Nonlinear Fatigue Damage Model Based on the Residual Strength Degradation Law*

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In this paper, a logarithmic expression to describe the residual strength degradation process is developed in order to fatigue test results for normalized carbon steel. The definition and expression of fatigue damage due to symmetrical stress with a constant amplitude are also given. The expression of fatigue damage can also explain the nonlinear properties of fatigue damage. Furthermore, the fatigue damage of structures under random stress is analyzed, and an iterative formula to describe the fatigue damage process is deduced. Finally, an approximate method for evaluating the fatigue life of structures under repeated random stress blocking is presented through various calculation examples.

Key Words : Fatigue Damage, Strength Degradation, Random Stress

1. Introduction

It is widely known that damage is an important parameter in describing the mechanical properties of materials in fatigue test. How to define the test and understand the degradation of damage is of great importance for studying fatigue disciplinarian, carrying out fatigue design of structural and estimating fatigue life. Under a constant amplitude stress cycle, the fatigue life can be calculated according to the stress–life curve of the material. For a nonconstant amplitude stress cycle, fatigue is usually studied based on the concept of damage accumulation, of which the main focus is the assumption of structural fracture when the total damage attains a critical level. Fatigue is a process of damage accumulation, where the mechanical properties of materials worsen gradually, i.e., the residual strength of the material will decrease gradually. Rotem (1988) and Flory (1980) reported that different materials have different residual strength degradation rules[1],[2]. Xie and Yu (1994) established a logarithm degradation model of the residual strength for normalized carbon steel[3], how-

ever, their model has a close connection with the maximum amplitude and its instance of occurrence in the external stress cycle, thus it is not useful for studying the damage process for a random stress cycle. Yu (1982) demonstrated an interesting property of fatigue[4], that is the damage increases slowly in the initial stage, and increases rapidly as it approaches the critical damage to fracture, Xie and Yu's logarithm degradation model of the residual strength reflects the above property of fatigue well.

We present another logarithm degradation model of the residual strength based on the test results of normalized carbon steel obtained by Xie and Yu (1994). In this model, the amplitude is not included in the residual strength expression, hence, it is convenient to extend to fatigue damage analysis for the random stress cycle. Furthermore, the corresponding definition and expression of damage are given according to the corresponding physics concept. One advantage of the model is that it can explain the nonlinear properties of the fatigue damage process at various stages of the stress level cycle. Finally, an approximate method of estimating the fatigue life for a repeated random stress block cycles is put forward.

2. Fatigue Damage Model under Constant Amplitude Stress Cycle

For normalized carbon steel excited under the symmetrical constant-amplitude stress cycle, the

(a) $\sigma=310$ MPa, $\qquad$ (b) $\sigma=395$ MPa,
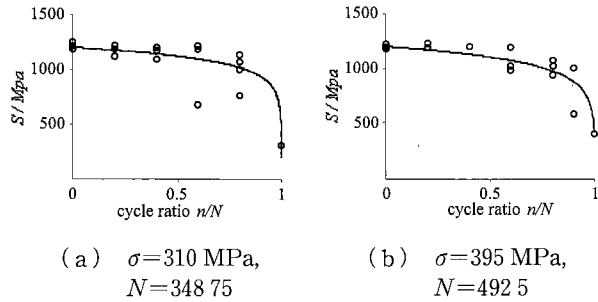$\quad N=348\,75$ $\qquad\qquad\quad N=492\,5$

Fig. 1 Residual strength degradation process

strength degradation rule has been tested by Xie and Yu (1994) and is shown in Fig. 1. After analyzing the test data, the expression describing the residual strength degradation is presented as follows.

$$S(n)=S_0\left[1+\frac{\ln\left(1-\frac{n}{N+1}\right)}{\ln(N+1)}\right] \qquad (1)$$

Here $S(n)$ is the residual strength after n stress cycles, S_0 is the initial strength, or the endurance limit of the material, and N is the fatigue life under the stress cycle. According to the material stress-life or S-N curve gives $N=N_0\sigma_{-1}^\varepsilon/\sigma^\varepsilon$, where σ_{-1} is the material fatigue limit, and N_0 is the fatigue life corresponding to σ_{-1}. For the nonsymmetrical constant-amplitude stress cycle, Hu and Xu (1995) pointed out that the equivalent stress amplitude σ_{eq} could be obtained through one of the following equations[5].

Goodman equation $\qquad \sigma_{eq}=\dfrac{\sigma_a}{\sigma_b-\sigma_m}\sigma_b \qquad (2)$

Gerber equation $\qquad \sigma_{eq}=\dfrac{\sigma_a}{\sigma_b^2-\sigma_m^2}\sigma_b^2 \qquad (3)$

Soderberg equation $\qquad \sigma_{eq}=\dfrac{\sigma_a}{\sigma_s-\sigma_m}\sigma_b \qquad (4)$

σ_s is the material tensile limit.

Bagci equation $\qquad \sigma_{eq}=\dfrac{\sigma_a}{\sigma_b^4-\sigma_m^4}\sigma_b^4 \qquad (5)$

In the above equivalent methods, the Goodman equation is the most widely applied in practice.

It can be proved from Eq.(1) that $S(0)=S_0$, and $S(N)=0$. According to the physics concept of damage, defining the ratio of the amount of strength degradation $(S_0-S(n))$ to initial strength of materials S_0, the fatigue damage after n stress cycles is

$$D(n)=\frac{S_0-S(n)}{S_0}=1-\frac{\ln(N+1-n)}{\ln(N+1)} \qquad (6)$$

It is found that $D(0)=0$, $D(N)=1$. Figure 2 shows the fatigue damage process corresponding to Fig. 1. Furthermore, the amount of fatigue damage for the n-th stress cycle is defined as

$$\delta(n)=D(n)-D(n-1)$$
$$=\frac{\ln(N+2-n)-\ln(N+1-n)}{\ln(N+1)} \qquad (7)$$
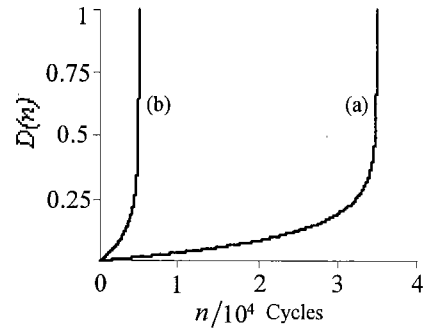
The above definition can describe the nonlinear prop-

Fig. 2 Fatigue damage process

(a) Under high-low $\qquad$ (b) Under low-high
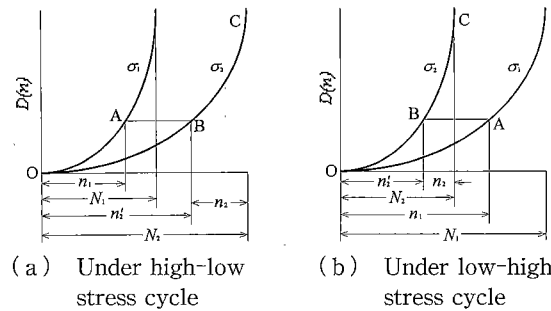$\quad$ stress cycle $\qquad\qquad$ stress cycle

Fig. 3 Fatigue damage process (OABC) under two different stages of stress cycle sequences

erties of fatigue damage. Assuming that the fatigue lives of the structure are N_1 and N_2 under constant amplitude stress cycles σ_1 and σ_2, respectively, the stress σ_1 cycles n_1 times first, then the stress σ_2 cycles n_2 times, and a structural fracture occurs. Figure 3 indicates that the amount of fatigue damage of the structural under stress σ_1 cycled n_1 times and σ_2 cycled (N_2-n_2) times are equal, and thus, Eq.(6) is rewritten as

$$1-\frac{\ln(N_1+1-n_1)}{\ln(N_1+1)}=1-\frac{\ln(1+n_2)}{\ln(N_2+1)} \qquad (8)$$

The above equation can also be expressed approximately as

$$\frac{\ln(N_1-n_1)}{\ln N_1}=\frac{\ln n_2}{\ln N_2} \qquad (9)$$

or

$$\frac{\ln(1-n_1/N_1)}{\ln N_1}=\frac{\ln(n_2/N_2)}{\ln N_2} \qquad (10)$$

When $\sigma_1>\sigma_2$, then $N_1<N_2$ and, $\ln(1-n_1/N_1>\ln(n_2/N_2)$, thus $\dfrac{n_1}{N_1}+\dfrac{n_2}{N_2}<1$. Otherwise when $\sigma_1<\sigma_2$, then $\dfrac{n_1}{N_1}+\dfrac{n_2}{N_2}<1$. The above deduction method can be extended to an arbitrary number of stress stage cycles, therefore for a high-low stress cycle $\sum_i\dfrac{n_i}{N_i}<1$ and for a low-high stress cycle $\sum_i\dfrac{n_i}{N_i}>1$.

3. Fatigue Damage Model under Random Stress Cycle

Let $D(i)$ represent the amount of fatigue damage of a structure under i random stress cycles, $\sigma_{e(i+1)}$ represent the structure attaining the same amount of fatigue damage as $D(i)$ under the symmetrical constant-amplitude stress $\sigma_{e(i+1)}$ cycled n_{i+1} times, that is

$$D(i)=1-\frac{\ln(N_{i+1}+1-n_{i+1})}{\ln(N_{i+1}+1)} \tag{11}$$

then

$$n_{i+1}=(N_{i+1}+1)\cdot[1-(N_{i+1}+1)^{-D(i)}] \tag{12}$$

On the other hand, the amount of fatigue damage of a structure under the $(i+1)$-th stress cycle is obtained from Eq.(7) as

$$\delta(i+1)$$
$$=\frac{\ln[N_{i+1}+2-(n_{i+1}+1)]-\ln[N_{i+1}+1-(n_{i+1}+1)]}{\ln(N_{i+1}+1)} \tag{13}$$

Substituting Eq.(12) into Eq.(13) gives

$$\delta(i+1)=-\frac{\ln[1-(N_{i+1}+1)^{D(i)-1}]}{\ln(N_{i+1}+1)} \tag{14}$$

Which is rewritten as

$$D(i+1)=D(i)-\frac{\ln[1-(N_{i+1}+1)^{D(i)-1}]}{\ln(N_{i+1}+1)} \tag{15}$$

The above expression provides an iterative method of calculating the fatigue damage process step by step. For a structure under repeated random stress block cycles, let m represent the number of stress block cycles, then the amount of fatigue damage $D(m)$ of the structure after the first stress block cycle can be calculated iteratively using Eq.(15). Assuming there exists an equivalent symmetrical constant-amplitude stress cycle, of which the first stress cycle leads to $D(m)$, then from Eq.(2) we can obtain

$$D(m)=1-\frac{\ln N_T}{\ln(N_T+1)} \tag{16}$$

or

$$N_T-(N_T+1)^{1-D(m)}=0 \tag{17}$$

Here N_T is the fatigue life of a structure under an equivalent stress cycle, which can be easily obtained using the above algebraic equation. Many examples indicate that N_T can be taken as an approximate value of the structural fatigue life under the original repeated random stress block cycles, and N_T represents the number of repetitions the stress block.

4. Example

Let $\sigma_b=610\,\text{MPa}$ and $\sigma_{-1}=218.6\,\text{MPa}$. Figure 4 shows the external stress history of an arbitrary stress block cycle. Here, the Rychlik rainflow counting method posed by Rychlik (1987) is introduced to demonstrate the distribution of the stress block cycle[6], and is presented in Fig. 5, where σ_{i1} and σ_{i2} are
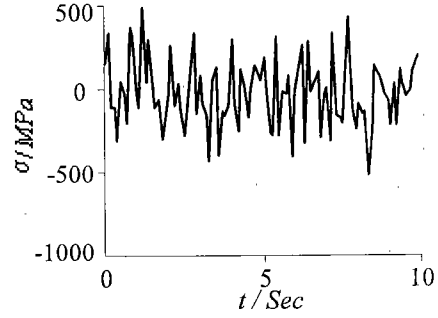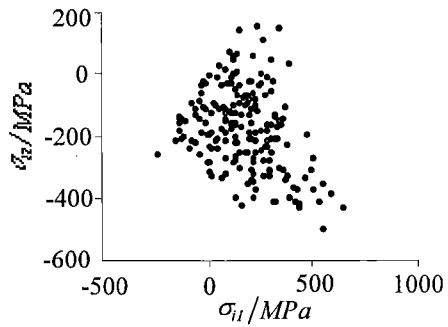

Fig. 4 Stress cycle history

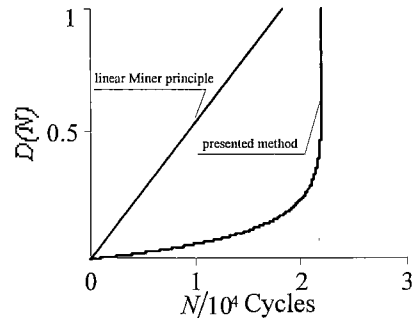Fig. 5 Rychlik rainflow counting distribution

Fig. 6 Fatigue damage process with stress block cycle

the peak value and lower value of the i-th stress cycle, respectively. The structural fatigue damage amount under the first stress block cycle is $D=4.5908 \times 10^{-6}$, which can be obtained using the iterative equation (15). The structural fatigue life is calculated through the linear Miner principle and the presented method, and the solutions are 18 353 and 21 974 stress block cycles respectively. The structural fatigue damage process for the stress block cycles is demonstrated in Fig. 6.

5. Conclusions

1) The authors present an expression that describes the residual strength degradation process of a normalized carbon steel structure under the symmetrical constant amplitude stress cycle. The definition and calculation method of the amount of

fatigue damage are also given. The corresponding damage expression can explain the nonlinear property of the fatigue damage process under different stages of stress sequences.

2) The iterative calculation method for the amount of structural fatigue damage is presented.

3) An approximate method of estimating the structural fatigue life under a repeated stress block cycle is achieved through numerous practical calculations.

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