, the streamwise vibration with a maximum vibration amplitude of $A_{xm} \approx 0.11D$ occurs, which is defined as the First Streamwise Vibration (FSV) in the present study. In the range of $4.2 < V_r < 11.1$, the Second Streamwise Vibration (SSV) appears and the maximum vibration amplitude ($A_{xm} \approx 0.56D$) occurs at $V_r = 7.9$. When the reduced velocity V_r is about 3.3, the transverse vibration begins and the vibration amplitude increases with the increasing V_r until $V_r = 6.2$ at which the maximum vibration amplitude ($A_{ym} \approx 1.02D$) occurs. When $V_r > 6.2$, the vibration amplitude begins decreasing and at $V_r = 8.8$ the vibration amplitude can be reduced to $0.28D$. The maximum amplitude of transverse vibration is 1.8 times as much as that of SSV and 9.3 times as that of FSV.

For the vibration frequency shown in Fig. 2b, the first streamwise vibration frequency is very close to the natural frequency of the cylinder, which locates in the range of $1.03f_n \sim 1.1f_n$. It is also indicated from the figure that the frequency of transverse vibration and second streamwise vibration increases slightly with increasing V_r. It is noted that the aforementioned conclusions are consistent with the results reported by Yang *et al*. (2008).

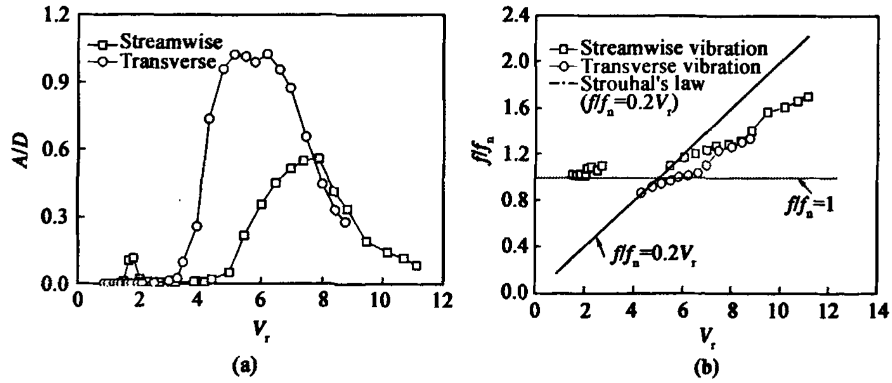


Fig. 2. The dynamic response of the cylinder. (a) Transverse and streamwise amplitude response versus the reduced velocity; (b) Transverse and streamwise frequency response versus the reduced velocity. ($D = 0.05$ m, $e_0/D = 1.02$, $m_x^* = 1.87$, $\zeta_x = 0.0392$, $m_y^* = 2.11$, $\zeta_y = 0.0681$, $K_{sx} = 0.1125$, $K_{sy} = 0.2118$, and $f_{nx}/f_{ny} = 0.75$).

4.2 Lee Wake Flow Pattern Around the Vibrating Cylinder

The visualization of wake flow pattern around the cylinder undergoing vortex-induced vibration was conducted with the method of hydrogen bubble. Fig. 3 shows the vortex shedding mode around the cylinder undergoing only streamwise vibration, whose vibration amplitude is $0.02D$ ($e_0/D = 1.02$). It is observed from the figure that the vortex shedding pattern is approximately symmetric during the course of streamwise vibration, i.e. the vortices from the top to bottom surfaces of the cylinder shed simultaneously in a cycle of vibration.

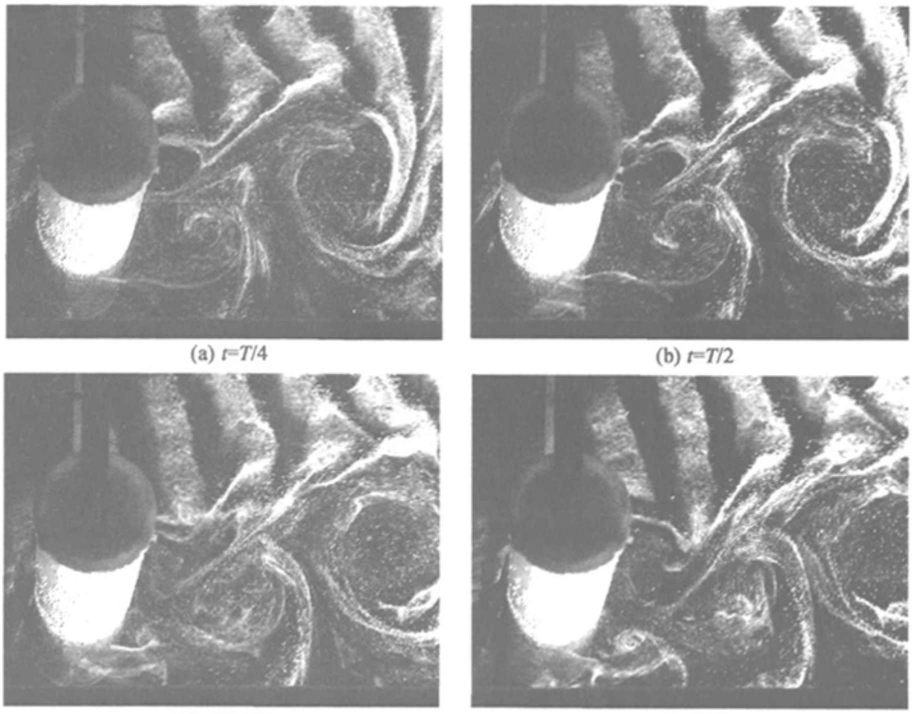
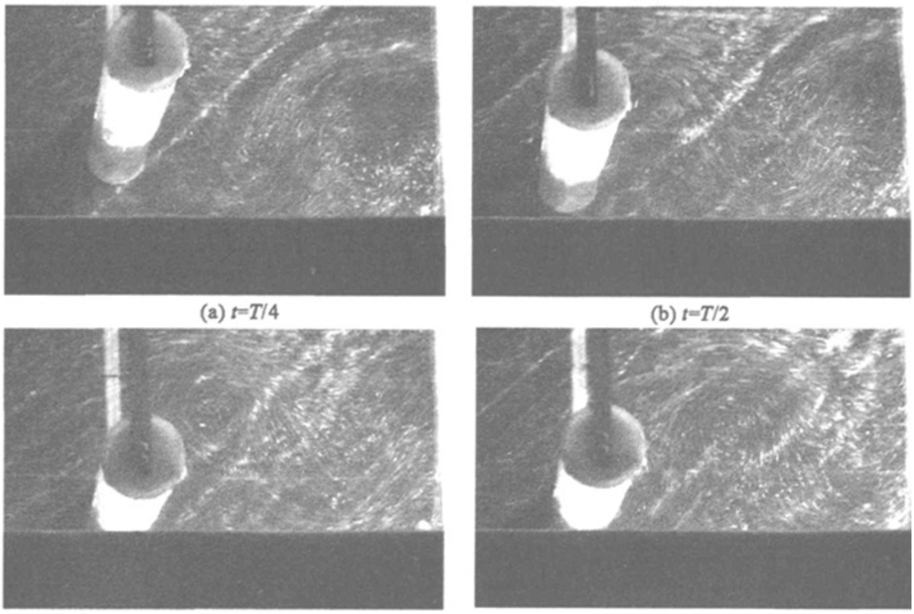

(a) $t=T/4$ (b) $t=T/2$ (c) $t=3T/4$ (d) $t=T$

Fig. 3. The visualization of wake flow around the cylinder undergoing only streamwise vortex-induced vibration ($A = 0.02D$, $V_r = 2$, $e_0/D = 1.02$).

Fig. 4 gives the typical vortex shedding pattern of the cylinder near a wall undergoing streamwise and transverse vibrations. The streamwise vibration amplitude of the cylinder with a gap-to-diameter ratio $e_0/D = 1.02$ is $0.22D$ and the transverse one is $0.74D$. It is indicated from the figure that the vortex shedding mode for the case of the cylinder undergoing VIV in two directions is different from that for the cylinder with only streamwise vibration. The shedding mode is not symmetric but alternate, i.e. the vortices from the top to bottom surfaces of the cylinder shed by turns in a cycle of vibration.

Considering the characteristics of vibration amplitude and vibration frequency of the cylinder, and the vortex shedding mode in the range of the first streamwise vibration, the reason of occurrence of the first streamwise vibration may be explained as follows: the fluctuation amplitude of the streamwise drag force acting on the cylinder is noticeable due to the symmetry of shedding vortices. Moreover, the fluctuation frequency of the streamwise drag force is close to the natural frequency of the cylinder. Therefore, the first streamwise vibration (FSV) appears. On the other hand, in the range of FSV the flow velocity is small and the magnitude of streamwise drag force is also small. Thus, the vibration amplitude of FSV is small. As for the SSV, the vortex shedding mode is changed to alternate shedding mode in this range of vibration, and the magnitude of fluctuating drag force increases due to the increase of flow velocity and the influence of transverse vibration, which leads to the augmentation of the SSV's vibration amplitude.



(a) $t=T/4$ (b) $t=T/2$

(c) $t=3T/4$ (d) $t=T$

Fig. 4. The visualization of wake flow around the cylinder undergoing transverse and streamwise vortex-induced vibrations ($V_{rx}=5.4$, $A_x=0.22D$, $V_{ry}=4.3$, $A_y=0.74D$, $e_0/D=1.02$).

4.3 Effects of Gap-to-Diameter Ratio (e_0/D)

For the investigation of the effect of the gap-to-diameter ratio (e_0/D) on the vibration amplitude and frequency of the cylinder, experiments are conducted under the condition of various gaps between the cylinder and the wall. The amplitude responses of vortex-induced vibration of the cylinder for various gap-to-diameter ratios are plotted in Fig. 5. Fig. 5a illustrates that the first streamwise vibration tends to disappear with increasing e_0/D. For the case of large gap-to-diameter ratios (e.g. $e_0/D=0.54\sim1.58$), the maximum amplitudes of the second streamwise vibration and transverse one increase with the increasing gap-to-diameter ratio. It can be seen from Fig. 5 that for the case of small gap-to-diameter ratios (e.g. $e_0/D=0.16$, 0.23), the variation of vibration amplitude of the cylinder with the reduced velocity (V_r) is distinctly different from that of large gap-to-diameter ratios case, which are both true for the second streamwise and transverse vibrations. For the small gap-to-diameter ratios case, the vibration amplitude of the cylinder increases slowly until the maximum amplitude at the initial stage (i.e. small V_r), and then decreases quickly at the last stage (i.e. large V_r).

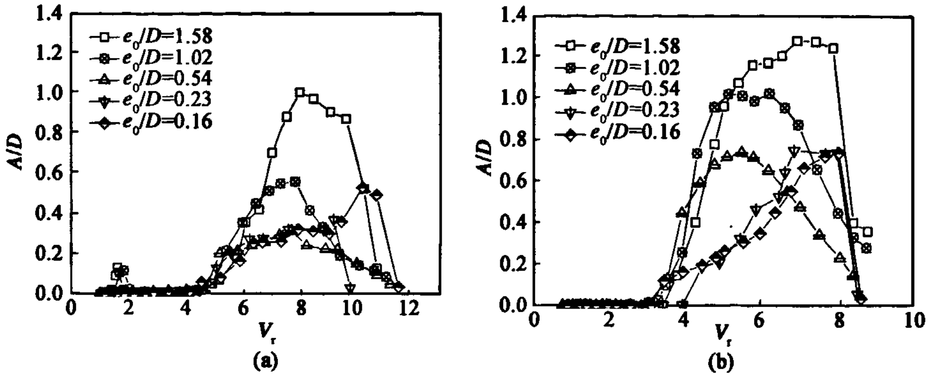


Fig. 5. The amplitude response of vortex-induced vibration of the cylinder with two degrees of freedom for various gap-to-diameter ratios ($D = 0.05$ m, $m_x^* = 1.87$, $\zeta_x = 0.0392$, $m_y^* = 2.11$, $\zeta_y = 0.0681$, $f_{nx}/f_{ny} = 0.75$, $K_{sx} = 0.1125$, and $K_{sy} = 0.2118$).

The effect of the gap-to-diameter ratio on the vibrating frequency of the cylinder is illustrated in Fig. 6. It is indicated from Fig. 6a that the gap-to-diameter ratio has a slight effect on the vibration frequency of the first streamwise vibration. In the range of the second streamwise vibration, the vibration frequency undergoes a jump at a certain value of V_r for the case of certain gap-to-diameter ratio (e.g. $e_0/D = 1.58$). The transverse vibration frequency of the cylinder increases steadily with V_r for various gap-to-diameter ratios in this study and does not undergo a jump. It also can be seen that the increasing rate of the vibration frequency of the cylinder with the increase of V_r for the case of the small gap-to-diameter ratio is larger than that for the large one. With the same value of V_r, the smaller the gap-to-diameter ratio is, the larger the vibration frequency of the cylinder will be.

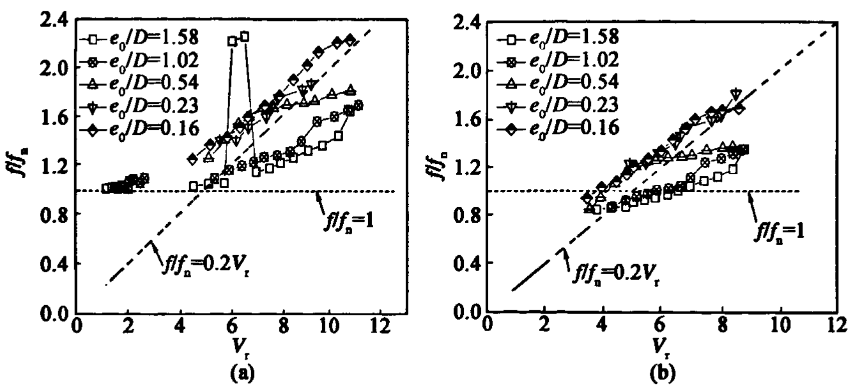


Fig. 6. The frequency response of vortex-induced vibration of the cylinder with two degrees of freedom for various gap-to-diameter ratios ($D = 0.05$ m, $m_x^* = 1.87$, $\zeta_x = 0.0392$, $m_y^* = 2.11$, $\zeta_y = 0.0681$, $f_{nx}/f_{ny} = 0.75$, $K_{sx} = 0.1125$, and $K_{sy} = 0.2118$).

4.4 Effects of Mass Ratio (m^*)

Figs. 7 and 8 present the influence of the mass ratio on the vibration amplitude and frequency of the cylinder with $e_0/D = 0.95$. The diameter of the cylinder with $f_{nx} = f_{ny}$ is 0.032 m, and the structural damping factors are $\zeta_x = 0.0568$ and $\zeta_y = 0.1102$. It can be seen from Fig. 7 that the streamwise and transverse vibration amplitudes of the cylinder with the small mass ratio is larger than those with the large one for the fixed value of V_r. The vibration range in terms of V_r tends to widen with the decrease of the mass ratio. Fig. 8a shows that the vibration frequency of the cylinder in the range of the first streamwise vibration varies slightly with the mass ratio. In the second streamwise vibration region, the vibration frequency of the cylinder with the small mass ratio (e.g. $m_x^* = 1.44$) undergoes a jump at a certain V_r. At the position of the jump, the vibration frequency of the cylinder is much larger than the Strouhal's frequency. At the non-jump position, the vibration frequency tends to increase with the increase of the mass ratio. For the transverse vibration frequency, it varies continuously and does not undergo the phenomenon of jump.

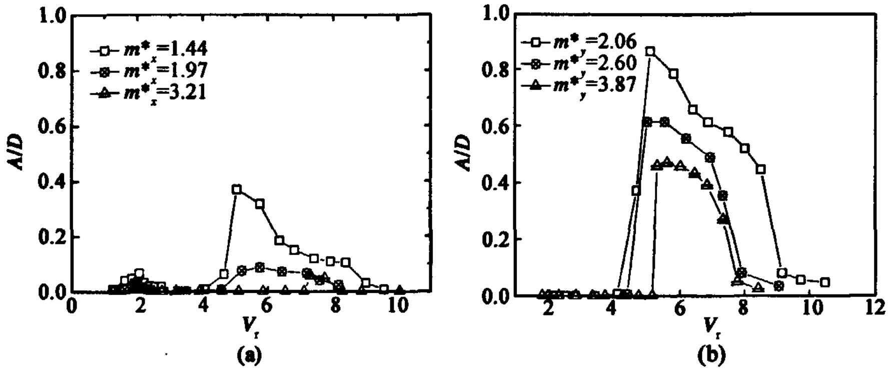


Fig. 7. The amplitude response of the vortex-induced vibration of the cylinder with two degrees of freedom for various mass ratios ($D = 0.032$ m, $e_0/D = 0.95$, $\zeta_x = 0.0568$, $\zeta_y = 0.1102$, and $f_{nx}/f_{ny} = 1$).

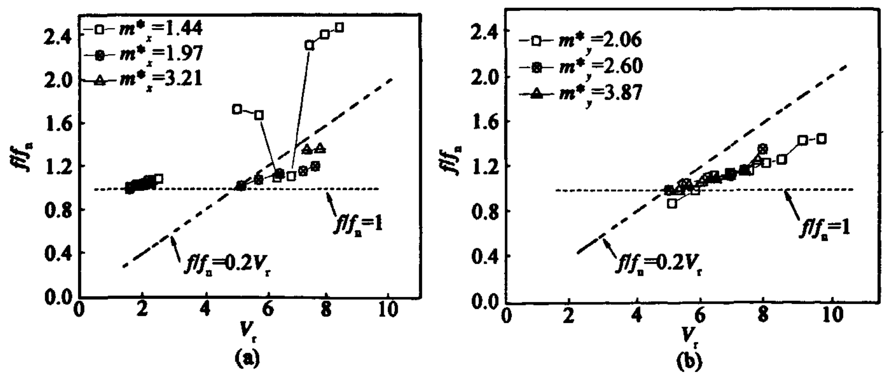


Fig. 8. The frequency response of vortex-induced vibration of the cylinder with two degrees of freedom for various mass ratios ($D = 0.032$ m, $e_0/D = 0.95$, $\zeta_x = 0.0568$, $\zeta_y = 0.1102$, and $f_{nx}/f_{ny} = 1$).

4.5 Comparison of Dynamic Responses of A Cylinder Between One Degree and Two Degrees of Freedom

For the wall-free cylinder, the results obtained by Moe and Wu (1990) with $f_{nx}/f_{ny} = 2$ and those obtained by Sarpkaya (1995) with $f_{nx}/f_{ny} = 1$ show that the maximum amplitudes of transverse vibration of the cylinder undergoing VIV in two directions occur at larger value of V_r, and have slightly higher values of amplitude than those of only transverse vibration. The experimental results obtained by Williamson and Jauvtis (2004) indicate that there is much difference between the response of the wall-free cylinder with one degree of freedom and that with two degrees of freedom for the case of $m^* < 6$. Fig. 9 presents the transverse amplitude and frequency response of the cylinder with one degree of freedom (transverse direction) and that with two degrees of freedom for the case of $e_0/D = 1.00$. The mass ratio of the cylinder undergoing only transverse vibration with a damping factor $\zeta = 0.0944$ is 1.36, and those undergoing VIV of two direction vibrations with the damping factors $\zeta_x = 0.0283$ and $\zeta_y = 0.0944$ are $m_x^* = 1.16$ and $m_y^* = 1.36$, respectively. It is indicated from Fig. 9a that the maximum amplitude of the transverse vibration for two degrees of freedom case is larger than that for one degree of freedom case, which is consistent with the results obtained by Moe and Wu (1990) and Sarpkaya (1995) for the wall-free cylinder case. The transverse vibration frequency of the cylinder with two degrees of freedom is lower than that with one degree of freedom (transverse direction), which can be observed in Fig. 9b.

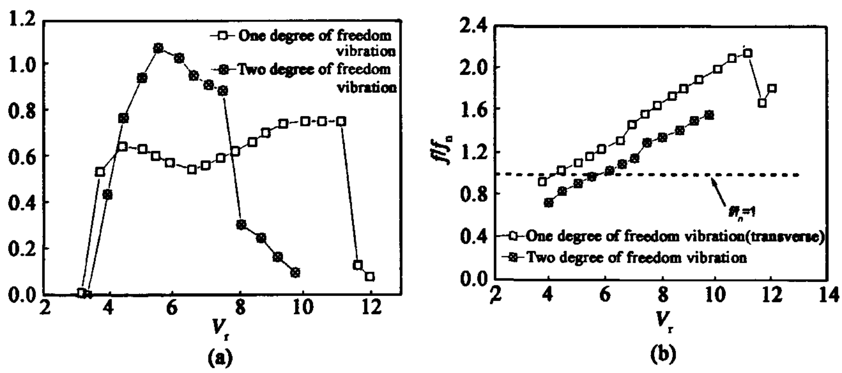
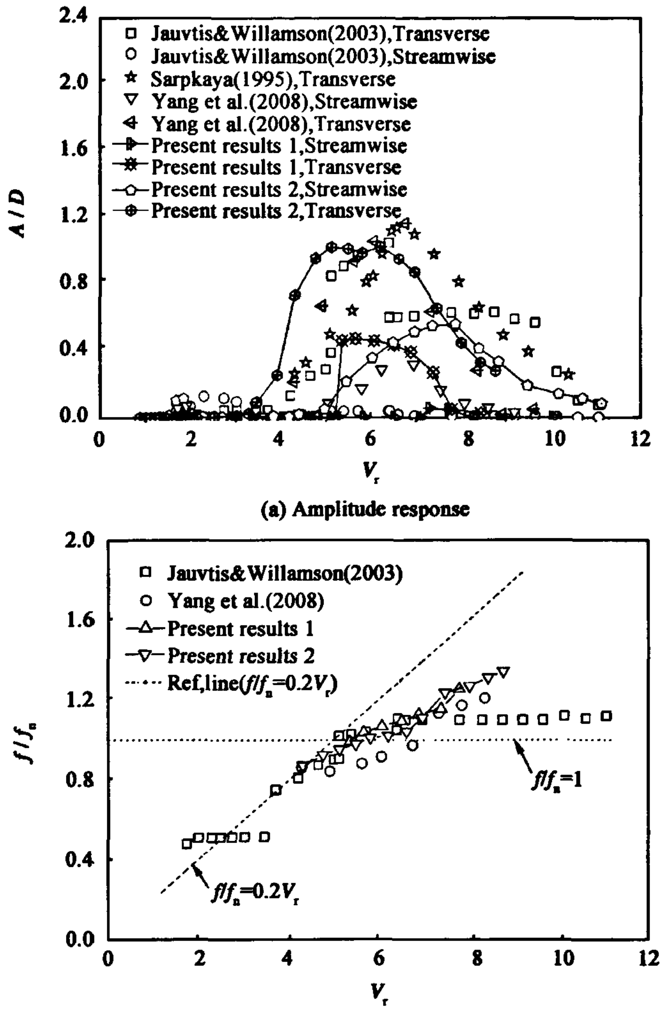


Fig. 9. Comparison of dynamic response of the cylinder between one degree (only transverse) and two degrees of freedom (streamwise and transverse) ($D = 0.050$ m, $e_0/D = 1.00$) (one degree: $m^* = 1.36$, $\zeta = 0.0944$; two degrees: $m_y^* = 1.36$, $\zeta_y = 0.0944$, $K_{sy} = 0.2228$, $m_x^* = 1.16$, $\zeta_x = 0.0283$, $K_{sx} = 0.0611$, and $f_{nd} = f_{nx} = f_{ny}$).

4.6 Comparisons of the Present Results with Previous Ones

Comparisons are made between the present experimental results and those of the vortex-induced vibration of a wall-free cylinder with two degrees of freedom obtained by Sarpkaya (1995), Jauvtis and Williamson (2003) and Yang *et al*. (2008), as shown in Fig. 10. In the results obtained by Sarpkaya (1995), the mass parameter and damping factor are not described, and only the natural frequen-



(a) Amplitude response

(b) Frequency response

Fig. 10. Comparisons between the present results and previous ones (Jauvtis and Williamson (2003): $f_{nx}/f_{ny} = 1$, $m^* = 6.9$, $(m^* + C_A) = 0.0115$; Sarpkaya (1995): $f_{nx}/f_{ny} = 1$; Yang *et al*. (2008): $m_x^* = 1.97$, $\zeta_x = 0.0431$, $m_y^* = 2.60$, $\zeta_y = 0.0973$, $e_0/D = 2.68$, and $f_{nx}/f_{ny} = 1$; Present study 1: $D = 0.032$ m, $e_0/D = 0.95$, $m_x^* = 3.21$, $\zeta_x = 0.0568$, $K_{sx} = 0.2391$, $m_y^* = 3.87$, $\zeta_y = 0.1102$, $K_{sy} = 0.5366$, and $f_{nx}/f_{ny} = 1$; Present study 2: $D = 0.05$ m, $e_0/D = 1.02$, $m_x^* = 1.87$, $\zeta_x = 0.0392$, $K_{sx} = 0.1125$, $m_y^* = 2.11$, $\zeta_y = 0.0681$, $K_{sy} = 0.2118$, and $f_{nx}/f_{ny} = 0.75$).

cy ratio $f_{nx} = f_{ny} = 1$ is provided. The experimental parameters in the results obtained by Jauvtis and Williamson (2003) are $m^* = 6.9$ and $(m^* + C_A)\ \zeta = 0.0115$, and those reported by Yang *et al*. (2008) are $m_x^* = 1.97$, $\zeta_x = 0.0431$, $m_y^* = 2.60$, $\zeta_y = 0.0973$, $e_0/D = 2.68$, $f_{nx}/f_{ny} = 1$, $K_{sx} = 0.1280$, and $K_{sy} = 0.3503$. The experimental parameters in the present study of first group are $D = 0.032$ m, $e_0/D = 0.95$, $m_x^* = 3.21$, $\zeta_x = 0.0568$, $K_{sx} = 0.2391$, $m_y^* = 3.87$, $\zeta_y = 0.1102$,

$K_{sy} = 0.5366$ and $f_{nx}/f_{ny} = 1$. The experimental parameters in the present study of the second group are $D = 0.05$ m, $e_0/D = 1.02$, $m_x^* = 1.87$, $\zeta_x = 0.0392$, $K_{sx} = 0.1125$, $m_y^* = 2.11$, $\zeta_y = 0.0681$, $K_{sy} = 0.2118$ and $f_{nx}/f_{ny} = 0.75$. It is indicated from Fig. 10a that the first streamwise vibration (FSV) mode defined in the present study is consistent with the SS (Streamwise Symmetric) vibration mode defined by Jauvtis and Williamson (2003). But the SA (Streamwise Antisymmetric) mode reported by Jauvtis and Williamson (2003) is not detected in the present study. The reason may be the cylinder's proximity to the wall in the present experiments. It can also be seen from the figure that the magnitude of the maximum amplitude of transverse vibration of the cylinder with small stability parameter in the present study for the second group is equivalent with those obtained by Sarpkaya (1995), Yang *et al*. (2008), and Jauvtis and Williamson (2003). The maximum transverse vibration amplitude for the large stability in the present study for the first group is much lower than those obtained by Sarpkaya (1995), Yang *et al*. (2008), and Jauvtis and Williamson (2003). The main reason may be the large stability parameter of the cylinder in the first group. It is observed from Fig. 10a that the lower branch of the transverse amplitude response reported by Jauvtis and Williamson (2003) is not clear in the results obtained by Sarpkaya (1995), Yang *et al*. (2008), and the present study. From the comparison of transverse vibration frequency between Jauvtis and Williamson (2003), and the present study in Fig. 10b, there exists a little difference of the trend of variation with V_r for the vibration frequency of the cylinder among them. The increase rate of the transverse vibration frequency of the cylinder in the present study is obviously larger than that obtained by Jauvtis and Williamson (2003), which may be due to the small mass ratio in the present study. But for the variation of vibration frequency between Yang *et al*. (2008) and the present study, there exists a similar trend.

5. Conclusions

Unlike most previous studies, which focused on the transverse vortex-induced vibration of a cylinder and concerned little with the vortex-induced vibration of a cylinder with two degrees of freedom (Williamson and Govardhan, 2004), this paper investigates experimentally the vortex-induced vibration of a cylinder with two degrees of freedom near a rigid wall. On the basis of the experimental observation and the results of a series of tests, the following conclusions can be drawn.

(1) There exist two types of streamwise vibration for a two-degree-of-freedom cylinder undergoing vortex-induced vibration near a wall, i.e. the first streamwise vibration (FSV) with a small amplitude which is consistent with the SS (Streamwise Symmetric) vibration mode defined by Jauvtis and Williamson (2003), and the second streamwise vibration (SSV) which coexists with a transverse vibration. The vortex shedding pattern for the FSV is approximately symmetric and that for the SSV is alternate.

(2) The first streamwise vibration tends to disappear with the decrease of e_0/D. For the case of large gap-to-diameter ratios (e.g. $e_0/D = 0.54 \sim 1.58$), the maximum amplitudes of the second streamwise vibration and transverse one increase with increasing gap-to-diameter ratio. For the case of small gap-to-diameter ratios (e.g. $e_0/D = 0.16, 0.23$), the vibration amplitude of the cylinder in-

creases slowly until the maximum amplitude at the initial stage (i.e. at small V_r), and then decreases quickly at the last stage (i.e. large V_r). During the course of SSV, the vibration frequency undergoes a jump at a certain value of V_r for the case of a certain gap-to-diameter ratio (e.g. $e_0/D = 1.58$). For the same value of V_r, the vibration frequency increases with the decrease of the gap-to-diameter ratio.

(3) Within the range of the examined small mass ratio ($m^* < 4$), both streamwise and transverse vibration amplitudes of the cylinder with a small mass ratio is larger than that with a large one for the same value of V_r. The vibration range (in terms of V_r) tends to widen with the decrease of the mass ratio. The vibration frequency of the cylinder in the range of the first streamwise vibration varies slightly with the mass ratio. In the second streamwise vibration region, the vibration frequency of the cylinder with a small mass ratio (e.g. $m_x^* = 1.44$) may undergo a jump at a certain V_r.

(4) The maximum amplitude of the transverse vibration for the two-degree-of-freedom vibration case is larger than that for the only transverse VIV. However, the transverse vibration frequency of the cylinder with two degrees of freedom is somewhat lower than that for the one-degree-of-freedom VIV (i.e. only transverse vibration).

References

Anand, N. M., 1985. *Free span vibrations of submarine pipelines in steady and wave flows*, Ph.D Thesis, The Norwegian Institue of Technology, Trondheim, Norway.

Bearman, P. W., 1984. Vortex shedding from oscillating bluff bodies, *Ann. Rev. Fluid Mech.*, **16**, 195 ~ 222.

Brika, D. and Laneville, A., 1993. Vortex-induced vibrations of a long flexible circular cylinder, *J. Fluid Mech.*, **250**, 481 ~ 508.

Blevins, R. D., 1977. *Flow-Induced Vibrations*, Van Nostrand Reinhold Company, New York.

Feng, C. C., 1968. *The measurement of vortex-induced effects on flow past stationary and oscillating circular and D-section cylinders*, M.Sc. Thesis, The University of British Columbia, Columbia.

Griffin, O. M. and Ramberg, S. E., 1982. Some recent studies of vortex shedding with application to marine tubulars and risers, *J. Energy Resour. Technol.*, ASME, **104**(1): 2 ~ 13.

Govardhan, R. and Williamson, C. H. K., 2000. Modes of vortex formation and frequency response for a freely-vibrating cylinder, *J. Fluid Mech.*, **420**, 85 ~ 130.

Jauvtis, N. and Williamson, C. H. K., 2003. Vortex-induced vibration of a cylinder with two degrees of freedom, *J. Fluids Struct.*, **17**(7): 1035 ~ 1042.

Khalak, A. and Williamson, C. H. K., 1996. Dynamics of a hydroelastic cylinder with very low mass and damping, *J. Fluids Struct.*, **10**(5): 455 ~ 472.

Khalak, A. and Williamson, C. H. K., 1997a. Investigation of the relative effects of mass and damping in vortex-induced vibration of a circular cylinder, *Journal of Wind Engineering and Industrial Aerodynamics*, **69-71**, 341 ~ 350.

Khalak, A. and Williamson, C. H. K., 1997b. Fluid forces and dynamics of a hydroelastic structure with very low mass and damping, *J. Fluids Struct.*, **11**(8): 973 ~ 982.

Khalak, A. and Williamson, C. H. K., 1999. Motions, forces and mode transitions in vortex-induced vibrations at low mass-damping, *J. Fluids Struct.*, **13**(7-8): 813 ~ 851.

Moe, G. and Wu, Z. J., 1990. The lift force on a cylinder vibrating in a current, *Journal of Offshore Mechanics and Arctic Engineering*, ASME, **112**(4): 297 ~ 303.

Parkinson, G., 1989. Phenomena and modeling of flow-induced vibrations of bluff bodies, *Progress in Aerospace Sciences*, **26**(2): 169 ~ 224.

Sarpkaya, T., 1979. Vortex-induced oscillations-A selective review, *Journal of Applied Mechanics*, Trans. of ASME, **46**, 241 ~ 258.

Sarpkaya, T., 1995. Hydrodynamic damping, flow-induced oscillations, and biharmonic Response, *Journal of Offshore Mechanics and Arctic Engineering*, ASME, **117**(4): 232 ~ 238.

Sumer, B. M. and Fredsoe, J., 1995. A review on vibrations of marine pipelines, *International Journal of Offshore and Polar Engineering*, **5**(2): 81 ~ 90.

Sumer, B. M. and Fredsoe, J., 1997. *Hydrodynamics around cylindrical structures*, Singapore: World Scientific, 527.

Tsahalis, D. T., 1984. Vortex-induced vibrations of a flexible cylinder near a plane boundary exposed to steady and wave-induced currents, *Journal of Energy Resources Technology*, Trans. of ASME, **106**, 206 ~ 213.

Williamson, C. H. K. and Govardhan, R., 2004. Vortex-induced vibrations, *Annual Review of Fluid Mechanics*, **36**, 413 ~ 455.

Williamson, C. H. K. and Jauvtis, N., 2004. A high-amplitude 2T mode of vortex-induced vibration for a light body in *xy* motion, *European Journal of Mechanics-B/Fluids*, **23**(1): 107 ~ 114.

Yang, B., Gao, F. P. and Wu, Y. X., 2008. Flow-induced vibrations of a cylinder with two degrees of freedom near a rigid plane boundary, *International Journal of Offshore and Polar Engineering*, **18**(4): 302 ~ 307.

Yang, B., Gao, F. P., Wu, Y. X., and Li, D. H., 2006. Experimental study on vortex-induced vibrations of submarine pipeline near seabed boundary in ocean currents, *China Ocean Eng.*, **20**(1): 113 ~ 121.