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A universal Biot number determining the susceptibility of ceramics to quenching

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Abstract – A universal Biot number of ceramics, which not only determines the susceptibility of the ceramics to quenching but also indicates the duration that the ceramics fail during thermal shock, is theoretically obtained. The present analysis shows that the thermal shock failure of the ceramics with a Biot number greater than this universal value is a very rapid process that just occurs in the initial regime of the heat conduction of the ceramics. This universal Biot number provides a guide to the selection of the ceramics applying to the thermostructural engineering including thermal shock.

Introduction. – Ceramics are renowned for their high resistances to the extreme environments such as ultra-high temperature and heavily chemical corrosion, but their poor resistances to thermal shock have been a major challenge in a lot of thermostructural applications for a long time [1,2]. The primary failure mechanism for ceramics during thermal shock is considered to be crack initiation when the stresses imposed by a thermal gradient exceed the strength of the materials [3–8]. Obviously, the failure of ceramics during thermal shock is associated intimately with the stress duration that is very sensitive to the transient temperature distribution produced in the ceramics [9]. Therefore, the stress duration of thermal shock actually plays a key role in the failure of ceramics in thermal shock. However, the duration of thermal shock has seldom been studied and its characteristic has not been well understood yet [10], despite there was a lot of literature to investigate the behaviors of ceramics during thermal shock.

In the existing study of thermal shock, Biot's number of ceramics, which compares the relative magnitudes of surface-convection and inter-conduction resistances to heat transfer, plays a key role in determining the transient thermal stress field of ceramics [7,9,11–13]. In this study, we present a universal Biot number that determines the susceptibility of ceramics to quenching and the duration characteristic of thermal stress produced in the ceramics during thermal shock.

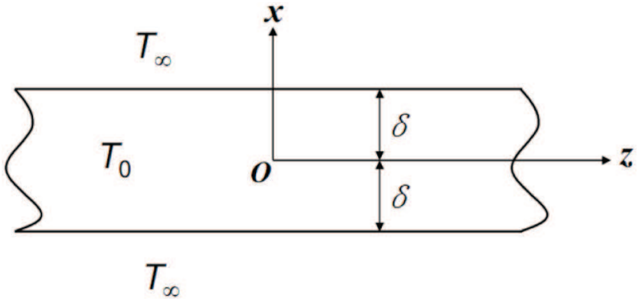

Fig. 1: An infinite plate of thickness 2δ and initial temperature T_0 suddenly exposed to a convective medium of temperature T_∞.

Results and discussion. – We investigate an infinite plate of thickness 2δ, which is initially at a uniform temperature T_0, and at time $\tau = 0$ its top and bottom surfaces are suddenly exposed to a convective medium of temperature T_∞, as shown in fig. 1. The condition of surface convection is written by

$$\mp k\frac{\partial T}{\partial x} = h(T_\infty - T) \quad \text{at } x = \pm\delta, \tag{1}$$

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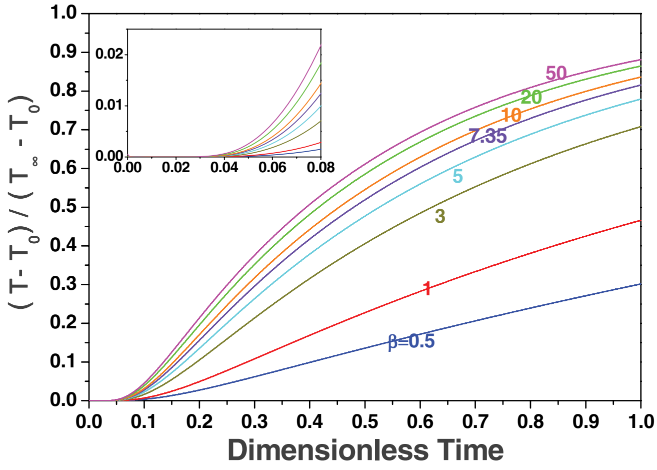

Fig. 2: (Colour on-line) The temperature profiles of the central plane of plate evolves under the condition of cold shock. Inset window shows the evolution of the temperature-wave penetration time.

where k is the thermal conductivity of the plate material, h is the surface heat transfer coefficient, and $T = T(x, \tau)$ is the transient temperature field in the plate, which satisfies the equation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a} \cdot \frac{\partial T}{\partial \tau}, \tag{2}$$

where $a = k/\rho c_p$ is the thermal diffusivity, ρ and c_p are the density and the specific heat under constant pressure for the material of the plate, respectively. Equation (2) is solved under the condition (1), giving [11]

$$\frac{T - T_0}{T_\infty - T_0} = 1 - 2\sum_{n=1}^{\infty} \exp\left(-\beta_n^2 \cdot f\right) \cdot \frac{\sin\beta_n \cos(\beta_n \cdot l)}{\beta_n + \sin\beta_n \cos\beta_n}, \tag{3}$$

where $l = x/\delta$ is the dimensionless length of the plate and $f = a\tau/\delta^2$ is the dimensionless time or the Fourier number of the plate, and β_n are the roots of the equation

$$\beta_n \tan\beta_n = \beta. \tag{4}$$

In eq. (4), $\beta = h\delta/k$ is the Biot number of the plate, which is a dimensionless characteristic including the physical and geometric properties of the plate.

First of all, we study the temperature-wave penetration time in the plate, f_p. The penetration time stands for the duration in which the temperature changes propagating from the surfaces of the plate just reach its central plane [12]. For the sake of convenience, here we take the penetration time as the duration that the temperature change of the central plane appears to reach 0.1% of the initial temperature of the plate. In terms of different Biot's modulus, the penetration time in the plate was obtained under the condition of cold shock, *i.e.* $T_0 > T_\infty$, as shown in fig. 2. It indicates that the temperature-wave penetration time associates intimately with the Biot number of the plate: the greater the Biot number is, the less the penetration time is.

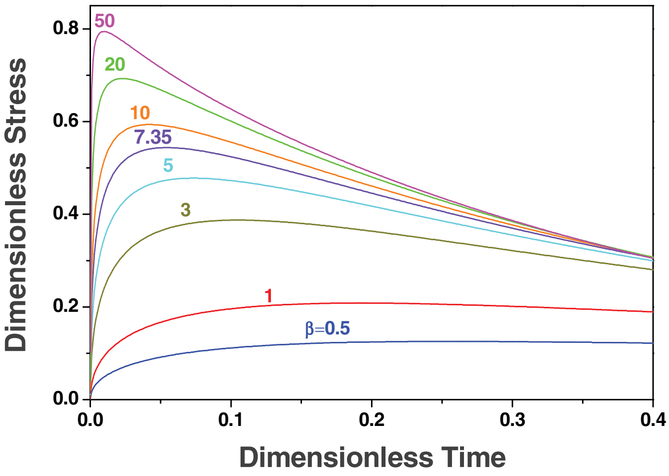

Fig. 3: (Colour on-line) The dimensionless surface stress of plate under the condition of cold shock changes with the different Biot numbers.

Secondly, the dimensionless thermal stress field of the plate during thermal shock is defined by [9,13]

$$\sigma^* = \frac{(1-\nu) \cdot \sigma(x,\tau)}{E\alpha(T_0 - T_\infty)}, \tag{5}$$

where E, ν and α are Young's modulus, Poisson's ratio and the coefficient of thermal expansion of the material, respectively; $\sigma(x, \tau)$ is the actual thermal stress field in the plate and is expressed as [14]

$$\begin{aligned}\sigma(x,\tau) = {} & \frac{\alpha E}{1-\nu} \cdot \left[-(T - T_0) + \frac{1}{2h} \right. \\ & \left. \times \int_{-\delta}^{\delta} (T - T_0) \mathrm{d}x + \frac{3x}{2h^3} \cdot \int_{-\delta}^{\delta} (T - T_0) x \mathrm{d}x \right].\end{aligned} \tag{6}$$

Here we only investigate the surface tensile stress during cooling and the tensile stress of the central plane during heating. It is because that ceramics are much weaker in tension than under compression, failure often occurs at the surface during cooling and at central plane during heating [7,9,13]. According to equations (3), (5) and (6), the dimensionless surface thermal stress of the plate is obtained at $x = \delta$ under the condition of cold shock, as shown in fig. 3. The results indicate that the maximum thermal stress in the plate, just as the penetration time, is intimately relevant to the Biot number and does not occur at the start of thermal shock except the Biot number equal to infinite. Obviously, in terms of each of different Biot's numbers of the plates, there is a dimensionless extremum time f^* at which the dimensionless surface thermal stress reaches its maximum value. From the theory of thermal shock fracture [3,5,13], as the thermal stress yielded in the plate during thermal shock is greater than the strength of the material of the plate, the failure of the plate occurs. Therefore, the failure of the ceramic plate takes

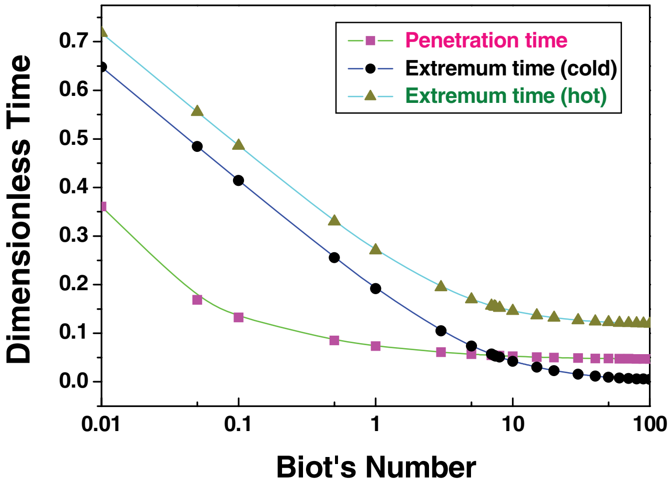

Fig. 4: (Colour on-line) The temperature-wave penetration time, the stress extremum time under cold shock and the stress extremum time under hot shock change with the Biot number of plate. There is an intersection of the penetration and cold extremum time, where the Biot number is 7.35 and dimensionless time is 0.0543. However, there is no intersection of the penetration and hot extremum time, and the hot extremum time is always greater than the penetration time.

consequentially place at or before the stress extremum time f^*.

By comparing the penetration time f_p with the stress extremum time f^*, we find that there is a critical value of the Biot number, $\beta_c = 7.35$, at which the two dimensionless time are the same, $f^* = f_p = 0.0543$, as shown in fig. 4. When the Biot number of the plate is greater than this critical value, $\beta > \beta_c$, the two types of the dimensionless time satisfy $f^* < f_p$; and when $\beta < \beta_c$, the dimensionless time $f^* > f_p$. This proved that the failure of ceramics during thermal shock consequentially occurs in the initial regime of the heat conduction process, *i.e.* before the temperature changes propagating from the surfaces reach the central plane of the plate [15], so far as the Biot number of ceramics is greater than this critical value. This is an indication that the critical Boit number determines the main duration characteristic for the ceramics subjected to thermal shock and, therefore, can be applied to evaluate the susceptibility of ceramics to thermal shock. Ceramic materials or structures, for example, are defined as the sensitive to thermal shock if their Biot numbers are greater than 7.35. Whereas, when the Biot numbers of ceramics are less than the critical value, the failure of the ceramics during thermal shock can theoretically occur at any time before its stress extremum time.

The critical Biot number of ceramics is presented under the condition of cold shock. However, under the condition of hot shock there is no point of intersection between the temperature-wave penetration time and the stress extremum time, as shown in fig. 4. It is because that the maximum thermal tensile stress appears just at the central plane of plate under the condition of hot shock [7,13], where the penetration time is always lower than the stress extremum time. Therefore, under the condition of hot shock the failure of the ceramics during thermal shock can theoretically occur at any time before its stress extremum time including the initial and regular regimes of heat conduction of the ceramics.

Conclusions. – As stated above, the universal Biot number, $\beta_c = 7.35$, not only determines the susceptibility of the ceramics to quenching but also indicates the duration that the ceramics fail during thermal shock. Obviously, this result can provide a guide to the selection of the ceramics applying to the thermstructural engineering with thermal shock, in particular, quenching into the environments of lower temperature.

* * *

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