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Vertical Bearing Capacity of a Partially-Embedded Pipeline on Tresca Soils

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ABSTRACT

Slip-line field solutions are presented for the ultimate load of submarine pipelines on a purely cohesive soil obeying Tresca yield criterion, taking into account of pipe embedment and pipe-soil contact friction. The derived bearing capacity factors for a smooth pipeline degenerate into those for the traditional strip-line footing when the embedment approaches zero. Parametric studies demonstrate that the bearing capacity factors for pipeline foundations are significantly influenced by the pipeline embedment and the pipe-soil frictional coefficient. With the increase of pipeline embedment, the bearing capacity factor N_c decreases gradually, and finally reaches the minimum value (4.0) when the embedment equals to pipeline radius. As such, if the pipeline is directly treated as a traditional strip footing, the bearing capacity factor N_c would be over evaluated. The ultimate bearing loads increase with increasing pipeline embedment and pipe-soil frictional coefficient.

KEY WORDS: Bearing Capacity; Slip-line Field Theory; Pipeline; Tresca Yield Criterion; Strip Footing

INTRODUCTION

In geotechnical engineering, the ultimate bearing capacity is the capacity of soil to support the loads applied to the ground, which is usually defined as the maximum average contact pressure between the foundation and the soil causing shear failure of the supporting soil immediately below and adjacent to the foundation. Till now, there are quite a few formulas for the ultimate bearing capacity developed for footings, but not especially for submarine pipelines (see Craig, 1997).

Under the action of its submerged weight, a submarine pipeline may penetrate into the soil with certain embedment. Unlike the traditional continuous strip footings, a pipeline holds circular cross-section. As such, the effective bearing width of the pipe-soil interface is a function of pipeline embedment, which would affect not only the bearing capacity of underlying soils but also the lateral stability of submarine pipelines in ocean currents and/or waves (Gao *et al*., 2007). The ultimate bearing load for the pipeline foundations is usually expressed with the maximum value of submerged weight of the pipeline per unit length. The existing formulas for the ultimate bearing capacity of traditional footing could not be efficiently employed for evaluating the ultimate load for the pipeline foundations.

In the submarine pipeline design practice, it is highly desired to efficiently evaluate the bearing capacity of pipeline foundations. During the past few decades, the bearing capacity of pipeline foundations has attracted much attention from many researchers. Small et al. (1971) treated the pipeline with certain submerged weight as an equivalent uniform distributed pressure upon a traditional rectangular footing, and proposed empirical formulas for the bearing capacity factors by modifying the solutions for a traditional strip footing. This treatment obviously could not consider the effects of the circular section of the pipeline. Karal (1977) applied the upper bound theorems of classical plasticity theory to develop a prediction of pipe penetration, idealizing the pipe as a rigid wedge indenter. The approximation of pipeline with wedge indenter might be reasonable at small embedment but error becomes significant with increasing embedment. Murff *et al.* (1989) analyzed the pipeline penetration into a cohesive soil using upper and lower bound theorems respectively. Aubeny *et al.* (2005) further investigated the collapse loads for a cylinder laid in a trench of cohesive soil. Due to the special circular cross-section of the submarine pipeline, the quantitative evaluations of ultimate load for pipeline foundations on various sediments and pipe-soil contact conditions are far from being fully achieved. The effects of pipe-soil friction coefficient and pipeline embedment on the bearing capacity of submarine pipelines are needed to be further investigated.

In this paper, the bearing capacity of the pipeline on Tresca soils is analyzed theoretically by employing the slip-line field theory. A parametric study is then performed to investigate the influential factors for the bearing capacity of the pipeline foundations.

SLIP-LINE FIELD SOLUTION

Slip-line Field Theory

The slip-line field theory is on the basis of equilibrium equations and the failure criterion of the material. For a weightless soil, the equilibrium equations are

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}=0, \quad (1a)$$

$$\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_y}{\partial y}=0. \quad (1b)$$

The soil is assumed to obey Tresca or maximum shear stress failure criterion, i.e.

$$(\sigma_x-\sigma_y)^2+4\tau_{xy}^2=4c^2. \quad (2)$$

Let $\sigma=(\sigma_1+\sigma_3)/2=(\sigma_x+\sigma_y)/2$ and refer to the Mohr circle, then

$$\sigma_x=\sigma-c\sin 2\theta, \quad (3a)$$

$$\tau_{xy}=c\cos 2\theta, \quad (3b)$$

$$\sigma_y=\sigma+c\sin 2\theta. \quad (3c)$$

Submitting Eqs. 3a～3c to Eq. 1, the characteristic functions for slip-lines can be derived as

$$\frac{dy}{dx}=\text{tg}\theta \quad (\alpha \text{ line}), \quad (4a)$$

$$\frac{dy}{dx}=-\text{ctg}\theta \quad (\beta \text{ line}), \quad (4b)$$

and the Hencky stress equations as

$$\sigma-2c\theta=const1 \quad (\text{along } \alpha \text{ line}), \quad (5a)$$

$$\sigma+2c\theta=const2 \quad (\text{along } \beta \text{ line}), \quad (5b)$$

where x and y are the coordinates for the point in the slip-line field, σ is the mean stress at a certain point in the slip-line field, θ is the angle between the α tangent line and the x- axis, c is the cohesion of soil. When the boundary conditions are given, the stress and the corresponding slip-lines can be determined (see Yan, 1988).

Basic Assumptions

The bearing capacity for a submarine pipeline laid upon the horizontally flat seabed can be treated as a plane-strain problem. The assumptions are given as follows (see Fig.1):

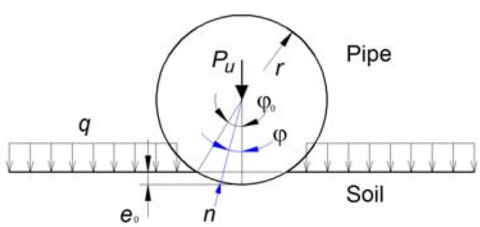

Fig. 1 Sketch map of pipeline embedment in soil

(1) The soil is a weightless, ideal rigid-plastic material;
(2) The Tresca yielding criterion is satisfied for the soil, such as the saturated soft clay under undrained conditions;
(3) There exists an embedment (e_0) of the pipe with radius of r. For the case of $e_0/r\le 1$, the uniform overburden load at the two sides of the pipe $q=0$. For the case of $e_0/r>1$, the pipe-soil contact condition can be treated as that for $e_0/r=1$, the weight of soil above the pipe center is replaced by an equivalent uniform surcharge pressure $q=(e_0-r)\gamma'$, where γ' is the effective (buoyant) unit weight of soil.

In this study, the pipe-soil contact friction is taken into account. The shear stress between the pipe and the surrounding soil is assumed as $a=\alpha c$ (Randolph & Houlsby, 1984). Note that, $0<\alpha<1$, α is the frictional coefficient. For a certain point E at the pipe-soil interface (see Fig. 2), the direction for the slip-line: $\theta_E=\pi/4-\varphi+\Delta/2$, in which $\Delta=\arcsin\alpha$.

Slip-line Field for the Pipeline Foundations

According to the slip-line field theory, the coordinates of the slip-lines can be obtained by solving the characteristic functions for slip-lines (Eqs. 4a and 4b) under certain boundary conditions using finite-differential method, then the mean stress σ and angle θ can be calculated from the Hencky stress equations (Eqs. 5a and 5b).

As shown in Fig. 2, the boundaries CG and CEB are the Riemann conditions for determining the uniform field CFG and the extrusion filed CBD, respectively; the boundaries CF and CD are the regressive Riemann conditions for determining the transition region CDF. Based on the stress analysis, on the line CG, the minimum stress can be determined with the magnitude of q and its direction is vertical. On the line CEB, the maximum stress is located, whose direction is perpendicular to the line CEB, and whose magnitude is to be determined. Lines CF and CD are the boundary for the filed CFD, whose solution can be determined from the results of the uniform field CFG and those of the extrusion filed CBD. By employing the finite-differential method, the slip-line fields for the pipeline foundations can be constructed. Fig. 2 gives the slip-line fields for the smooth pipeline ($\alpha=0$) and the rough pipeline ($\alpha=0.5$), respectively. As indicated in the figure, the whole slip-line field can be divided into three regions, i.e. the uniform region CFG, the extrusion region CBD, and the transition region CDF. The magnitude of the slip-line field for the case of the rough pipelines is larger than that of the smooth pipes.

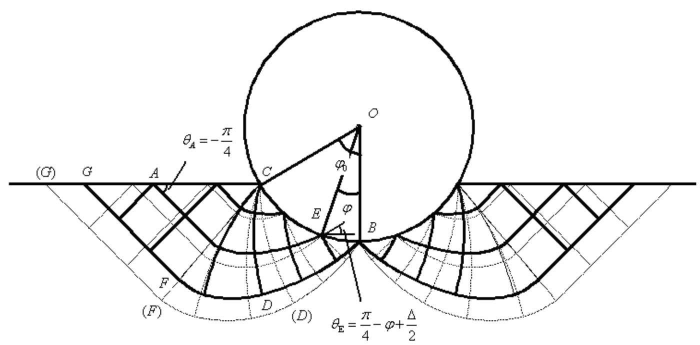

Fig. 2 Slip-line field of the pipeline foundation (Real lines: smooth pipeline ($\alpha=0$); dash lines: rough pipeline ($\alpha=0.5$))

Ultimate Load for Pipeline Foundations: Derivation

On the basis of the aforementioned basic assumptions and the constructed slip-line fields, the ultimate load for pipeline foundations can be further derived. The ultimate bearing load P_u is expressed in the form of integral as

$$P_u=2\int_0^{\varphi_0} r\sigma_{E,y}\,d\varphi, \quad (6)$$

where $\sigma_{E,y}$ is the vertical component of the pipe-soil contact force;

φ_0 is the embedment angle $\angle$BOC (see Fig. 2): $\varphi_0 = \arccos(1-e_0/r)$. As shown in Fig. 2, points A and E are along the same α line, and let $\angle\text{BOE}=\varphi$. Submitting the values of σ and θ at points A and E into the Hencky stress equations (i.e. Eqs. 5a and 5b), then

$$\sigma_{\rm A}\text{-}2c\theta_{\rm A} = \sigma_{\rm E}\text{-}2c\theta_{\rm E}, \tag{7}$$

That is,

$$\sigma_{\rm E} = \sigma_{\rm A} + 2c(\theta_{\rm E} - \theta_{\rm A}) = \sigma_{\rm A} + 2c(\frac{\pi}{2}+\frac{\Delta}{2}-\varphi). \tag{8}$$

As $\sigma_{\rm E} = \sigma_{{\rm E},1} - c$ ($\sigma_{{\rm E},1}$ is the first principal stress at point-E along the pipe-soil contact arc), then

$$\sigma_{E,1} = \sigma_E + c = \sigma_A + 2c(\frac{\pi}{2}+\frac{\Delta}{2}-\varphi)+c, \tag{9}$$

in which $\sigma_A = q + c$. At the point-E along the pipe-soil contact arc, the vertical component of the pipe-soil contact force $\sigma_{E,y}$ can be expressed as

$$\sigma_{E,y} = \sigma_{E,1}\cos(\varphi - \Delta/2). \tag{10}$$

Submitting Eqs. 9 ~ 10 into Eq. 6, the ultimate bearing load P_u can be derived as

$$\begin{aligned} P_u &= 2\int_0^{\varphi_0}[c(\pi+\Delta+2\text{-}2\varphi)+q]\cos(\varphi-\frac{\Delta}{2})rd\varphi \\ &= 2[cr(2+\pi+\Delta)+qr][\sin(\varphi_0-\frac{\Delta}{2})+\sin(\frac{\Delta}{2})] \\ &\quad -4cr[\varphi_0\sin(\varphi_0-\frac{\Delta}{2})+\cos(\varphi_0-\frac{\Delta}{2})-\cos(\frac{\Delta}{2})]. \end{aligned} \tag{11}$$

Referring to the formula of the bearing capacity for traditional strip footings, the bearing capacity for pipeline foundations may be expressed in the following form:

$$\frac{P_u}{2r\sin\varphi_0} = cN_c + qN_q, \tag{12}$$

where "$2r\sin\varphi_0$" is the width of the pipe-soil interface.

Submitting Eq. 11 into Eq. 12, the bearing capacity factor for cohesion (N_c) and the bearing capacity factor for distributed load (N_q) can thereby be obtained:

$$\begin{aligned} N_c &= \frac{1}{\sin\varphi_0}(2+\pi+\Delta)[\sin(\varphi_0-\frac{\Delta}{2})+\sin(\frac{\Delta}{2})] \\ &\quad -\frac{2}{\sin\varphi_0}[\varphi_0\sin(\varphi_0-\frac{\Delta}{2})+\cos(\varphi_0-\frac{\Delta}{2})-\cos(\frac{\Delta}{2})], \end{aligned} \tag{13a}$$

$$N_q = \frac{\sin(\varphi_0-\frac{\Delta}{2})+\sin(\frac{\Delta}{2})}{\sin\varphi_0}. \tag{13b}$$

PARAMETRIC STUDIES

Comparison of Bearing Capacity between Pipeline and Traditional Strip Footings

In the analysis on the general shear failure mechanism of a strip footing on weightless soils, e.g. Prandtl-Reissner solution, the smooth strip footing carries a uniform pressure on the surface of a mass of homogeneous, isotropic soil; the shear strength parameters for the soil are c and ϕ; and a surcharge pressure q acting on the soil surface is considered. The following exact solution has been widely used for the ultimate bearing capacity of a strip footing on the surface of a weightless soil (see Craig, 1997):

$$\frac{P_u}{b} = cN_c + qN_q, \tag{14}$$

where N_c and N_q are the bearing capacity factors, i.e.

$$N_c = (N_q - 1)\cot\phi, \tag{15a}$$

$$N_q = e^{\pi\tan\phi}\tan^2(\frac{\pi}{4}+\frac{\phi}{2}), \tag{15b}$$

b is the width of the traditional strip footing (note: for the pipeline foundation, $b = 2r\sin\varphi_0$ (see Fig. 2)), ϕ is the internal angle of soils. For a pure cohesive soil (i.e. $\phi = 0$),

$$N_q = 1, \tag{16a}$$

$$N_c = \pi + 2. \tag{16b}$$

For the case of the partially-embedded pipeline on Tresca soils, if the pipeline surface is fully smooth (Δ=0), then the bearing capacity factors (13a) and (14b) are simplified as

$$N_c = (\pi+2) + 2(\frac{1-\cos\varphi_0}{\sin\varphi_0} - \varphi_0), \tag{17a}$$

$$N_q = 1. \tag{17b}$$

Now to examine the two extrema of N_c (see Eq. (17a)):

$$\lim_{\varphi_0\to 0} N_c = (\pi+2)+2(\frac{1-\cos\varphi_0}{\sin\varphi_0}-\varphi_0) = \pi+2, \tag{18a}$$

$$\lim_{\varphi_0\to\frac{\pi}{2}} N_c = 4. \tag{18b}$$

Fig. 3 gives the variation of N_c with e_0/r for smooth pipes. When $\varphi_0 \to 0$ (i.e. the pipeline just touches the soil surface e_0/r =0), the bearing capacity factor N_c for pipeline foundations (see Eq. 18a) matches that for the traditional strip footings (see Eq.16b). This indicates that, when the pipeline embedment approaches zero, the formulae for the bearing capacity of pipeline foundations degenerate into those for the traditional strip footings. With the increase of the pipeline embedment, the value of N_c decreases gradually and finally reaches 4.0 when the pipeline is half buried (see Fig. 3). Therefore, if pipeline foundations are directly simplified as traditional strip footings, the bearing capacity factor N_c would be over evaluated, whose error may be up to 28.5%.

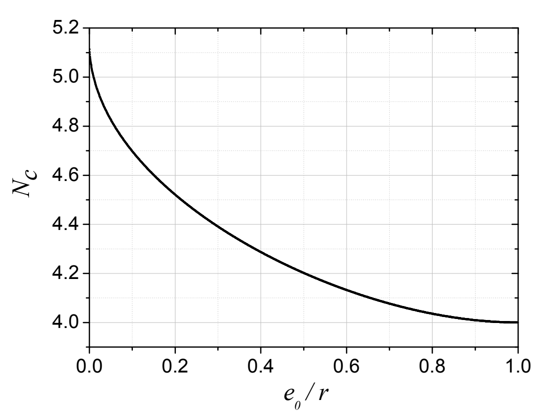

Fig. 3 Variation of N_c with e_0/r for smooth pipes.

Parametric Study

Based on the derived formulae for the bearing capacity of the partially embedded pipeline on Tresca soils, i.e. Eqs. 13a and 13b, the relationship between the bearing capacity factors (N_c , N_q) and the non-dimensional pipeline embedment (e_0/r), and the pipe-soil frictional coefficient (α) can be established. Fig. 4 gives the variation of N_c with the parameters e_0/r and α . As shown in Fig. 4, when $e_0/r<0.6$, the values of N_c initially increases to a maximum value, then decreases continuously with the increase of the frictional coefficient α ; when $e_0/r>0.6$, the values of N_c increases with increasing α . The effect of the pipe-soil frictional coefficient (α) on N_c increased with the increase of pipeline embedment (e_0/r). When α =1 and $e_0/r=1$, the maximum value of N_c emerges, i.e. N_c = 7.30 (see Fig. 4).

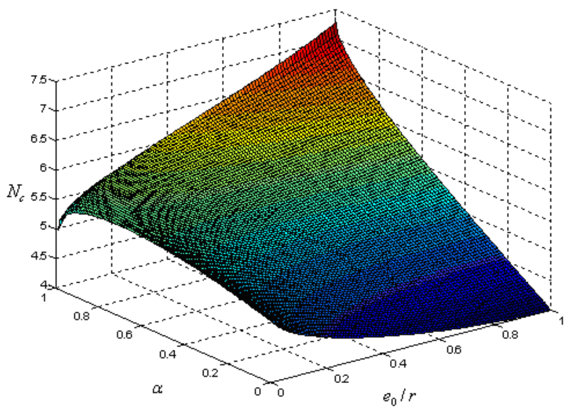

Fig. 4 Variation of N_c with α and e_0/r

Fig. 5 gives the variation of N_q with the parameters e_0/r and α . When $e_0/r<0.2$, N_q decreases with increasing α . Nevertheless, when $e_0/r>0.2$, N_q increases with increasing α . When α =1 and $e_0/r=1$, the maximum value of N_q emerges, i.e. N_q = 1.42 (see Fig. 5).

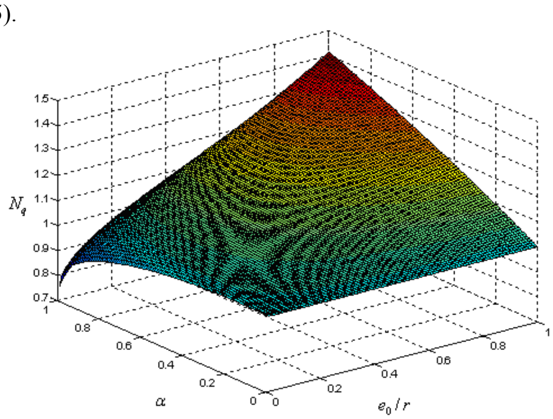

Fig. 5 Variation of N_q with α and e_0/r

For better understanding the bearing capacity of pipeline foundations, the dimensionless ultimate bearing load P_u/cr is introduced. Eq. (12) is thereby rewritten as

$$P_u/cr = 2\sin\varphi_0\left(N_c+\frac{q}{c}N_q\right), \qquad (19)$$

in which, the bearing capacity factors N_c and N_q are calculated with Eqs. 13a and 13b; φ_0 is the embedment angle (see Fig. 1). Fig 6 gives the variation of P_u/cr with the dimensionless pipeline embedment (e_0/r) and the pipe-soil frictional coefficient (α), under the condition that the embedment is less than the pipeline radius ($q=0$). For the fixed value of α , P_u/cr increases with increasing e_0/r . For the fixed valus of e_0/r , P_u/cr increases with increasing α ; the effects of α on P_u/cr are higher for larger values of e_0/r . When α =1 and $e_0/r=1$, P_u/cr reaches its maximum value (see Fig. 6).

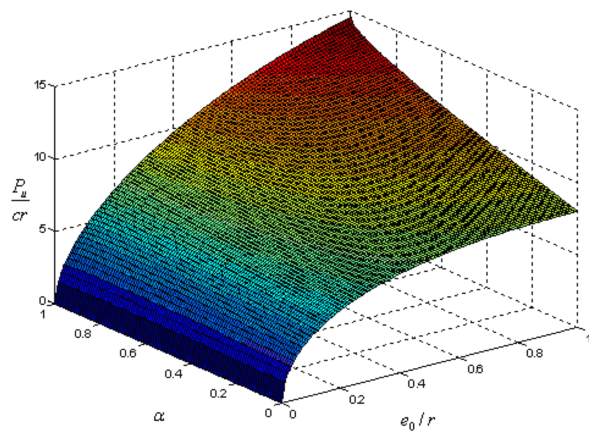

Fig. 6 Variation of P_u/cr with α and e_0/r ($q=0$)

CONCLUSIONS

Based on the slip-line field theory, the theoretical solutions are derived for the bearing capacity of a partially-embedded pipeline on Tresca soils, taking into account the pipe-soil contact frictional coefficient and the pipeline embedment, etc. Parametric studies are performed on the bearing capacity factors and the dimensionless ultimate load. The following conclusions can be drawn:

(1) Due to the special circular arc shape of the pipeline foundations, the constructed slip-line fields for the partially-embedded pipeline on Tresca soils are different from those of traditional strip footings. The pipe-soil contact friction also has much effect on the magnitude of the slip-line fields. The slip-line field for the rough pipeline is larger than that for the smooth pipeline.

(2) The bearing capacity factors for pipeline foundations, i.e. N_c and N_q, are derived. When the embedment of a smooth pipeline approaches zero, the bearing capacity factors degenerate into those for the traditional strip-line footing (Prandtl's mechanism). With the increase of the pipeline embedment, the bearing capacity factor N_c decreases gradually, and finally reaches the minimum value (4.0) when the embedment equals to pipeline radius. As such, if pipeline foundations are directly simplified as traditional strip footings, the bearing capacity factor N_c would be over evaluated.

(3) The bearing capacity factors for pipeline foundations are significantly influenced by non-dimensional pipeline embedment and pipe-soil frictional coefficient.

(4) A dimensionless ultimate bearing load P_u/cr is introduced. Under the condition that the embedment is less than pipeline radius, P_u/cr increases with increasing pipeline embedment (e_0/r) for the fixed value of the pipe-soil frictional coefficient (α); P_u/cr increases with increasing the pipe-soil frictional coefficient (α) for the fixed value of e_0/r. The effects of α on P_u/cr get higher for larger values of e_0/r.

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