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HYDRODYNAMIC INTERACTION BETWEEN TWO VERTICAL CYLINDERS IN WATER WAVES*

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Abstract

The hydrodynamic interaction between two vertical cylinders in water waves is investigated based on the linearized potential flow theory. One of the two cylinders is fixed at the bottom while the other is articulated at the bottom and oscillates with small amplitudes in the direction of the incident wave. Both the diffracted wave and the rediation wave are studied in the present paper. A simple analytical expression for the velocity potential on the surface of each cylinder is obtained by means of Graf's addition theorem. The wave-excited forces and moments on the cylinders, the added masses and the radiation damping coefficients of the oscillating cylinder are all expressed explicitly in series form. The coefficients of the series are determined by solving algebraic equations. Several numerical examples are given to illustrate the effects of various parameters, such as the separation distance, the relative size of the cylinders, and the incident angle, on the first-order and steady second-order forces, the added masses and radiation-damping coefficients as well as the response of the oscillating cylinder.

Key words wave-excited force, added masses, radiation damping, drift force, articulated cylinder, two vertical cylinders

I. Introduction

With the construction of large offshore structures, wave diffraction and radiation problems caused by several bodies become increasingly important. Large offshore platforms, wave-power extraction devices, large storage facilities and offshore floating airports[1] have been proposed or constructed with several elementary members or legs such as cylinders. Thus, the interactions among several solid bodies in fluids, such as the hydrodynamic interaction between offshore structures and floating ice masses and the collision between two ships, need to be investigated for accurate and efficient theoretical predictions. The interaction between two cylinders is, perhaps, the simplest problem to be investigated.

To consider the interaction among several bodies in an incident wave, it is necessary to account for not only the diffraction of each body, but also the multiple scattering due to other

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bodies. The diffraction and radiation waves of each body can affect other bodies, and in the mean time, they are reflected by other bodies. This process goes on and on. In principle, this type of problems can be solved by a number of reliable numerical methods[2]. However, the computation is quite cumbersome as the number of bodies increases. Therefore, it is desirable to have some analytical methods to reduce the complication of computation.

We shall adopt the approach developed by Záviska[3] and rediscovered by Spring and Monkmeyer[4] for water waves. In this approach, the direct matrix method is used to solve for the amplitudes of wave components around each body simultaneously, subjected to the boundary conditions for the respective body. Ohkusu[5] applied the two-dimensional scattering of acoustic radiation by an array of circular cylinders to a structure that is composed of three vertical circular cylinders. Later, Simon[6] developed a plane-wave approximation (or large-spacing approximation) and applied it to axisymmetric bodies in heaving motion. Kagemoto and Yue[7] combined the features of the matrix method and the multiple-scattering technique to develop an interaction theory to solve the complete problem. They determined wave excited forces, hydrodynamic coefficients and second-order drift forces based algebraically on the diffraction characteristics of every single body. Fang and Kim[8] determined the seakeeping characteristics of two adjacent ships advancing on parallel course. They employed the two-dimensional integral-equation method to consider the hydrodynamic interaction between freely floating cylinders in beam sea. Linton and Evans[9] simplified the expression of the velocity potential. They all dealt with the diffraction of fixed uniform vertical cylinders, but no radiation wave was considered. Recently, Kim[10] extended the diffraction theory to radiation problem and calculated the added masses and damping coefficients for four cylinders with the same diameter located at corners of a square. On the other hand, Landweber, Chwang and Guo[11] developed an irrotational-flow model in which time-varying added masses were determined from the solutions of integral equations for source distributions on the surfaces of two bodies. Forces on bodies were obtained from Lagrange's form of equations of motion, but there were no waves.

In the present paper, we shall consider the interaction between an incident wave and two circular cylinders. One of them is fixed at the bottom and the other is articulated at the bottom and can oscillate in the wave direction. Both the diffraction wave and the radiation wave are considered. In Section II, an exact interaction theory is formulated by means of an addition theorem for wave potentials. A simple analytical expression for the velocity potential on the surface of each cylinder is also given. In Section III, the wave excited forces and moments are obtained by integrating the pressure on both cylinders. Added masses and damping coefficients of the oscillating cylinder are determined. In Section IV, several numerical examples are presented. The effects of various parameters, such as the separation distance, the relative size of the cylinders, and the incident angle, on wave excited forces, the steady drift forces, the added masses and damping coefficients are discussed. In principle, the method presented here can be used to solve the problem with N bodies. The generalization is not difficult but tedious. The only change is that the total velocity potential is summed over all N bodies instead of two bodies.

II. Formulation

Let us consider the irrotational motion of an incompressible and inviscid fluid around two surface-piercing circular cylinders. In a Cartesian coordinate system with the z axis pointing vertically upward, the mean free surface is located at $z=0$ and the bottom of the fluid is at $z=-h$. Cylinder 1 with radius a_1 is fixed at the bottom and Cylinder 2 with radius a_2 is articulated at the bottom and can oscillate in the x direction which is the direction of the incident wave. The motion of the two cylinders can be described in two cylindrical coordinate systems (r_i, θ_i, z) $(i=1, 2)$ with the origins O_1 and O_2 being in the $z=0$ plane (Fig. 1). The coordinates of O_1 and O_2 are $(x_1, y_1, 0)$ and $(x_2, y_2, 0)$ respectively.

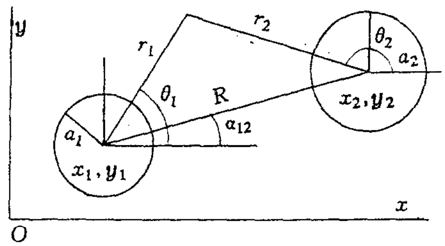

Fig. 1 Coordinate systems for two cylinders

The velocity potential $\Phi(x, y, z, t)$ satisfies the Laplace equation

$$\nabla^2\Phi=0 \tag{2.1}$$

and the linearized boundary conditions for Φ are

$$\Phi_{tt}+g\Phi_z=0 \qquad \text{on } z=0 \tag{2.2}$$

$$\Phi_z=0 \qquad \text{on the bottomon } z=-h \tag{2.3}$$

$$\Phi_n=0 \qquad \text{on } C_1: r_1=a_1 \tag{2.4}$$

$$\Phi_n=\frac{d\Psi}{dt}(z+h)\cos\theta_2 \qquad \text{on } C_2: r_2=a_2 \tag{2.5}$$

where Ψ is the small angular deviation of cylinder 2 from the z axis and in the linear theory, equation (2.5) can be satisfied on the surface of cylinder 2 in the static equilibrium position $\Psi=0$. The velocity potential Φ should also satisfy the radiation condition at infinity.

We assume that all motions are time harmonic with angular frequency ω, then Φ can be expressed as

$$\Phi(x,y,z,t)=\phi(x,y.z)e^{-i\omega t} \tag{2.6}$$

where

$$\omega^2=gk\tanh(kh) \tag{2.7}$$

k is the wave number and g is the gravitational constant. The spatial velocity potential of the incident wave ϕ_I may be expressed as

$$\phi_I=-\frac{igA}{\omega}f_0(z)e^{ikx}=-\frac{igA}{\omega}I_{ji}\exp[ikr_j\cos\theta_j]f_0(z) \tag{2.8}$$

where $I_{ji}=\exp[ikx_j]$ is a phase factor associated with cylinder j, A represents the amplitude of the incident wave and (x_j, y_j) are the coordinates for the center of cross-sections of cylinder j and

$$f_0(z)=\frac{\cosh k(z+h)}{\cosh kh} \tag{2.9}$$

Equation (2.8) can be written as

$$\phi_I=-\frac{igA}{\omega}I_{ji}\sum_{n=-\infty}^{\infty}J_n(kr_j)\exp\left[in\left(\frac{\pi}{2}-\theta_j\right)\right]f_0(z)\qquad (j=1,2)\tag{2.10}$$

where $J_n(kr_j)$ is the Bessel function of the first kind and of order n and argument kr_j.

The general form of the diffraction wave potential due to the incident wave is given by

$$\phi_{dj}=-\frac{igA}{\omega}\sum_{n=-\infty}^{\infty}A_{nj}Z_{nj}H_n(kr_j)\exp[in\theta_j]f_0(z)\qquad (j=1,2)\tag{2.11}$$

where $H_n(kr_j)=H_n^{(1)}(kr_j)$ is the Hankel function of the first kind and of order n. A_{nj} are unknown coefficients to be determined and

$$Z_{nj}=\frac{J_n'(ka_j)}{H_n'(ka_j)}\tag{2.12}$$

The prime denotes differentiation with respect to the argument ka_j.

The radiation potential due to the oscillation of cylinder 2 is

$$\phi_{rj}=i\omega\Psi\sum_{n=-\infty}^{\infty}\Big[B_{n0j}Z_{nj}H_n(kr_j)f_0(z)$$

$$+\sum_{m=1}^{\infty}B_{nmj}X_{nmj}K_n(k_mr_j)f_m(z)\Big]\exp[in\theta_j]\qquad (j=1,2)\tag{2.13}$$

where Ψ is the amplitude of the overturning motion of the oscillating cylinder,

$$\Psi=\psi e^{-i\omega t}\tag{2.14}$$

$$X_{nmj}=\frac{I_n'(k_ma_j)}{K_n'(k_ma_j)}\tag{2.15}$$

$I_n(k_ma_j)$ is the modified Bessel function of the first kind, $K_n(k_mr_j)$ is the modified Bessel function of the second kind, k_m are the solutions of the equation

$$\omega^2+gk_m\tan(k_mh)=0\tag{2.16}$$

$$f_m(z)=\frac{\cos k_m(z+h)}{\cos(k_mh)}\tag{2.17}$$

B_{n0j} and B_{nmj} are unknown coefficients to be determined.

The total velocity potential is

$$\phi=\phi_I+\sum_{j=1}^{2}(\phi_{dj}+\phi_{rj})\tag{2.18}$$

This velocity potential Φ, after being substituted into (2.6), satisfies the governing equation (2.1), the boundary conditions (2.2) and (2.3) and the relevant radiation condition at infinity. All the unknown coefficients in (2.18) will be determined by applying the boundary conditions (2.4) and (2.5). The only difficulty is that equation (2.18) is expressed in terms of two cylindrical coordinates. Using Graf's addition theorem for Bessel functions[12],we have

$$H_n(kr_1)\exp[in(\theta_1-\alpha_{12})]=\sum_{l=-\infty}^{\infty}H_{n+l}(kR)J_l(kr_2)\exp[il(\pi+\alpha_{12}-\theta_2)]\tag{2.19}$$

$$K_n(kr_1)\exp[in(\theta_1-\alpha_{12})]=\sum_{l=-\infty}^{\infty}K_{n+l}(kR)I_l(kr_2)\exp[il(\pi+\alpha_{12}-\theta_2)] \qquad (2.20)$$

Then the velocity potential can be expressed in terms of the coordinates $(r_j,\ \theta_j,\ z)$ as

$$\phi(r_j,\theta_j,z)=-\frac{igA}{\omega}I_{ji}\sum_{n=-\infty}^{\infty}J_n(kr_j)\exp\left[in\left(\frac{\pi}{2}-\theta_j\right)\right]f_0(z)$$

$$-\frac{igA}{\omega}\sum_{n=-\infty}^{\infty}f_0(z)\Big[\exp[in\theta_j]A_{nj}Z_{nj}H_n(kr_j)$$

$$+\sum_{l=-\infty}^{\infty}A_{ni}Z_{ni}H_{n+l}(kR)J_l(kr_j)\exp[il(\pi-\theta_j)]\exp[i(n+l)\alpha_{ij}]\Big]$$

$$+i\omega\psi\sum_{n=-\infty}^{\infty}[B_{n0j}Z_{nj}H_n(kr_j)f_0(z)\exp[in\theta_j]$$

$$+\sum_{m=1}^{\infty}B_{nmj}X_{nmj}K_n(k_mr_j)f_m(z)\exp[in\theta_j]$$

$$+\sum_{l=-\infty}^{\infty}B_{n0i}Z_{ni}H_{n+l}(kR)J_l(kr_j)\exp[il(\pi-\theta_j)]\exp[i(n+l)\alpha_{ij}]f_0(z)$$

$$+\sum_{l=-\infty}^{\infty}\sum_{m=1}^{\infty}B_{nmi}X_{nmi}K_{n+l}(k_mR)I_l(k_mr_j)\exp[il(\pi-\theta_j)]\exp[i(n+l)\alpha_{ij}]f_m(z)]$$

$$(i\neq j,\ i=1,\ 2,\ j=1,\ 2) \qquad (2.21)$$

where

$$R=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2},\quad \alpha_{ij}=\arctan\left(\frac{y_j-y_i}{x_j-x_i}\right)$$

Equation (2.21) is valid for $r_j<R$. Therefore it is an expression valid near cylinder j. Replacing l by $-l$ in (2.21), substituting (2.21) into boundary conditions (2.4) and (2.5) and using the orthogonality relations for $\cos n\theta$, $\sin n\theta$ and $f_n(z)$, we have

$$I_{ji}\exp\left[il\frac{\pi}{2}\right]+A_{lj}+\sum_{n=-\infty}^{\infty}A_{ni}Z_{ni}H_{n-l}(kR)\exp[i(n-l)\alpha_{ij}]=0,$$

$$-\infty<l<\infty,\ i\neq j,\ j=1,\ 2 \qquad (2.22)$$

$$B_{l01}+\sum_{n=-\infty}^{\infty}B_{n02}Z_{n2}H_{n-l}(kR)\exp[i(n-l)\alpha_{21}]=0,\qquad -\infty<l<\infty \qquad (2.23a)$$

$$B_{l02}+\sum_{n=-\infty}^{\infty}B_{n01}Z_{n1}H_{n-l}(kR)\exp[i(n-l)\alpha_{12}]=\begin{cases}0, & l\neq\pm1,\ -\infty<l<\infty\\ \pm C_{10}/2, & l=\pm1\end{cases}$$

$$(2.23b)$$

$$B_{lm1}+\sum_{n=-\infty}^{\infty}B_{nm2}X_{nm2}K_{n-l}(k_mR)\exp[il\pi]\exp[i(n-l)\alpha_{21}]=0,$$
$$-\infty<l<\infty,\quad 1\leqslant m<\infty \tag{2.24a}$$

$$B_{lm2}+\sum_{n=-\infty}^{\infty}B_{nm1}X_{nm1}K_{n-l}(k_mR)\exp[i(n-l)\alpha_{12}]\exp[il\pi]$$
$$=\begin{cases}0, & l\neq\pm1,\ -\infty<l<\infty\\ C_{1m}/2, & l=\pm1,\ 1\leqslant m<\infty\end{cases} \tag{2.24b}$$

where

$$C_{10}=\frac{-2\cosh kh(kh\sinh kh-\cosh kh+1)}{k^2J_1'(ka_2)(\sinh kh\cosh kh+kh)} \tag{2.25a}$$

$$C_{1m}=\frac{-2\cos k_mh(k_mh\sin k_mh+\cos k_mh-1)}{k_m^2I_1'(k_ma_2)(\sin k_mh\cos k_mh+k_mh)} \tag{2.25b}$$

$(A_{l1},\ A_{l2})$, $(B_{l01},\ B_{l02})$ and $(B_{lm1},\ B_{lm2})$ are completely determined by linear equations (2.22), (2.23) and (2.24) respectively. Unfortunately, they form an infinite system. In order to evaluate them, (2.22) and (2.23) are truncated to a $2(2L+1)$ system of equations with $2(2L+1)$ unknowns and (2.24) is truncated to a $2M(2L+1)$ system of equations in $2M(2L+1)$ unknowns.

In order to simplify the expression for the velocity potential, we interchange the summation order and replace l by $-l$, and thus we have

$$\sum_{n=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}A_{ni}Z_{ni}H_{n+l}(kR)J_l(kr_j)\exp[il(\pi-\theta_j)]\exp[i(n+l)\alpha_{ij}]$$
$$=\sum_{l=-\infty}^{\infty}\Big[\sum_{n=-\infty}^{\infty}A_{ni}Z_{ni}H_{n-l}(kR)\exp[i(n-l)\alpha_{ij}]\Big]\exp[il\theta_j]J_l(kr_j) \tag{2.26}$$

$$\sum_{n=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\sum_{m=1}^{\infty}B_{nmi}X_{nmi}K_{n+l}(k_mR)I_l(k_mr_j)\exp[-il(\pi-\theta_j)]\exp[i(n+l)\alpha_{ij}]f_m(z)$$
$$=\sum_{l=-\infty}^{\infty}\sum_{m=1}^{\infty}\Big[\sum_{n=-\infty}^{\infty}B_{nmi}X_{nmi}K_{n-l}(k_mR)\exp[i(n-l)\alpha_{ij}]\exp[il\theta_j]\Big]$$
$$\cdot\exp[il\pi]I_l(k_mr_j)f_m(z) \tag{2.27}$$

By using equations (2.22) to (2.24), we can simplify $\phi(r_j,\ \theta_j,\ z)$ as

$$\phi(r_j,\theta_j,z)=-\frac{igA}{\omega}\sum_{n=-\infty}^{\infty}A_{nj}[Z_{nj}H_n(kr_j)-J_n(kr_j)]f_0(z)\exp[in\theta_j]$$
$$+i\omega\psi\sum_{n=-\infty}^{\infty}B_{n0j}[Z_{nj}H_n(kr_j)-J_n(kr_j)]f_0(z)\exp[in\theta_j]$$
$$+i\omega\psi\sum_{n=-\infty}^{\infty}\sum_{m=1}^{\infty}B_{nmj}[X_{nmj}K_n(k_mr_j)-I_n(k_mr_j)]f_m(z)\exp[in\theta_j]$$

$$+i\omega\psi\delta_{j2}[C_{10}J_1(kr_j)f_0(z)+\sum_{m=1}^{\infty}C_{1m}I_1(k_mr_j)f_m(z)]\cos\theta_2 \qquad (2.28)$$

where $\delta_{12}=0$ and $\delta_{22}=1$. In particular, the velocity potential on the jth cylinder becomes

$$\phi(a_j,\theta_j,z)=-\frac{2gA}{\pi\omega ka_j}\sum_{n=-\infty}^{\infty}\frac{A_{nj}}{H_n'(ka_j)}f_0(z)\exp[in\theta_j]$$

$$+i\omega\psi\sum_{n=-\infty}^{\infty}\left[\frac{-2i}{\pi ka_j}\frac{B_{n0j}}{H_n'(ka_j)}f_0(z)\exp[in\theta_j]+\sum_{m=1}^{\infty}\frac{B_{nmj}f_m(z)}{k_ma_jK_n'(k_ma_j)}\exp[in\theta_j]\right]$$

$$+i\omega\psi\delta_{j2}\left[C_{10}J_1(ka_j)f_0(z)+\sum_{m=1}^{\infty}C_{1m}I_1(k_ma_j)f_m(z)\right]\cos\theta_2 \qquad (2.29)$$

where the following Wronskian relations for Bessel functions[12] have been used:

$$J_n'(ka_j)H_n(ka_j)-H_n'(ka_j)J_n(ka_j)$$
$$=-i[J_{n+1}(ka_j)Y_n(ka_j)-J_n(ka_j)Y_{n+1}(ka_j)]=-\frac{2i}{\pi ka_j}$$

$$I_n'(k_ma_j)K_n(k_ma_j)-I_n(k_ma_j)K_n'(k_ma_j)=\frac{1}{k_ma_j}$$

For a single fixed circular cylinder, we assume that the center of the cylinder is at the origin and $\Psi=0$. Then, from equations (2.22) to (2.24), we have $A_{n1}=-i^n$, $B_{n01}=B_{nm1}=0$. Equation (2.29) reduces to the result of McCamy and Fuchs[13]. For a single oscillating circular cylinder, moving the origin to the axis of the cylinder, we have $A_{n2}=-i^n$, $B_{102}=C_{10}/2$, $B_{1m2}=C_{1m}/2$, $B_{n02}=B_{nm2}=0\,(n\neq 1)$. This recovers the result of Drake et al.[14]. If $\Psi=0$, both cylinders are fixed and $B_{nm1}=B_{nm2}=0$. Equation (2.22) is the same as that of Linton and Evans[9].

III. Forces and Moments

The first-order horizontal wave excited forces $\mathrm{Re}(f_{xj}e^{-i\omega t})$, $\mathrm{Re}(f_{yj}e^{-i\omega t})$ and base overturning moment $\mathrm{Re}(M_{yj}e^{-i\omega t})$ on the jth cylinder are given by integrating the pressure, which is induced by incident and diffraction waves, over the surface of the cylinder. Thus

$$\begin{Bmatrix} f_{xj} \\ f_{yj} \end{Bmatrix}=\int_0^{2\pi}\int_{-h}^{0}-\rho i\omega\left(\phi_I+\sum_{j=1}^{2}\phi_{dj}\right)\begin{Bmatrix}\cos\theta_j\\ \sin\theta_j\end{Bmatrix}a_j dz d\theta_j$$

$$=\frac{2i\rho gA\tanh kh}{k^2H_1'(ka_j)}(A_{1j}\mp A_{-1j})\begin{Bmatrix}1\\ i\end{Bmatrix} \qquad (3.1a)$$

$$M_{yj}=\int_0^{2\pi}\int_{-h}^{0}-\rho i\omega\left(\phi_I+\sum_{j=1}^{2}\phi_{dj}\right)\cos\theta_j a_j(z+h)dzd\theta_j$$

$$=\frac{2i\rho gA}{kH_1'(ka_j)}(A_{1j}-A_{-1j})\frac{kh\sinh kh-\cosh kh+1}{k^2\cosh kh} \qquad (3.1b)$$

The forces $\mathrm{Re}(f_{rxj}e^{-i\omega t})$, $\mathrm{Re}(f_{ryj}e^{-i\omega t})$ and moment $\mathrm{Re}(M_{ryj}e^{-i\omega t})$ due to radiation can be expressed as

$$\begin{Bmatrix} f_{rxj} \\ f_{ryj} \end{Bmatrix}=\int_0^{2\pi}\int_{-h}^{0}-\rho i\omega\sum_{j=1}^{2}\phi_{rj}\begin{Bmatrix}\cos\theta_j \\ \sin\theta_j\end{Bmatrix}a_j dz d\theta_j$$

$$=-\frac{2i\rho\omega^2\psi\tanh kh}{k^2H_1'(ka_j)}(B_{10j}\mp B_{-10j})\begin{Bmatrix}1\\ i\end{Bmatrix}+\sum_{m=1}^{\infty}\frac{\rho\omega^2\psi\pi\tan k_m h}{k_m^2K_1'(k_m a_j)}(B_{1mj}\pm B_{-1mj})\begin{Bmatrix}1\\ i\end{Bmatrix}$$

$$+\delta_{j2}\rho\omega^2\pi a_j\psi\left[\frac{\tanh kh}{k}C_{10}J_1(ka_j)+\sum_{m=1}^{\infty}\frac{\tan k_m h}{k_m}C_{1m}I_1(k_m a_j)\right]\begin{Bmatrix}1\\ 0\end{Bmatrix}$$

$$=-\omega^2\begin{Bmatrix}\bar{A}_{1\psi j}\\ \bar{A}_{2\psi j}\end{Bmatrix}\psi-i\omega\begin{Bmatrix}\bar{B}_{1\psi j}\\ \bar{B}_{2\psi j}\end{Bmatrix}\psi \tag{3.2a}$$

$$M_{ryj}=\int_0^{2\pi}\int_{-h}^{0}-\rho i\omega\sum_{j=1}^{2}\phi_{rj}\cos\theta_j a_j(z+h)dz d\theta_j$$

$$=\left[-\frac{2i\rho\omega^2\psi}{kH_1'(ka_j)}(B_{10j}-B_{-10j})+\rho\omega^2 a_j\delta_{j2}\psi\pi C_{10}J_1(ka_j)\right]\frac{kh\sinh kh-\cosh kh+1}{k^2\cosh kh}$$

$$+\left[\rho\omega^2\psi\sum_{m=1}^{\infty}\frac{\pi(B_{1mj}+B_{-1mj})}{k_mK_1'(k_m a_j)}+\rho\omega^2 a_j\psi\delta_{j2}\pi C_{1m}I_1(k_m a_j)\right]$$

$$\frac{k_m h\sin k_m h+\cos k_m h-1}{k_m^2\cos k_m h}$$

$$=-\omega^2\bar{A}_{5\psi j}\psi-i\omega\bar{B}_{5\psi j}\psi \tag{3.2b}$$

where $\bar{A}_{k\psi j}$, $\bar{B}_{k\psi j}$ are the added masses and radiation damping coefficients due to the pitch motion of cylinder 2 respectively.

Cylinder 2 is freely pinned at a fixed point at the bottom and remains upright in still water due to the excess buoyancy in the cylinder. The linearized equation for the overturning motion of cylinder 2 can be written as

$$I\ddot{\Psi}+C\Psi=(M_{y2}+M_{ry2})e^{-i\omega t} \tag{3.3}$$

where I is the moment of inertia of cylinder 2 about the articulated point, M_{y2} is the wave induced pitching moment on cylinder 2 and M_{ry2} is the hydrodynamic moment on cylinder 2 due to radiation. The stiffness coefficient C arises from the balance of moments induced by buoyancy and weight forces acting on centers of buoyancy and gravity of cylinder 2. It is given by

$$C=\frac{1}{2}\rho g\pi a_2^2h^2-M_2gC_g \tag{3.4}$$

In the present computations, $M_2/\rho\pi a_2^2h=0.5$ and $C_g=0.5h$.

The above equation is valid for small Ψ and can be solved by the frequency domain analysis. After the response of pitch motion of cylinder 2 is determined, we can calculate the second-order forces. Actually, the total force is

$$\begin{Bmatrix} F_{xj} \\ F_{yj} \end{Bmatrix} = -\int_0^{2\pi}\int_{z_1}^{z_2} a_j\{-\rho gz+\varepsilon(p^{(1)}+\delta_{j2}\rho g a_j\cos\theta_j\Psi^{(1)})+\varepsilon^2[p^{(2)}$$

$$+\delta_{j2}(z+h)\Psi^{(1)}\frac{\partial p^{(1)}}{\partial x}-\delta_{j2}a_j\cos\theta_j\Psi^{(1)}\frac{\partial p^{(1)}}{\partial z}+\delta_{j2}\rho g a_j\cos\theta_j\Psi^{(2)}$$

$$+\delta_{j2}\frac{\rho g}{2}(2z+h)(\Psi^{(1)})^2]\}\begin{Bmatrix} \cos\theta_j \\ \sin\theta_j \end{Bmatrix} dz d\theta_j+O(\varepsilon^3) \tag{3.5a}$$

where

$$z_1=-h+\delta_{j2}\varepsilon a_j\cos\theta_j\Psi^{(1)}+\delta_{j2}\varepsilon^2 a_j\cos\theta_j\Psi^{(2)}+O(\varepsilon^3)$$

$$z_2=\varepsilon(\zeta^{(1)}+\delta_{j2}a_j\cos\theta_j\Psi^{(1)})+O(\varepsilon^2)$$

$$=\varepsilon\left(-\frac{1}{g}\frac{\partial\Phi^{(1)}}{\partial t}\bigg|_{z=0}+\delta_j a_j\cos\theta_j\Psi^{(1)}\right)+O(\varepsilon^2)$$

The superscripts (1) and (2) denote the first and the second order respectively.

$$M_{yj}=-\int_0^{2\pi}\int_{z_1}^{z_2} a_j\{-\rho gz+\varepsilon(p^{(1)}+\delta_{j2}\rho g a_j\cos\theta_j\Psi^{(1)})+\varepsilon^2[p^{(2)}$$

$$+\delta_{j2}(z+h)\Psi^{(1)}\frac{\partial p^{(1)}}{\partial x}-\delta_{j2}a_j\cos\theta_j\Psi^{(1)}\frac{\partial p^{(1)}}{\partial z}+\delta_{j2}\rho g a_j\cos\theta_j\Psi^{(2)}$$

$$+\delta_{j2}\frac{\rho g}{2}(2z+h)(\Psi^{(1)})^2]\}\cos\theta_j(z+h)dz d\theta_j+O(\varepsilon^3) \tag{3.5b}$$

For the mean drift forces and moment, the second-order potential are not needed, as will be seen later.

According to Drake et al. (1984), for an articulated cylinder, the second-order forces are

$$F_{xj}^{(2)}=\rho a_j\int_0^{2\pi}\Bigg[\int_{-h}^0\Bigg[\frac{\partial\Phi^{(2)}}{\partial t}+(z+h)\Psi^{(1)}\left(\frac{\partial^2\Phi^{(1)}}{\partial r_j\partial t}\cos\theta_j-\frac{\partial^2\Phi^{(1)}}{a_j\partial\theta_j\partial t}\sin\theta_j\right)$$

$$-a_j\cos\theta_j\Psi^{(1)}\frac{\partial^2\Phi^{(1)}}{\partial z\partial t}+\frac{1}{2}|\nabla\Phi^{(1)}|^2\Bigg]dz-\frac{1}{2}g\zeta_r^{(1)2}$$

$$-a_j\Psi^{(1)}\frac{\partial\Phi^{(1)}}{\partial t}\bigg|_{z=-h}\cos\theta_j\Bigg]\cos\theta_j d\theta_j \tag{3.6}$$

where

$$\zeta_r=\eta_r e^{-i\omega t}=\frac{i\omega}{g}\Phi+\delta_{j2}\Psi a_j\cos\theta_j$$

For the mean second-order forces (time-averaged forces), the above expressions become

$$\begin{Bmatrix} \bar{F}_{xj}^{(2)} \\ \bar{F}_{yj}^{(2)} \end{Bmatrix}=\frac{1}{4}\rho a_j\int_0^{2\pi}\Bigg\{-g\eta_r\eta_r^*+\int_{-h}^0\nabla\phi\cdot\nabla\phi^* dz-\delta_{j2}2i\omega\psi^*\int_{-h}^0\Bigg[(z+h)\left(\frac{\partial\phi}{\partial r_j}\cos\theta_j\right.$$

$$\left.-\frac{\partial\phi}{a_j\partial\theta_j}\sin\theta_j\right)-a_j\frac{\partial\phi}{\partial z}\cos\theta_j\Bigg]dz-\delta_{j2}2i\omega a_j\psi^*\phi\bigg|_{z=-h}\cos\theta_j\Bigg\}$$

$$\cdot\begin{Bmatrix}\cos\theta_j\\ \sin\theta_j\end{Bmatrix}d\theta_j \tag{3.7a}$$

$$\bar{M}^{(2)}_{\psi j}=\frac{1}{4}\rho a_j\int_0^{2\pi}\cos\theta_j d\theta_j\Big\{-g\eta_r\eta_r^* h+\int_{-h}^0\nabla\phi\cdot\nabla\phi^*(z+h)dz-\delta_{j2}2i\omega\psi^*$$

$$\cdot\int_{-h}^0\Big[(z+h)^2\Big(\frac{\partial\phi}{\partial r_j}\cos\theta_j-\frac{\partial\phi}{a_j\partial\theta_j}\sin\theta_j\Big)-a_j\frac{\partial\phi}{\partial z}(z+h)\cos\theta_j\Big]dz\Big\} \tag{3.7b}$$

where the superscript * denotes the complex conjugate and the superscript (1) is omitted in the above expressions.

IV. Discussion of Results

Some numerical examples are shown in this section to illustrate the effects of various parameters on forces, added masses, radiation damping coefficients and the response of the system. Unless it is stated otherwise, the values of the parameters invloved in the present study are $R=8$m, $a_1=a_2=2$m, $k=0.2$/m, and $h=8$m. In order to evaluate the accuracy of our computation, we have calculated two examples given by McIver and Evans (1984) and Drake et al. (1984) for the wave excited force in the diffraction problem and the steady drift force in the radiation problem respectively. Our results agree completely with their results. In our calculation, $M=L=8$ is accurate enough. In all the following presentations, the horizontal wave excited forces and the steady drift forces are nondimensionalized by ρgAa_2h and ρgA^2a_2 respectively. The dimensionless added masses and radiation damping coefficients $a_{i\psi j}$ and $b_{i\psi j}$ due to the overturning motion of cylinder 2 are defined by

$$a_{1\psi j}=\frac{\bar{A}_{1\psi j}}{\rho\pi a_j^2h^2/2},\quad a_{5\psi j}=\frac{\bar{A}_{5\psi j}}{\rho\pi a_j^2h^3/2},\qquad (j=1,\ 2) \tag{4.1a}$$

$$b_{1\psi j}=\frac{\bar{B}_{1\psi j}}{(\pi/2)\rho a_j^2h^2\sqrt{g/h}},\quad b_{5\psi j}=\frac{\bar{B}_{5\psi j}}{(\pi/2)\rho a_j^2h^3\sqrt{g/h}},\qquad (j=1,\ 2) \tag{4.1b}$$

where $\bar{A}_{i\psi j}$ and $\bar{B}_{i\psi j}$ are defined in equations (3.2a) and (3.2b) respectively.

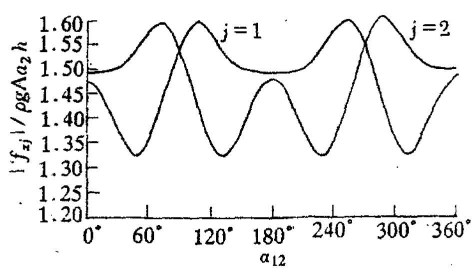

Fig. 2 Wave excited forces in the x direction versus the incident angle α_{12}

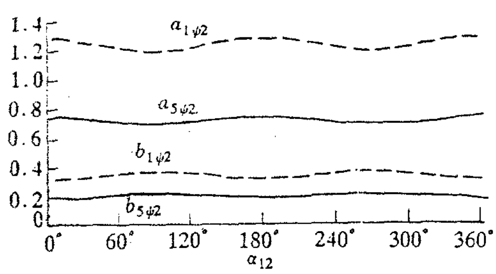

Fig. 3 Added mass and damping coefficients of the articulated cylinder versus the incident angle α_{12}

The dependence of the wave excited forces in the x direction on the incident angle α_{12} is shown in Fig. 2. It is noted from Fig. 2 that the wave excited forces are symmetric about $\alpha_{12}=$

π. Because the radii of the two circular cylinders are equal, $|f_{x1}(\alpha_{12})|=|f_{x2}(\pi-\alpha_{12})|$ according to physical intuition. It is valuable to note that the maximum magnitudes of the wave excited forces are not at $\alpha_{12}=0$ or $\pi/2$, but at some incident angle between them. The wave force components perpendicular to the wave direction are small in comparison with those parallel to the wave direction at any incident angle α_{12}. Therefore, they can be neglected in computing the magnitudes of the wave excited forces. The dependence of the added masses and radiation damping coefficients on the incident angle is shown in Fig. 3. We note from Fig. 3 that the added masses have minimum values and the radiation damping coefficients have maximum values when the incident angle $\alpha_{12}=\pi/2$.

The variation of the wave excited forces with the dimensionless separation distance kR is shown in Fig. 4. At zero incident angle, the wave excitied force acting on the fixed cylinder (cylinder 1) oscillates with a larger amplitude than that on the articulated cylinder (cylinder 2). The amplitude of oscillation decreases as the separation distance increases. As kR approaches infinity, the wave excited forces tend to that of an isolated cylinder.

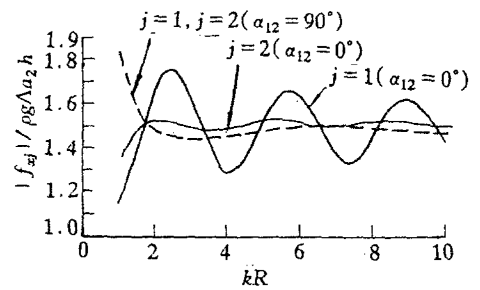

Fig. 4 Wave excited forces versus the separation distance kR

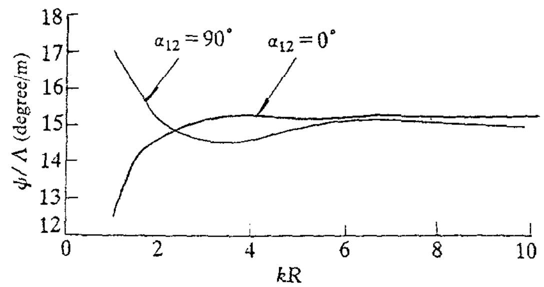

Fig. 6 Response of the articulated cylinder

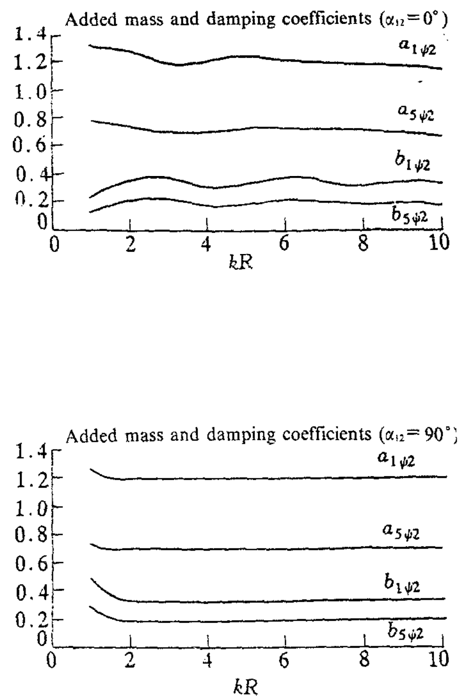

Fig. 5 Added mass and damping coefficients of the articulated cylinder versus the separation distance kR

Though at zero incident angle, the added masses and damping cofficient of the articulated cylinder oscillates with the separation distance kR as shown in Fig. 5, the amplitude is very small. They are constant at $\alpha_{12}=\pi/2$, and increases very rapidly as the two cylinders become closer. So, the added masses and the radiation damping are almost independent of the separation distance kR. This is the reason why a plane-wave approximation (or large-spacing approximation) developed by Simon[6] can be applied to the case with normal spacing. We note from Fig. 6 that the response of the articulated cylinder is almost unchanged when kR

becomes large. For small kR, however, the response of the articulated cylinder decreases rapidly due to the shadow effect of the leading cylinder at $\alpha_{12}=0$, and becomes large due to the interaction between two cylinders at $\alpha_{12}=\pi/2$.

As the size of the cylinder increase, the wave excited forces increase until they reach maximum values near $ka=0.5$ (Fig. 7), not near $ka=1$ as in the case for an isolated cylinder. Then they decrease as well as oscillate with further increase in ka at $\alpha_{12}=0$ and the amplitude for the leading fixed cylinder is large then that for the back articulated cylinder. As shown in Fig. 8, the steady drift force increases fast when $ka>1$. The resonance for the articulated cylinder occurs near $ka=0.2$ (Fig. 9) where the viscous effect should be take into consideration.

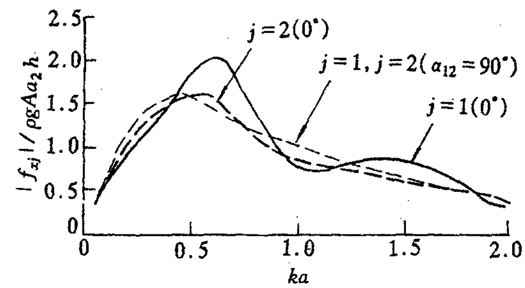

Fig. 7 **Wave excited forces versus the size of the cylinder**

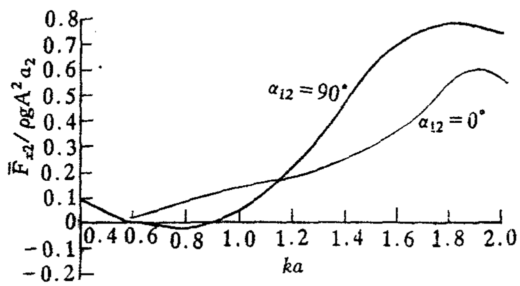

Fig. 8 **Drift forces on the articulated cylinder**

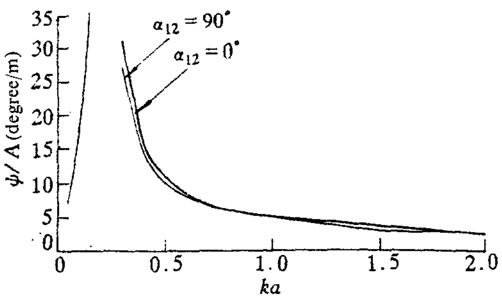

Fig. 9 **Response of the articulated cylinder**

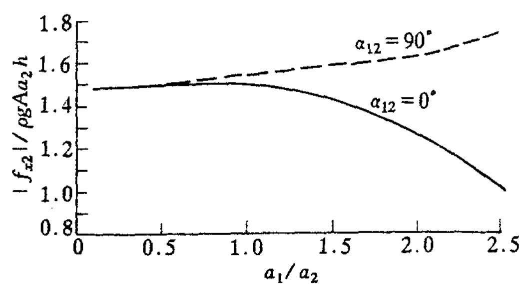

Fig. 10 **Wave excited forces on the articulated cylinder versus the size ratio**

It is obvious that the wave excited force on the fixed cylinder increases as its size increases. However, as the relative size a_1/a_2 increases, the wave excited force on the back cylinder decreases at $\alpha_{12}=0$ as shown in Fig. 10, because of the large wake of the front cylinder. When $\alpha_{12}=\pi/2$, the wave excited force on the articulated cylinder increases slowly. Since the separation distance between the centers of two cylinders is fixed, the gap between two cylinders becomes small when the radius of the fixed cylinder becomes large. Thus, the interaction between two cylinders is strong, and the component of the wave excited force and the steady second-order force perpendicular to the wave direction may not be negligible.

V. Conclusions

Based on the present results, it is found that the magnitudes of wave excited forces on cylinders depend on the incident angle which is the angle between the incident wave direction and the line joining two cylinder centers. The wave force component perpendicular to the wave direction is very small in comparison to that parallel to the wave direction. The maximum wave force occurs at a certain incident angle which is neither zero nor right angle. The wave excited forces also depend on the separation distance between the two cylinders. At zero incident angle, the wave force actint on the front fixed cylinder oscillates with a larger amplitude than that on the back articulated cylinder and the amplitude of oscillation decreases as the separation distance increases. For large separation distances, the wave forces on both cylinders approach to those for isolated cylinders respectively. If the front cylinder becomes large, the wave force on the back one decreases at zero incident angle because of the large wake of the front cylinder. On the other hand, the wave force on the articulated cylinder increases slowly if the incident angle is 90 degrees.

The added masses and radiation damping cofficients for articulated cylinder corresponding to the excited force in the wave direction and the overturning moment are considered. It is found that the added masses have minimum values and the radiation damping coefficients have maximum values when the incident angle is 90 degrees. The radiation damping coefficient and the added masses oscillate with the separation distance at zero incident angle while they are almost independent of the separation distance at $\alpha=\pi/2$. Resonance for the articulated cylinder occurs at certain wavelength.

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