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SEMI-WEIGHT FUNCTION METHOD ON COMPUTATION OF STRESS INTENSITY FACTORS IN DISSIMILAR MATERIALS *

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Abstract: *Semi-weight function method is developed to solve the plane problem of two bonded dissimilar materials containing a crack along the bond. From equilibrium equation, stress and strain relationship, conditions of continuity across interface and free crack surface, the stress and displacement fields were obtained. The eigenvalue of these fields is lambda. Semi-weight functions were obtained as virtual displacement and stress fields with eigenvalue-lambda. Integral expression of fracture parameters,* K_{I} *and* K_{II}*, were obtained from reciprocal work theorem with semi-weight functions and approximate displacement and stress values on any integral path around crack tip. The calculation results of applications show that the semi-weight function method is a simple, convenient and high precision calculation method.*

Key words: dissimilar material; interface crack; stress intensity factor; semi-weight function method; plane fracture problem

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Introduction

On the study of interface crack problems, Williams[1] was the firstly to analyze stress field near crack tip through the method of eigenfunction expansion and discovered the existence of oscillatory character. Rice and Sih[2] formulated stress field of interface crack with the conception of stress intensity factor. From then on, to solve the stress intensity factor of specific structure and loads became the key of interface crack problems. Among those methods to calculate stress intensity factors of interface crack, boundary element method, contour integral method, weight-function method and boundary collocation method were commonly used.

Bueckner presented weight function method in 1970[3]. Weight function method gave approach to decouping the influences between geometry and loads of crack. When weight function

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of typical crack body was obtained, the stress intensity factors can be obtained by simply calculating an integral along any path around the crack tip. To interface crack problems, Gao[4] used weight function method to solve anisotropic interface crack problem, Banks-Sills[5] solved some specific problem with weight function method, Shen Lian-xi, Yu Shou-wen[6] got explicit expression of weight function near interface crack tip of infinite body with semi-infinite interface crack problem. However, because weight function is relevant to geometric size of crack body, in the application process of weight function method, weight function must be solved according to specific geometric shape, its degree of difficulty is almost equivalent to solving a specific crack problem. If the geometric shape is very complex, it is difficult to get the weight function. Similarly, because of the difficulty of satisfying boundary condition of finite body, it is hard to get a perfect weight function.

Liu Chen-tu and Zhang Duan-zhong[7] put forward the semi-weight function in 1991. The form of semi-weight function is similar to weight function. The stress intensity factor can be deduced from an integration formula. The semi-weight function is independent of geometric size of crack body and boundary condition. This method had a perfect application in planar problem of mode Ⅰ. This method is extended in this paper to treat the problem of two bonded dissimilar materials containing a crack along the bond. Stress and displacement fields near crack tip are firstly deduced through Williams expansion. The singularity eigenvalue is λ. The virtual stress and displacement fields which singularity eigenvalue is $-\lambda$ are also deduced through Williams expansion. These virtual fields are called semi-weight functions. From the principle of reciprocal work theorem, the stress intensity factors K_{I} and K_{II} of interface crack have integration formula along any path around crack tip from bottom crack surface to upper crack surface. Approximate values on the path are needed. Because the singularity of crack tip is avoided when far field values are adopted in integration, even these approximate values are solved through rough model or method the stress intensity factors can get high precision. Compared with the deduction of weight function, the same semi-weight functions can be used in any situation of interface crack. The solution range can be extended and the calculation difficulty can be reduced.

1 Displacement and Stress Fields on Crack Tip and Semi-Weight Functions

1.1 Displacement and stress fields on crack tip

To the problem of two bonded dissimilar materials containing a crack along the bond, the stress function has the form

$$U_i = r^{\lambda+1}F_i(\theta,\lambda) + \mathrm{c.c.} \qquad (i = 1,2), \tag{1}$$

where F_i is

$$F_i = A_i\sin(\lambda+1)\theta + B_i\cos(\lambda+1)\theta + C_i\sin(\lambda-1)\theta + D_i\cos(\lambda-1)\theta, \tag{2}$$

A_i, B_i, C_i, D_i are undetermined complex coefficients, λ can be determined by characteristic equation, c.c. is conjugate item of last one, accordingly stress and displacement fields can be determined by

$$\sigma_{irr} = \frac{1}{r}\frac{\partial U_i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 U_i}{\partial \theta^2},\ \sigma_{i\theta\theta} = \frac{\partial^2 U_i}{\partial r^2},\ \sigma_{ir\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial U_i}{\partial \theta}\right), \tag{3}$$

$$\begin{cases} u_{ir} = \dfrac{r^\lambda}{8G_i\lambda}(-4\lambda(\lambda+1)F_i + (k_i+1)(F_i'' + (\lambda+1)^2F_i)) + \mathrm{c.c.}, \\ u_{i\theta} = \dfrac{r^\lambda}{8G_i\lambda(\lambda-1)}(-4\lambda(\lambda-1)F_i' - (k_i+1)(F_i''' + (\lambda+1)^2F_i')) + \mathrm{c.c.}, \end{cases} \tag{4}$$

they are

$$\begin{cases}\sigma_{irr} = -\lambda r^{\lambda-1}(A_i(\lambda+1)\sin(\lambda+1)\theta + B_i(\lambda+1)\cos(\lambda+1)\theta + \\ \qquad C_i(\lambda-3)\sin(\lambda-1)\theta + D_i(\lambda-3)\cos(\lambda-1)\theta) + \text{c.c.}, \\ \sigma_{i\theta\theta} = \lambda(\lambda+1)r^{\lambda-1}(A_i\sin(\lambda+1)\theta + B_i\cos(\lambda+1)\theta + \\ \qquad C_i\sin(\lambda-1)\theta + D_i\cos(\lambda-1)\theta) + \text{c.c.}, \\ \sigma_{ir\theta} = -\lambda r^{\lambda-1}(-B_i(\lambda+1)\sin(\lambda+1)\theta + A_i(\lambda+1)\cos(\lambda+1)\theta - \\ \qquad D_i(\lambda-1)\sin(\lambda-1)\theta + C_i(\lambda-1)\cos(\lambda-1)\theta) + \text{c.c.}, \\ u_{ir} = -r^{\lambda}(A_i(\lambda+1)\sin(\lambda+1)\theta + B_i(\lambda+1)\cos(\lambda+1)\theta + \\ \qquad C_i(\lambda-k_i)\sin(\lambda-1)\theta + D_i(\lambda-k_i)\cos(\lambda-1)\theta)/(2G_i) + \text{c.c.}, \\ u_{i\theta} = -r^{\lambda}(-B_i(\lambda+1)\sin(\lambda+1)\theta + A_i(\lambda+1)\cos(\lambda+1)\theta - \\ \qquad D_i(\lambda+k_i)\sin(\lambda-1)\theta + C_i(\lambda+k_i)\cos(\lambda-1)\theta)/(2G_i) + \text{c.c.},\end{cases} \tag{5}$$

where $k_i = \begin{cases} 3-4\nu_i & \text{(plane strain)}, \\ (3-\nu_i)/(1+\nu_i) & \text{(plane stress)}, \end{cases}$

G_i is shear modulus.

Crack surface conditions are

$$\sigma_{1\theta\theta}\big|_{\theta=-\pi} = \sigma_{1r\theta}\big|_{\theta=-\pi} = 0,\ \sigma_{2\theta\theta}\big|_{\theta=\pi} = \sigma_{2r\theta}\big|_{\theta=\pi} = 0. \tag{6}$$

Continuity conditions on bond of two materials are

$$\begin{cases} u_{1r}\big|_{\theta=0} = u_{2r}\big|_{\theta=0},\ u_{1\theta}\big|_{\theta=0} = u_{2\theta}\big|_{\theta=0}, \\ \sigma_{1\theta\theta}\big|_{\theta=0} = \sigma_{2\theta\theta}\big|_{\theta=0},\ \sigma_{1r\theta}\big|_{\theta=0} = \sigma_{2r\theta}\big|_{\theta=0}. \end{cases} \tag{7}$$

Putting Eq. (5) into conditions (6) ~ (7) and define complex stress intensity factor by

$$\bar{K} = K_{\mathrm{I}} - \mathrm{i}K_{\mathrm{II}} = \lim_{r\to 0}(\sigma_{\theta\theta} - \mathrm{i}\sigma_{r\theta})\big|_{\theta=0} r^{1-\lambda}. \tag{8}$$

Then the undetermined complex coefficients are

$$A_1 = \frac{-\mathrm{i}A(R+\lambda)}{2(1+\lambda)},\ B_1 = \frac{A(R-\lambda)}{2(1+\lambda)},\ C_1 = \frac{\mathrm{i}A}{2},\ D_1 = \frac{A}{2};$$

$$A_2 = \frac{-\mathrm{i}A(1+R\lambda)}{2(1+\lambda)},\ B_2 = \frac{A(1-R\lambda)}{2(1+\lambda)},\ C_2 = \frac{\mathrm{i}AR}{2},\ D_2 = \frac{AR}{2},$$

where $R = (G_1 + G_2k_1)/(G_2 + G_1k_2)$ is material constant of two bonded dissimilar materials;

$\lambda = \dfrac{1}{2} + \mathrm{i}\dfrac{\ln R}{2\pi}$ is eigenvalue;

$A = \dfrac{\bar{K}}{(1+R)\lambda}$.

On the interface in front of the crack tip we can get the stress field

$$\sigma_{\theta\theta} + \mathrm{i}\sigma_{r\theta}\big|_{\theta=0} = Kr^{-\lambda}. \tag{9}$$

Relative displacement between crack surface

$$\delta_r + \mathrm{i}\delta_\theta = (u_r + \mathrm{i}u_\theta)\big|_{-\pi}^{\pi} \sim r^{\lambda}. \tag{10}$$

Because λ is complex number, as $r \to 0$ and we approach the crack tip the stress field (9) exhibit an oscillatory singularity and crack surfaces (10) are embed each other. When $G_1 = G_2$, $\nu_1 = \nu_2$, the stress and displacement fields can be back to expressions of single material.

Consider the continuity on interface, according to interface crack on infinite plate Ref. [2] took continuity condition of stress component ε_x. However, in Ref. [2] this condition is not naturally satisfied, stress component on x-direction of each material is added to make ε_x

continue. Refs. [8], [10] and [11] followed this additional condition on calculation of finite plate. This limited the scope and precision of solution. In this paper, ε_x near crack tip on interface is naturally satisfied, no additional condition is needed. The continuity far from crack tip can be considered in approximate calculations. The scope and precision of solution can be improved. When bi-materials constant R equals 1, σ_x near crack tip on interface is continuous.

1.2 Semi-weight functions with form of virtual displacement and stress fields

Changing λ into $-\lambda$ in Eq. (5) and putting it into crack surface conditions (6) and continuity conditions (7), we get expressions of semi-weight functions

$$\left\{\begin{aligned} \sigma_{irr}^{(s)} &= \lambda r^{-\lambda-1}(A_i^{(s)}(\lambda-1)\sin(\lambda-1)\theta - B_i^{(s)}(\lambda-1)\cos(\lambda-1)\theta + \\ &\quad C_i^{(s)}(\lambda+3)\sin(\lambda+1)\theta - D_i^{(s)}(\lambda+3)\cos(\lambda+1)\theta) + \text{c.c.}, \\ \sigma_{i\theta\theta}^{(s)} &= \lambda(\lambda-1)r^{-\lambda-1}(-A_i^{(s)}\sin(\lambda-1)\theta + B_i^{(s)}\cos(\lambda-1)\theta - \\ &\quad C_i^{(s)}\sin(\lambda+1)\theta + D_i^{(s)}\cos(\lambda+1)\theta) + \text{c.c.}, \\ \sigma_{ir\theta}^{(s)} &= \lambda r^{-\lambda-1}(-B_i^{(s)}(\lambda-1)\sin(\lambda-1)\theta - A_i^{(s)}(\lambda-1)\cos(\lambda-1)\theta - \\ &\quad D_i^{(s)}(\lambda+1)\sin(\lambda+1)\theta - C_i^{(s)}(\lambda+1)\cos(\lambda+1)\theta) + \text{c.c.}, \\ u_{ir}^{(s)} &= -r^{-\lambda}(A_i^{(s)}(\lambda-1)\sin(\lambda-1)\theta - B_i^{(s)}(\lambda-1)\cos(\lambda-1)\theta + \\ &\quad C_i^{(s)}(\lambda+k_i)\sin(\lambda+1)\theta - D_i^{(s)}(\lambda+k_i)\cos(\lambda+1)\theta)/(2G_i) + \text{c.c.}, \\ u_{i\theta}^{(s)} &= -r^{-\lambda}(-B_i^{(s)}(\lambda-1)\sin(\lambda-1)\theta - A_i^{(s)}(\lambda-1)\cos(\lambda-1)\theta - \\ &\quad D_i^{(s)}(\lambda-k_i)\sin(\lambda+1)\theta - C_i^{(s)}(\lambda-k_i)\cos(\lambda+1)\theta)/(2G_i) + \text{c.c.} \end{aligned}\right. \tag{11}$$

The parameters of semi-weight functions are not unique, with proper parameters we can get integration form of stress intensity factors directly. In the expressions of parameters, superscript (s) means they are parameters of semi-weight functions, first number of subscript means material number, the second one means group number of semi-weight functions.

Parameters of first group are

$$\left\{\begin{aligned} A_{11}^{(s)} &= \frac{-\mathrm{i}Q(\lambda-R)}{2(\lambda-1)},\ B_{11}^{(s)} = \frac{Q(\lambda+R)}{2(\lambda-1)},\ C_{11}^{(s)} = \frac{\mathrm{i}Q}{2},\ D_{11}^{(s)} = -\frac{Q}{2}, \\ A_{21}^{(s)} &= \frac{-\mathrm{i}Q(R\lambda-1)}{2(\lambda-1)},\ B_{21}^{(s)} = \frac{Q(R\lambda+1)}{2(\lambda-1)},\ C_{21}^{(s)} = \frac{\mathrm{i}QR}{2},\ D_{21}^{(s)} = -\frac{QR}{2}. \end{aligned}\right. \tag{12}$$

Parameters of second group are

$$\left\{\begin{aligned} A_{12}^{(s)} &= \frac{Q(\lambda-R)}{2(\lambda-1)},\ B_{12}^{(s)} = \frac{\mathrm{i}Q(\lambda+R)}{2(\lambda-1)},\ C_{12}^{(s)} = -\frac{Q}{2},\ D_{12}^{(s)} = -\frac{\mathrm{i}Q}{2}, \\ A_{22}^{(s)} &= \frac{Q(R\lambda-1)}{2(\lambda-1)},\ B_{22}^{(s)} = \frac{\mathrm{i}Q(R\lambda+1)}{2(\lambda-1)},\ C_{22}^{(s)} = -\frac{QR}{2},\ D_{22}^{(s)} = -\frac{\mathrm{i}QR}{2}, \end{aligned}\right. \tag{13}$$

where

$$Q = \frac{G_1 G_2}{\pi(G_1 + G_2 k_1)}.$$

2 Stress Intensity Factors $\mathbf{K}_{\mathrm{I}}$ and $\mathbf{K}_{\mathrm{II}}$ Expressed by Semi-Weight Functions

Consider a plane structure with interface crack to arbitrary loads. We consider an arbitrary region Ω (Fig. 1), including crack surface. The boundary of this region is $\partial V = C_s + \Gamma$, where C_s is the crack surface and Γ is another boundary. If we cut out a circle with radius R, and the boundary of the circle is C_R. The region after cutting out is $\bar{\Omega}$, the crack surface is $\bar{C}_S$ and outer boundary is Γ. Then we get

$$\begin{cases} \lim\limits_{R\to 0}\bar{\Omega} = \Omega, \\ \lim\limits_{R\to 0}\bar{C}_S = C_S. \end{cases} \tag{14}$$

From the principle of virtual work (reciprocal work theorem), we get

$$\int_{\bar{\Omega}} f_i^{(s)} u_i \mathrm{d}\Omega + \int_{\Gamma+\bar{C}_S+C_R} p_i^{(s)} u_i \mathrm{d}s = \int_{\bar{\Omega}} f_i u_i^{(s)} \mathrm{d}\Omega + \int_{\Gamma+\bar{C}_S+C_R} p_i u_i^{(s)} \mathrm{d}s, \tag{15}$$

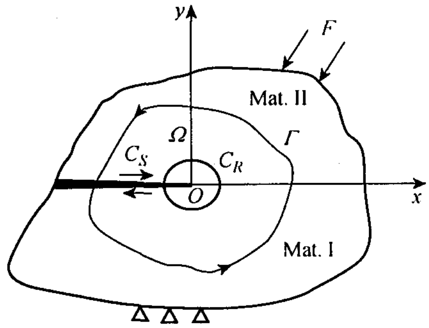

Fig. 1 Crack and integration contour in dissimilar media

where (u_i, p_i, f_i) and $(u_i^{(s)}, p_i^{(s)}, f_i^{(s)})$ are two sets of displacements, tractions and volume forces, respectively. Expanding and transforming the above expression along integral path and ignoring volume force, we get

$$\int_{C_R} (p_i^{(s)} u_i - p_i u_i^{(s)}) \mathrm{d}s + \int_{\bar{C}_S} (p_i^{(s)} u_i - p_i u_i^{(s)}) \mathrm{d}s = \int_{\Gamma} (p_i u_i^{(s)} - p_i^{(s)} u_i) \mathrm{d}s. \tag{16}$$

Assume that (u_i, p_i) are the real approximate displacements and tractions, there must be $p_i = 0$ on crack surface. We set $(u_i^{(s)}, p_i^{(s)})$ as semi-weight functions, on crack surface $p_i^{(s)} = 0$ too. The above expression (16) by absence of integration on C_S can be changed to

$$\int_{C_R} (p_i^{(s)} u_i - p_i u_i^{(s)}) \mathrm{d}s = \int_{\Gamma} (p_i u_i^{(s)} - p_i^{(s)} u_i) \mathrm{d}s. \tag{17}$$

In polar coordinates, there is

$$\int_{C_R} (\sigma_{rr}^{(s)} u_r + \sigma_{r\theta}^{(s)} u_\theta - \sigma_{rr} u_r^{(s)} - \sigma_{r\theta} u_\theta^{(s)}) \mathrm{d}s = \int_{\Gamma} (\sigma_{rr}^{(s)} u_r + \sigma_{r\theta}^{(s)} u_\theta - \sigma_{rr} u_r^{(s)} - \sigma_{r\theta} u_\theta^{(s)}) \mathrm{d}s. \tag{18}$$

Putting Eqs. (5) and (11) into Eq. (18), noticing the piecewise integration in different materials. When R approaches 0, we get the integration form of stress intensity factors with respect to semi-weight functions group 1 or group 2.

$$\begin{cases} K_{\mathrm{I}} = \int_{\Gamma} (\sigma_{rr1}^{(s)} u_r + \sigma_{r\theta 1}^{(s)} u_\theta - \sigma_{rr} u_{r1}^{(s)} - \sigma_{r\theta} u_{\theta 1}^{(s)}) \mathrm{d}s, \\ K_{\mathrm{II}} = \int_{\Gamma} (\sigma_{rr2}^{(s)} u_r + \sigma_{r\theta 2}^{(s)} u_\theta - \sigma_{rr} u_{r2}^{(s)} - \sigma_{r\theta} u_{\theta 2}^{(s)}) \mathrm{d}s. \end{cases} \tag{19}$$

Notice that the second subscript is the group number of semi-weight function.

Compared with the weight function method, this method provides applicable analytical expressions of semi-weight functions and in less restrict conditions. The same semi-weight functions can be used in crack bodies with different geometric shape. Two groups of semi-weight functions presented in this paper can be used to solve out stress intensity factors of interface crack in general situations. With the existence of semi-weight functions, the singularity on crack tip can be avoided from the weight integration form of stress intensity factors. Even the low precision reference solutions on far field integral path are used; the high precision stress intensity factors can be get from weight integration. It needs pointing out that the singularity on crack tip and special characteristic on interface are only local phenomenon, the influence of these phenomenon will be reduced with the increase of r and departure from interface, so the stress intensity factors from integration around crack tip can be reached to demanded calculate precision.

3 Applications

In our applications, we use FEA analysis software ANSYS to calculate the approximate values on integral path. The finite element is PLANE82—2-D 8-Node Structural Solid Element. No singular elements are used in modeling. Integral path is a section of arch from lower surface of crack to upper surface of crack. The radius of the arch is larger than maximum element size on crack tip. We select this kind of integral path only because of its convenience, different kind of integral path can be used according to different situations. K_{I} and K_{II} can be solved out with the coordinate transformed approximate values of displacements and stresses on the path being put into Eq.(19).

In a general way, the finer mesh size we choose, the more precisen result we can get. However, if we choose a proper integral path, the improvement of precision with fine mesh size is not apparent. We can choose coarse mesh size and proper integral path to reduce the amount of FEA calculation. According to the characteristics of stress and displacement and experience of calculations, if the calculation precision of another part of structure can be satisfied, we can choose the radius of integral path (here is an arch) larger than maximum size of element on this place and integral path not close to boundary of structure, the calculation result of stress intensity factors can be acceptable. To simplify the construct and calculation of model in our applications, we adopted the auto mesh facility of ANSYS, and chose integral path as arch with radius larger than element size near crack tip.

3.1 Central interface crack on infinite plate

To approximate the infinite plate of plane stress, we set the size of plate be 20 times of crack length on FEA modeling (Fig.2), where $a = 1$ m, $\sigma = 1$ kN/m^2, $E_1 = 1$ kN/m^2, $\nu_1 = \nu_2 = 0.3$. We simplified this problem with symmetry. The results were listed in Table 1 compared with Refs.[2] and [9] to different elastic modulus ratio. Data outside bracket is $K_{\mathrm{I}}\sqrt{2\pi}/\sigma\sqrt{\pi a}$, inside is $K_{\mathrm{II}}\sqrt{2\pi}/\sigma\sqrt{\pi a}$. The modeling and calculation were simple and convenient; the results were close to references.

Table 1 Stress intensity of center-crack tension infinite bimaterial plate

E_2/E_1	This paper	Ref.[9]	Analytic solution[2]
1	1.000	1.009	1.000
3	0.995(−0.074 6)	0.999(−0.082 2)	0.988(−0.072 4)
10	0.973(−0.117 8)	0.981(−0.128 9)	0.968(−0.117 1)
100	0.949(−0.136 2)	0.968(−0.140 1)	0.953(−0.139 1)
100 0	0.945(−0.138 0)	0.957(−0.153 5)	0.952(−0.141 5)

3.2 Single side skew interface crack on rectangular plate

We calculated the stress intensity factors of single side skew interface crack (for plane strain) on rectangular plate, which did not appear in references. The loads of the plate were uniform tension on both edges parallel to the crack (Fig.3). Different material constants R and angle of crack β were considered. Where $b = 2$ m, $\sigma = 1$ N/m^2, $E_1 = 2.0 \times 10^{11}$ N/m^2, $\nu_1 = \nu_2 = 0.3$.

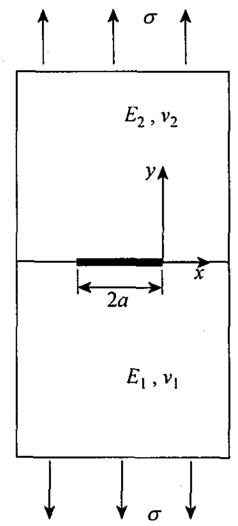

Fig.2 Center-crack tension infinite bimaterial plate

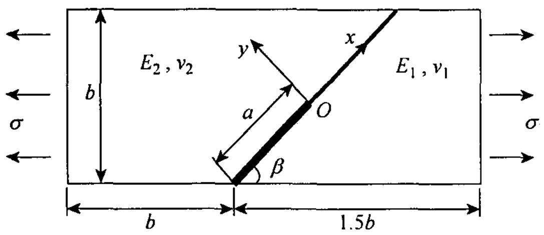

Fig.3 Skew-single-side-crack tension rectangle bimaterial plate

Change R to the form $R = (1 + k_1 G_2/G_1)/(G_2/G_1 + k_2)$, when poisson's ratio is constant, the value of R is up to elastic modulus ratio. The limited range of R is $(\min(k_1, 1/k_2), \max(k_1, 1/k_2))$. To this application, we selected elastic modulus ratio from 1 to 100, then the numeric area of R was (1, 1.778). We plotted the results (Fig.4) according to angles 45°, 60° and 75°, respectively, where $F = |K|\sqrt{2\pi}/\sigma\sqrt{\pi a}$. From the curves we can see that the modulus of complex stress intensity factors decrease with R increasing to a given angle, while absolute values of $K_{\mathrm{II}}/K_{\mathrm{I}}$ increase. To different angles, these changes with R are different, as a whole, when the angle is small the modulus of complex stress intensity factors changes rapidly, and $K_{\mathrm{II}}/K_{\mathrm{I}}$ changes slowly. When $R = 1$, they are the same to behaviors in single material. To the case of single side skew crack on rectangular plate subjected to uniform tension on edges, there are owned composite stress intensity factors of modes I and II. The existence of interface

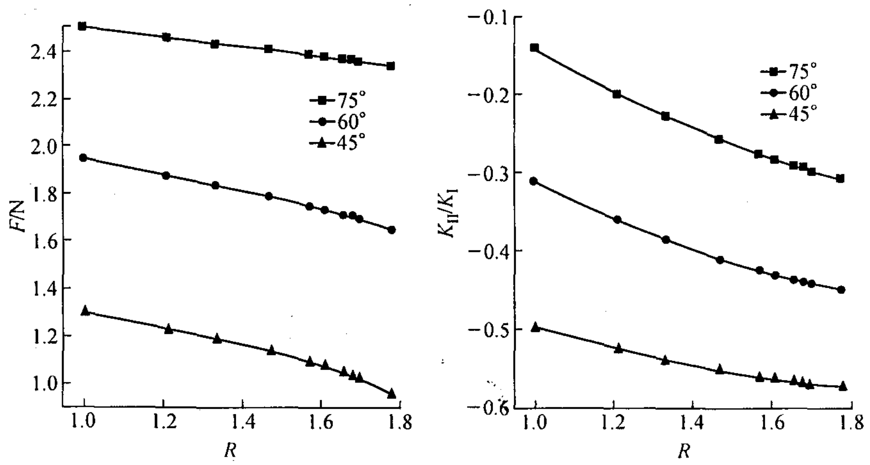

Fig.4 Stress intensity of skew-single-side-crack tension rectangle bimaterial plate

strengthens the shear effect K_{II}. It is apparent with this effect when the crack angle is large, or approaches level crack. In this situation the shear effect from skew angle is not the main. When the angle is small, the shear effect from skew angle itself is the main, while that from interface decreases. This indicates that although there is an oscillatory singularity of stress on crack tip. K_{I} and K_{II} can also appear the performance of tension and shear, the existence of interface strengthens the effect of K_{II}.

4 Conclusions

1) Analysis of general plane problem of interface crack is given in this paper. The expressions of stress and displacement fields on crack tip are presented. The continuity of strain component expressions ε_x on interface is naturally satisfied.

2) Two groups of semi-weight functions according to different materials are deduced. The independent expressions of K_{I} and K_{II} with the form of weight integration are solved.

3) Because the oscillatory singularity of stress on crack tip is avoided in this method, it can be focused on the modeling of whole structure when numerical methods such as FEA are used to calculate far field values. A rather rough model can get precision results.

4) The results of applications indicate the variety of K_{I} and K_{II} with different material constant. Although there is oscillatory singularity of stress on crack tip, K_{I} and K_{II} can also appear the performance of tension and shear.

5) The semi-weight function method used in this paper is simple and reliable. It has high precision and perfect practicability.

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