RESEARCH PAPERS

Difference schemes on non-uniform mesh and their application*

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Abstract High order accurate schemes are needed to simulate the multi-scale complex flow fields to get fine structures in simulation of the complex flows with large gradient of fluid parameters near the wall, and schemes on non-uniform mesh are desirable for many CFD (computational fluid dynamics) workers. The construction methods of difference approximations and several difference approximations on non-uniform mesh are presented. The accuracy of the methods and the influence of stretch ratio of the neighbor mesh increment on accuracy are discussed. Some comments on these methods are given, and comparison of the accuracy of the results obtained by schemes based on both non-uniform mesh and coordinate transformation is made, and some numerical examples with non-uniform mesh are presented.

Keywords: **scheme on non-uniform mesh, high order accurate scheme, compact difference approximation.**

There are two ways to improve the resolution of solutions in computational fluid dynamics: one is to refine the mesh grid system, the other is to construct difference approximation with high order accuracy. Selection of methods to improve the resolution of solutions depends on the problem to be solved. From the point of view of the scales of flow structures, we can divide the practical problems into two classes: the flow problem with large scale structures and the problem with wide range of scales. For the first class of problems the grid generation techniques have made great successes. We can use structured grid system for simple flow configuration and unstructured grid system for complex flow configuration. Development of unstructured grid generation techniques can greatly improve the resolution for the first class of problems. However, because of the complex grid point distribution for the unstructured grid generation, it is difficult to construct high order accurate approximation.

We are interested in improving the resolution of solutions for the second class of problems. One example is the simulation of the turbulent flows. With the recent development of computer technology, many turbulent model problems with low Reynolds number have been solved, but it is difficult even to directly simulate the simple model turbulent flows with high Reynolds number. To solve many model turbulent flows we have to develop high order accurate difference methods.

For many typical model turbulent flows, like shear flow, channel flow and turbulent boundary layer, the physical parameters have large gradient in the small narrow sub-domains. To solve this kind of problem we have to refine the mesh grid in the subdomain, for example, near the wall region, to capture the fine structures. There are two ways to obtain schemes on refined mesh in the near wall region, the method of coordinate transformation and construction of scheme on non-uniform mesh. In Ref. [1] Rai and Moin solved a fully developed incompressible turbulent channel flow with fifth order accurate traditional upwind biased difference approximation on non-uniform mesh, and in Refs. [2, 3] compact schemes on non-uniform mesh were used to simulate the compressible turbulent channel flows. Both analysis and practice show that it is better to use schemes on non-uniform meshes to simulate problems like boundary layer. Higher order accurate schemes based on non-uniform meshes can be obtained by using coefficient match method, Lagrange interpolating polynomial, Hermite interpolating polynomial, residual correction method, and direct Taylor expansion. In section 2

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some methods of scheme construction on non-uniform mesh are presented, and the accuracy analysis and numerical experiments are given in section 3.

1 Scheme construction on non-uniform mesh

Consider the following model equation and its semi-discrete difference approximation

$$\frac{\partial u}{\partial t}+\frac{\partial f}{\partial x}=\mu\frac{\partial^2 u}{\partial x^2},\quad f=cu,\quad c=\text{const.}\tag{1}$$

$$\frac{\partial u_j}{\partial t}+F_j=\mu S_j,\tag{2}$$

where

$$F_j=(\partial f/\partial x)_j,\quad S_j=(\partial^2 u/\partial x^2)_j.$$

Define

$$h_j=x_{j+1}-x_j,\quad \theta_j=h_{j-1}/h_j,$$
$$\delta_x^{\pm}f_j=\mp(f_j-f_{j\pm1}),$$
$$h_j=\sigma_j h,\quad \frac{h_j}{h_{j+1}}=1+O(h).\tag{3}$$

There are many ways to construct the difference approximations on non-uniform mesh, and in this section we will briefly represent some.

1.1 Coefficient match method

Now we consider the scheme construction of traditional difference approximation on non-uniform mesh, which means that the derivative can be directly expressed by the linear combination of the point function. Supposing we have a point function

$$u_j=u(x_j)=u(x+\xi_j),$$
$$x_j=x+\xi_j,\quad \xi_j=\sum_{k=0}^{j-1}h_k,$$
$$h_k=x_{k+1}-x_k,\quad x_0=x,$$

the function $u(x+\xi_j)$ can be expressed by the Taylor expansion series

$$u_j=u(x_j)=u(x+\xi_j)=\sum_{p=0}^{P}\frac{\xi_j^p}{p!}\frac{\partial^p u(x)}{\partial x^p}+\cdots.$$

The nth derivative of function $u(x)$ can also be expressed by the linear combination of point function $u(x+\xi_j)$:

$$\frac{\partial^m u(x)}{\partial x^m}=\sum_{j=-J}^{J'}\alpha_j u(x+\xi_j).\tag{4}$$

After series expansion we have

$$\frac{\partial^m u(x)}{\partial x^m}=\Big(\sum_j\alpha_j\Big)u(x)+\Big(\sum_j\alpha_j\xi_j\Big)\frac{\partial u(x)}{\partial x}+\cdots+\Big(\sum_j\frac{\alpha_j}{m!}\xi_j^m\Big)\frac{\partial^m u(x)}{\partial x^m}+\cdots.$$

To obtain the approximation with required accuracy, the following equations must be satisfied:

$$\sum_j\alpha_j=0,\quad \sum_j\alpha_j\xi_j=0,\quad\cdots,$$
$$\sum_j\frac{\alpha_j}{(m-1)!}\xi_j^{m-1}=0,\quad \sum_j\frac{\alpha_j}{m!}\xi_j^m=1.\tag{5}$$

Here we have $m+1$ equations. $J+J'=m+1$ is the minimum number of grid points to obtain the consistent difference approximation. Using more mesh grid points, we can obtain a higher order accurate approximation. Using the given grid point distribution from Eq. (5), we can obtain the coefficients α_j. The simplest second order accurate difference approximations are

$$F(f_j)=\frac{\theta_j}{1+\theta_j}\frac{\delta_x^+f_j}{h_j}+\frac{1}{1+\theta_j}\frac{\delta_x^-f_j}{h_{j-1}}\tag{6}$$

for the first derivative and

$$S(u_j)=\frac{2}{h_j+h_{j-1}}\Big[\frac{u_{j+1}-u_j}{h_j}-\frac{u_j-u_{j-1}}{h_{j-1}}\Big]\tag{7}$$

for the second derivative. Using this method we can obtain any order accurate traditional difference approximation for any order derivatives.

1.2 Method with Lagrange interpolating polynomial

Using the given mesh grid distribution we can construct Lagrange interpolating polynomial

$$v(x)=\sum_{j=-J}^{J}\varphi_j(x)u_j,\tag{8}$$

where

$$\varphi_j(x_{i+k})=\delta_{j,k},\quad j,k=-J,\cdots,J',$$
$$(\delta_{j,k}=\text{Kronecker delta}),$$
$$\varphi_j(x)=\frac{P_j(x)}{P_j(x_{i+j})},\tag{9}$$
$$P_j(x)=\prod_{\substack{l=-J\\ l\neq j}}^{J'}(x-x_{i+l}).\tag{10}$$

Defining

$$v^{(m)}(x)=\sum_{j=-J}^{J'}\frac{P_j^{(m)}(x)}{P_j(x_{i+j})},\tag{11}$$

we can take $v^{(m)}(x)$ as the difference approximation for the mth derivative of function $u(x)$ at the point x. Rai and Moin obtained the fifth order accurate upwind biased scheme with Lagrange interpolating polynomial and solved the incompressible turbulent channel flow[1]. Details can be found in Ref. [4].

1.3 Method with Hermitian relation

The generalized Hermitian relation is expressed as

$$H^{(0,1,\cdots,m)}\equiv\sum_{j=-J}^{J'}(a_j^{(0)}u_j^{(0)}+a_j^{(1)}u_j^{(1)}+\cdots+a_j^{(m)}u_j^{(m)})=0,$$

$$J \geqslant 0, \quad J' \geqslant 0, \tag{12}$$

where $u_j^{(0)}$ is the original point function, and $u_j^{(k)}$ is the difference approximation of kth derivative. After Taylor series expansion and solving the obtained system of linear algebraic equations, we can define the coefficients $a_j^{(k)}$. The simple Hermitian relation at three grid points x_{i-1}, x_i, x_{i+1} is

$$H^{(1)} \equiv \sum_{j=-1}^{1} (a_j u_j + b_j u_j') = 0, \tag{13}$$

from which we can obtain the simpler compact difference approximations. In the case $\xi_j = jh$, we can obtain

$$a_{-1} = \frac{3\alpha - 2\beta}{h}, \quad a_0 = \frac{4\beta}{h}, \quad a_1 = -\frac{3\alpha + 2\beta}{h},$$
$$b_{-1} = \alpha - \beta, \quad b_0 = 4\alpha, \quad b_1 = \alpha + \beta. \tag{14}$$

To define the first derivative requires $\sum_j b_j = 6\alpha \neq 0$. Higher order accurate compact difference approximations are obtained in this way[5].

1.4 Residual correction

Usually we can obtain a higher order difference approximation in a simpler way by using the known lower order approximation. We constructed the fourth order accurate compact scheme on uniform mesh with residual correction method in 1992[6]. Lerad et al. using the same method constructed the fourth order traditional scheme on uniform mesh in 1996[7,8]. Recently we presented the new high order accurate difference approximations with residual correction.

Considering a first order accurate approximation for the first derivative, after Taylor series expansion we have

$$\frac{f_{j+1} - f_j}{h_j} = \frac{\partial f}{\partial x} + \frac{h_j}{2} \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) + \cdots. \tag{15}$$

Defining $F = \partial f / \partial x$ and discretizing the residual term in Eq. (15),

$$[\partial(\partial f/\partial x)/\partial x]_j = \partial F/\partial x \sim (F_{j+1} - F_j)/(h_j),$$

we can easily obtain the second order Pade scheme

$$(F_{j+1} + F_j)/2 = (f_{j+1} - f_j)/h_j. \tag{16}$$

Considering that the approximation at three grid points, and the difference approximations Eq. (6) for the first derivative and the approximation of Eq. (7) for the second derivative are known, after Taylor series expansion of Eq. (6) for $F(f_j)$, we have

$$\frac{\theta_j}{1+\theta_j} \frac{\delta_x^+ f_j}{h_j} + \frac{1}{1+\theta_j} \frac{\delta_x^- f_j}{h_{j-1}} = \left(\frac{\partial f}{\partial x}\right)_j + \frac{1}{3!} \frac{1}{1+\theta_j} [\theta_j h_j^2 + h_{j-1}^2] \left(\frac{\partial^3 f}{\partial x^3}\right)_j + \cdots. \tag{17}$$

Define $\partial f / \partial x = F$ and $\partial^3 f / \partial x^3 = \partial^2 (F) / \partial x^2 = S(F)$. Using Eq. (7) for $S(F)$ and neglecting the term with $O(h^4)$ for the first derivative, we can obtain a fourth order accurate compact difference approximation on non-uniform mesh

$$\alpha_j F_{j-1} + F_j + \gamma_j F_{j+1} = a_j \frac{\delta_x^+ f_j}{h_j} + b_j \frac{\delta_x^- f_j}{h_{j-1}}, \tag{18}$$

$$\alpha_j = \frac{1}{2(1+\theta_j)}, \quad \gamma_j = \frac{\theta_j}{2(1+\theta_j)},$$
$$a_j = \frac{3\theta_j}{2(1+\theta_j)}, \quad b_j = \frac{3}{2(1+\theta_j)}. \tag{19}$$

It can be seen that Eq. (18) is obtained by using the known lower order approximation without solving the algebraic equation.

In the same way we can construct the third order upwind compact difference approximation for the case $c > 0$ as expressed in Eq. (18), but with

$$a_j = \frac{\theta_j^2}{(1+\theta_j)^2}, \quad b_j = \frac{2 + 3\theta_j}{(1+\theta_j)^2},$$
$$\alpha_j = \frac{1}{1+\theta_j}, \quad \gamma_j = 0, \tag{20}$$

and the third order upwind compact difference approximation for the case of $c < 0$ with coefficients

$$a_j = \frac{\theta_j(3 + 2\theta_j)}{(1+\theta_j)^2}, \quad b_j = \frac{1}{(1+\theta_j)^2},$$
$$\gamma_j = \frac{\theta_j}{1+\theta_j}, \quad \alpha_j = 0. \tag{21}$$

We can also construct the fourth order accurate compact approximation for the second derivative.

1.5 Direct Taylor series expansion

The super compact difference approximations on uniform mesh were constructed in Ref. [9] and on nonuniform mesh in Ref. [10]. Using the Taylor expansion series we can give the basic equation

$$(\alpha\delta_x^+ + \beta\delta_x^-) f_j = \frac{1}{1!}[\alpha + \beta\theta_j] f_j^{\langle 1 \rangle} + \frac{1}{2!}[\alpha - \beta\theta_j^2] f_j^{\langle 2 \rangle} + \cdots + \frac{1}{N!}[\alpha + (-1)^{N+1}\beta\theta_j^N] f_j^{\langle N \rangle},$$

and the auxiliary equation

$$\frac{1}{\theta_{j-1}^l} f_{j-1}^{\langle l \rangle} - 2 f_{j+1}^{\langle l \rangle} + \theta_{j+1}^l f_{j+1}^{\langle l \rangle} = \frac{1}{1!}[1 - \theta_j] f_j^{\langle l+1 \rangle} + \frac{1}{2!}[1 + \theta_j^2] f_j^{\langle l+2 \rangle}$$

$$+\cdots+\frac{1}{(N-l)!}[1+(-1)^{N-1}\theta_j^{N-1}]f_j^{\langle N\rangle}.$$

Defining

$$f_j^{\langle k\rangle}=h_j^k\left(\frac{\partial^k f}{\partial x^k}\right)_j, \tag{22}$$

$$\bar{\boldsymbol{F}}=[f^{\langle 1\rangle},f^{\langle 2\rangle},\cdots,f^{\langle N\rangle}]^{\mathrm{T}},$$

$$\boldsymbol{e}_1=[1,0,\cdots,0]^{\mathrm{T}},$$

$$\boldsymbol{L}(\theta)=\begin{bmatrix}0&0&\cdots&0&0\\ \theta&0&\cdots&0&0\\ 0&\theta^2&\cdots&0&0\\ \cdots&\cdots&\cdots&\cdots&\cdots\\ 0&0&\cdots&\theta^{N-1}&0\end{bmatrix}, \tag{23}$$

$$\bar{\boldsymbol{A}}=\frac{1}{2}\begin{bmatrix}\frac{\alpha+\beta\theta}{1!}&\frac{\alpha-\beta\theta^2}{2!}&\frac{\alpha+\beta\theta^3}{3!}&\cdots&\frac{\alpha+(-1)^{N+1}\beta\theta^N}{N!}\\ 0&\frac{1-\theta}{1!}&\frac{1+\theta^2}{2!}&\cdots&\frac{1+(-1)^{N-1}\theta^{N-1}}{(N-1)!}\\ 0&0&\frac{1-\theta}{1!}&\cdots&\frac{1+(-1)^{N-2}\theta^{N-2}}{(N-2)!}\\ \cdots&\cdots&\cdots&\cdots&\cdots\\ 0&0&0&0&\frac{1-\theta}{1!}\end{bmatrix}, \tag{24}$$

we get a simpler super compact difference approximation on non-uniform mesh

$$-\frac{1}{2}\boldsymbol{L}(1/\theta_{j-1})\bar{\boldsymbol{F}}_{j-1}+[\boldsymbol{L}(\theta_j)+\bar{\boldsymbol{A}}(\theta_j)]\bar{\boldsymbol{F}}_j-\frac{1}{2}\boldsymbol{L}(\theta_{j+1})\bar{\boldsymbol{F}}_{j+1}=\frac{1}{2}(\alpha\delta_x^++\beta\delta_x^-)\boldsymbol{f}_j\boldsymbol{e}_1, \tag{25}$$

where α and β are free parameters, and $f_j^{\langle k\rangle}/h_j^k$ approximates $(\partial f^k/\partial x^k)_j$ with the accuracy order of $N-k+1$. The super compact difference approximation gives more information, has smaller stencil of grid points and higher accuracy compared with schemes with the same order of approximation, but a system of equations has to be solved.

2 Accuracy analysis

The accuracy of schemes is analyzed in two ways in this paper. First we consider the influence of the stretch ratio of the grid size on accuracy with Fourier analysis for the case of $\theta_j=\theta$, and then we will discuss the accuracy of methods based on non-uniform mesh, and coordinate transformation with uniform mesh in the computational domain by numerical experiments.

2.1 Fourier analysis

As mentioned above, the scheme keeps high order accuracy under the condition of

$$\theta_j=1+O(h).$$

If this condition is not satisfied, the order of accuracy will be reduced. Supposing we have the relation

$$\theta_j=\theta\neq 1,$$

considering a single Fourier mode, after putting the single mode into the difference approximation, we can obtain the modified wave number[11]. For the second order symmetrical difference approximation Eq. (6), we have $K_e=K_r+\mathrm{i}K_i$,

$$K_r=\frac{1}{1+\theta}\left\{\frac{1}{\theta}[1-\cos(\alpha\theta)]-\theta[1-\cos(\alpha)]\right\}, \tag{26}$$

$$K_i=\frac{1}{1+\theta}\left[\theta\sin(\alpha)+\frac{1}{\theta}\sin(\alpha\theta)\right], \tag{27}$$

where K_r represents the dissipation characteristics of the numerical solutions, and K_i represents the spread direction of numerical waves. For the third order accurate upwind compact difference approximation, we have

$$K_r=\frac{AC+BD}{AA+BB},\quad K_i=\frac{AD-BC}{AA+BB}, \tag{28}$$

$$A=1+\frac{\cos(\alpha\theta)}{\theta(1+\theta)},$$

$$C=\frac{(2+3\theta)[1-\cos(\alpha\theta)]-\theta[1-\cos(\alpha)]}{(1+\theta)^2},$$

$$B=-\frac{\sin(\alpha\theta)}{\theta(1+\theta)},$$

$$D=\frac{(2+3\theta)\sin(\alpha\theta)+\theta\sin(\alpha)}{(1+\theta)^2}.$$

Figure 1 gives the variations of K_i and K_r as functions of α with different parameters θ for the second order symmetrical approximation Eq. (6), compared with the exact solution. It can be seen from Fig. 1(a) that the accuracy is reduced for the case of $\theta\neq 1$, the scheme is SLW (slow) for the case of $\theta=1.2$, and MXD (mixed) for the case of $\theta=0.8$[11], which can tell us that the property of the scheme can be changed from one class of schemes into another according to the group velocity by changing the parameter θ. It can be seen from Fig. 1(b) that the scheme is non-dissipative for the case of $\theta=1$, dissipative for the case of $\theta=0.8$, and anti-dissipative for the case of $\theta=1.2$, which can tell us that the scheme may change its dissipativity with the change of the parameter θ.

Figure 2 gives the variations of K_i as a function of α with different parameters θ for the third order traditional upwind difference approximation and upwind compact difference approximation Eq. (20). We can see that with the same parameter θ the third order accurate upwind compact difference approximation can give better resolution.

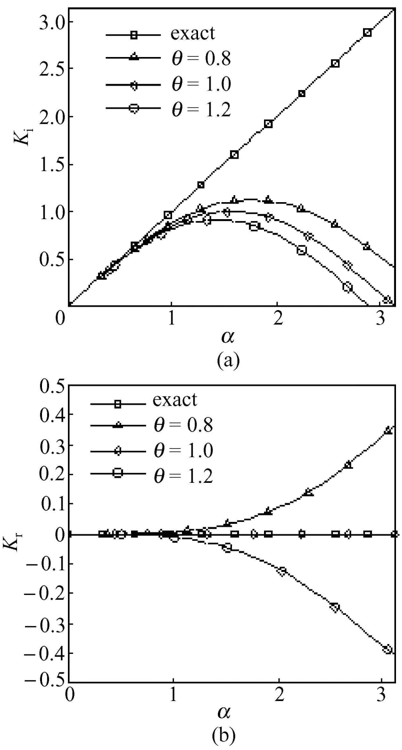


Fig. 1. Variation of (a) K_i and (b) K_r for second order symmetrical approximation.

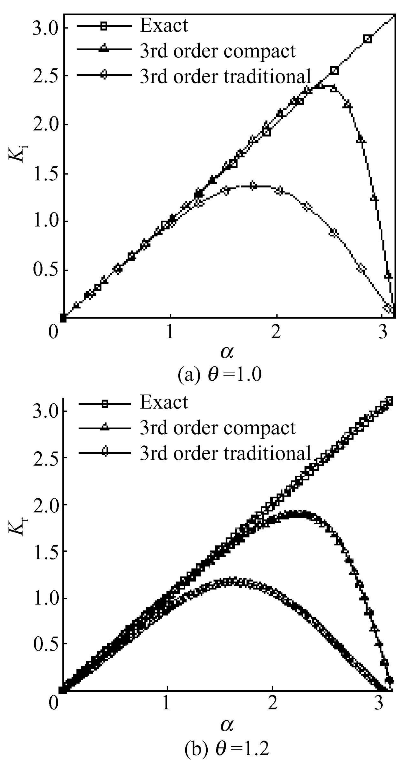


Fig. 2. Variation of K_i for third order upwind approximation with (a) $\theta = 1.0$ and (b) $\theta = 1.2$.

2.2 Numerical tests

When the variation of grid size from point to point is not serious, there is not much difference of accuracy between the schemes obtained by coordinate transformation and by construction on non-uniform mesh. If variation of the grid size is large and high order accurate schemes with large stencil are used to simulate the complex flow field like the boundary layer, and if the grid points are strongly one side biased, the scheme on non-uniform mesh is preferred. Now we consider the following two examples.

Example 1

We have a function and its derivative.

$$u(x) = \mathrm{th}(ax)/\mathrm{th}(a), \quad u' = \mathrm{d}u(x)/\mathrm{d}x.$$

The grid distribution is obtained by the following function

$$\zeta = \mathrm{th}(bx)/\mathrm{th}(b), \quad -1 \leqslant x \leqslant 1, \quad -1 \leqslant \zeta \leqslant 1.$$

In computation with $a = 20$, $b = 2$, the number of grid points is $N = 301$. The function $u(x)$, and its derivative are given in Fig. 3. The derivative is computed by two methods. One uses non-uniform mesh, and the other uses coordinate transformation. The errors between the exact derivative and the numerical solution obtained by fourth and sixth order compact

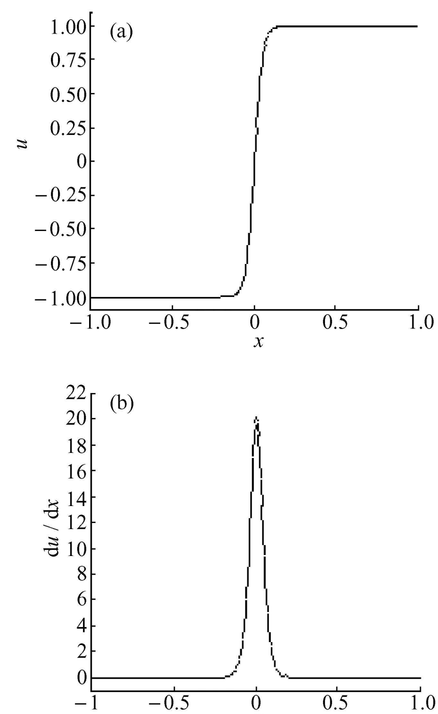
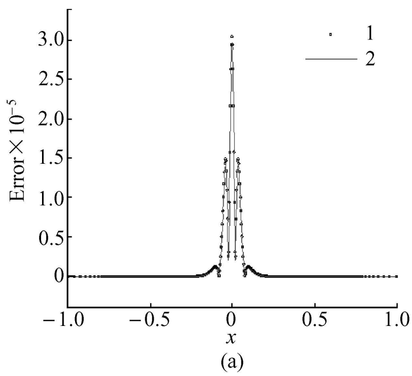


Fig. 3. (a) The function $u = u(x)$ and (b) its derivative $\mathrm{d}u/\mathrm{d}x$.

difference approximation are given in Fig. 4. We can see that the errors of numerical derivative obtained by approximation on non-uniform mesh and by coordinate transformation are almost the same.



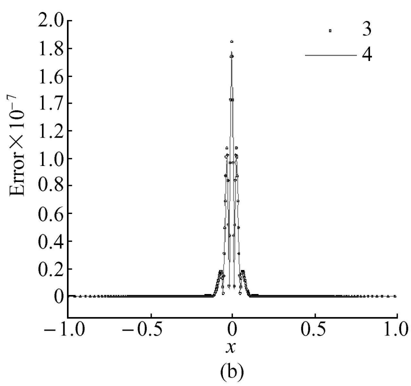


Fig. 4. Errors of solution for (a) fourth order and (b) sixth order difference approximation. (1) and (3), Non-uniform mesh; (2) and (4), coordinate transformation.

Example 2

Suppose we have a function $u(x)=\sin(\pi x)$ defined within $[-1, 1]$, from which $\partial u/\partial x$ can be calculated exactly. The non-uniform mesh grid is generated by

$$x_j = \tanh(b\eta_j)/\tanh(b), \tag{29}$$

where b is a parameter for mesh stretching. The parameters in Eq. (29) used in computation are $b=3$, $\eta_j = jH/N - 1$, $H=2$, $N=160$, where H is the computational domain in x derection, and N is the number of grid points. The error is defined by $\varepsilon_j = F_j - \pi\cos(\pi x)$. The errors for fifth order upwind compact scheme obtained by both construction on non-uniform mesh and method with coordinate transformation are given in Table 1. We can see that the results by approximation on non-uniform mesh with stencil strongly one side biased is much better.

Table 1. Errors of the fifth order scheme

ε_j	Coor. Trans.	Non-uniform Mesh
$j=2$	$+0.275^{-2}$	-0.108^{-6}
$j=3$	-0.153^{-4}	-0.972^{-12}
$j=4$	$+0.780^{-7}$	-0.435^{-15}

Example 3

Now we briefly present some results for 2-D boundary layer problem. The effect of mesh grid distribution is studied numerically. The flow parameters in the computation are: The Reynolds number $Re = 635000$ inch, the incoming Mach number $M_\infty = 2.25$, the wall temperature $Tw = 1.9$, and the domain of computation is $0 \leqslant x \leqslant 5.0$ inch, $0 \leqslant y \leqslant 3.0$ inch. The upstream boundary is located at $x=0$, where the uniform incoming boundary conditions are given, and the leading edge of plan is at $x=0.05$ inch. To verify our calculation, the computed results are compared with Blasius solution. The computations are also carried out on different mesh, and the results are given in Fig. 5. The results of different mesh scales are nearly the same, which means that the grid number is enough. Curve 3 agrees well with curve 1, obtained by using coordinate transformation with uniform mesh near the wall, which can tell us that the result is also quite good if coordinate transformation with uniform mesh near the wall is used. Curve 4 is worse compared with other results obtained by using coordinate transformation with non-uniform mesh.

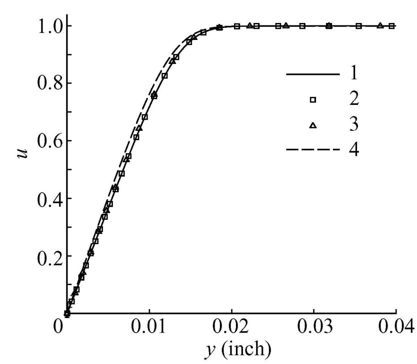


Fig. 5. Velocity profiles for different mesh grid distributions. (1) on direct non-uniform mesh with 401×412; (2) on direct non-uniform mesh with 201×206; (3) coordinate transformation with uniform mesh near the wall; (4) coordinate transformation on direct non-uniform mesh with 201×206.

2.3 Remarks

Schemes based on non-uniform mesh have reasonable accuracy, and it can be constructed on arbitrarily distributed mesh grid points. The advantage of using approximation on non-uniform mesh is near the wall boundary with large gradient of flow parameters. Ref. [4] presented a result of approximation on randomly distributed grid points for the first derivative, but it is difficult to construct schemes for two or three dimensional problems on non-uniform mesh in all the coordinate directions simultaneously.

The approximation based on coordinate transformation is much simpler. If the coordinate transformation is chosen to make sure that it has uniform mesh near the wall in physical plan, we can also obtain better resolution near the boundary wall. If coordinate transformations are used, it is better to use the coordinate transformation with continuous mapping function with continuous derivatives for simulation of complex flow fields with wide range of scales.

From Fourier analysis we can see that the accuracy of schemes on non-uniform mesh is reduced. This analysis is carried out under the suggestion $\theta_j = \theta \neq 1$ (for the boundary layer problem $\theta_j = \theta < 1$), which means that the mesh size will increase to infinity all the time $[0, +\infty]$ with the same rule. Practical applications show that the accuracy of scheme on non-uniform mesh is reasonable.

3 Summary

Some methods of scheme construction and several schemes on non-uniform mesh with high order accuracy have been presented. The effect of stretch ratio of mesh increment is discussed with Fourier analysis. From analysis and numerical experiments, it is shown that in the region near the wall boundary where the gradient of flow parameters is high, better resolution can be obtained by using high order accurate schemes on non-uniform mesh or by using coordinate transformation with continuously mapping function with uniform mesh in physical plan near the wall. The high order accurate upwind compact difference scheme on non-uniform mesh is used to simulate practical problems, and good results are obtained.

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