A Geometry Model for Tortuosity of Flow Path in Porous Media *

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(Received 27 April 2004)

A simple geometry model for tortuosity of flow path in porous media is proposed based on the assumption that some particles in a porous medium are unrestrictedly overlapped and the others are not. The proposed model is expressed as a function of porosity and there is no empirical constant in this model. The model predictions are compared with those from available correlations obtained numerically and experimentally, both of which are in agreement with each other. The present model can also give the tortuosity with a good approximation near the percolation threshold. The validity of the present tortuosity model is thus verified.

PACS: 47.55.Mh, 47.15.−x, 47.55.−t

The characteristics of material transport in porous media, such as the hydraulic permeability or electrical conductivity, have received much attention[1−12] due to many unresolved problems in science and engineering applications. These macroscopic transport parameters (e.g. the hydraulic permeability or electrical conductivity) are usually related to the tortuosity of tortuous path followed by transported materials. However, it is known that the tortuous path a transported material follows is microscopically very complicated. Therefore, the tortuosity for flow path in porous media is conventionally determined in experiments[13,14] and the experimental data are then correlated as correlations with one or more empirical constants. In addition, the tortuosity for flow path in porous media can be also determined by applying the lattice gas (LG) cellular automaton method.[3,4] However, an analytical solution for tortuosity of flow path in porous media is currently not available to our knowledge.

The definition of tortuosity is often given by[1,2]

$$\tau = L_e/L, \tag{1}$$

where L_e and L are the actual length of flow path and the straight length or thickness of a sample/unit cell along the macroscopic pressure gradient, respectively. Some researchers define tortuosity as $\tau = (L_e/L)^2$.[1,2] Some other researchers[1] define the tortuosity as $\tau = (L/L_e)^2$ and found the values of $(L/L_e)^2 = 0.4$. Carman defines the tortuosity as $\tau = L/L_e$, and he found the empirical value $L/L_e = 0.71$,[1] and other values reported in the literature for L/L_e vary in the range of 0.56–0.8.[1] In this work, Eq. (1) is applied for the definition of tortuosity.

Koponen *et al.*[3] applied the LG method to solve numerically a creeping flow of Newtonian incompressible fluid in a two-dimensional porous substance constructed by randomly placed rectangles of equal size and with unrestricted overlap. They obtained a correlation between the average tortuosity of flow path and the porosity ϕ as

$$\tau = 0.8(1-\phi) + 1. \tag{2}$$

In their later work, Koponen *et al.*[4] considered the percolation threshold ϕ_c (found to be 0.33 in their system) and again applied the LG method to solve the flow of a Newtonian incompressible fluid in a two-dimensional porous substance constructed by randomly placed rectangles of equal size and with unrestricted overlap. Their LG simulations were carried out at porosities 0.40–0.90 for tortuosity and numerical results were fitted by the following correlation:

$$\tau = 1 + a\frac{(1-\phi)}{(\phi-\phi_c)^m}, \tag{3}$$

where $\phi > \phi_c$, $a(=0.65)$ and $m(=0.19)$ are the fitting parameters.

A correlation of the form

$$\tau = 1 + P\ln(1/\phi) \tag{4}$$

was obtained by the experiments on flow through beds packed with cubes,[14] and $P(=0.63)$ is the empirical/fitting constant obtained by fitting the experimental data. Equations (2)–(4) all satisfy the condition $\tau = 1$ for $\phi = 1$, this is consistent with the physical situation. However, they all have one or more empirical constants obtained by matching the numerical results or experimental data.

This work focuses on the derivation of a simple and approximate expression for tortuosity of flow path in porous media. The results predicted from the proposed model are compared with those from the available correlations Eqs. (2)–(4).

* Supported by the National Natural Science Foundation of China under Grant No 10272052.
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Figure 1 displays the two possible configurations for flow through porous media of two-dimensional square particles. For configuration a, see Fig. 1(a), we have the total volume V_t of the unit cell shown as the dashed squares:

$$V_t = (b + BF)^2, \tag{5}$$

where b and BF are the particle size and gap size between particles, respectively. The total pore volume in the unit cell is

$$V_p = (BF)^2 + 2BF \times b. \tag{6}$$

Thus, the porosity is

$$\phi = 1 - \Big(\frac{b}{BF + b}\Big)^2. \tag{7}$$

From Eq. (7), we obtain

$$BF = b\Big(\frac{1}{\sqrt{1-\phi}} - 1\Big). \tag{8}$$

Equation (8) indicates that when $\phi \to 0$, $BF \to 0$, and when $\phi \to 1$, $BF \to \infty$. This is expected because when $\phi \to 0$, the gap size between particles tends to be zero, and when $\phi \to 1$, no particle is in the medium, corresponding to the gap BF being infinite.

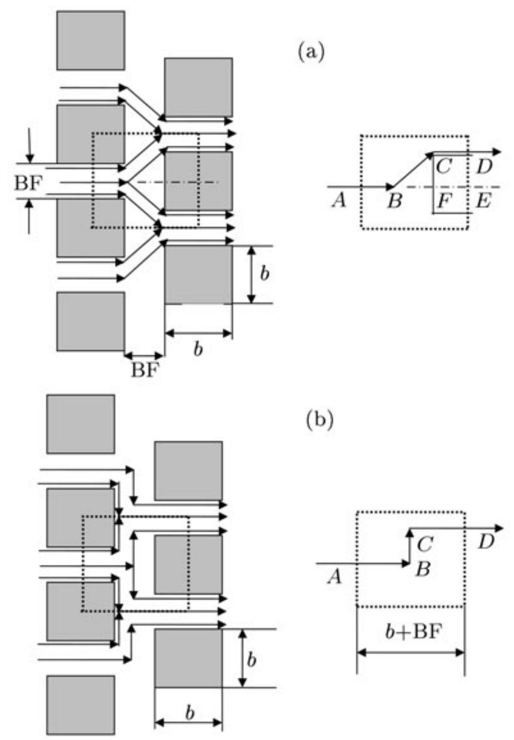

Fig. 1. Two idealized configurations of streamlines for creeping flow in porous media.

From Fig. 1(a), the length of streamline BC satisfies

$$(BC)^2 = b^2\Big(\frac{1}{\sqrt{1-\phi}} - 1\Big)^2 + \frac{b^2}{4}, \tag{9}$$

from which the length of streamline BC can be obtained by

$$BC = b\sqrt{\Big(\frac{1}{\sqrt{1-\phi}} - 1\Big)^2 + \frac{1}{4}}. \tag{10}$$

Equation (10) shows that when $\phi \to 0$, the length $BC = b/2$, and when $\phi \to 1$, the length BC tends to be infinite. This is expected because when $\phi \to 0$, the gap size between particles tends to be zero, resulting in BC tending to be equal to $b/2$ (Fig. 1(a)). When $\phi \to 1$, no particle is in the medium, the length BC thus tends to be infinite.

Since the particles in actual porous media are randomly distributed, this means that some particles overlap each other, while the others do not. Koponen *et al.*,[3,4] in their numerical simulations, constructed a two-dimensional porous substance by randomly placed rectangles of equal size and with unrestricted overlap. Therefore, in the present model, the effect of 'unrestricted overlap' is also taken into account. For this purpose, we argue that since some particles *randomly* overlap each other in real porous media, the period of streamlines may not be the same as that shown in Fig. 1(a) and it may be in the scale of pore size (BF). Therefore, according to the definition Eq. (1), we have the tortuosity for flow through unrestrictedly overlapped particles (referring to configuration a) as

$$\tau_a = \frac{BC}{BF} = \sqrt{1-\phi}\,\frac{\sqrt{\Big(\frac{1}{\sqrt{1-\phi}} - 1\Big)^2 + \frac{1}{4}}}{1 - \sqrt{1-\phi}}. \tag{11}$$

For a porous medium with un-overlapped particles, we assume that the flow path follows the idealized pathway as shown in Fig. 1(b), configuration b. Since the streamline BC is perpendicular to the horizontal direction, we take the whole length of streamline A–B–C–D to calculate the tortuosity in the unit cell. According to the definition of Eq. (1), we obtain the tortuosity for configuration b as

$$\tau_b = \frac{BC + b + BF}{b + BF} = 1 + \frac{b/2}{b + b(1/\sqrt{1-\phi} - 1)} = 1 + \frac{1}{2}\sqrt{1-\phi}. \tag{12}$$

where $BC = b/2$ [see Fig. 1(b)].

The real tortuosity can be obtained by averaging over all possible configurations for flow path in porous media. For simplicity, this work roughly considers two idealized configurations, i.e. some particles are overlapped and the others are not. The averaged tortuosity is thus given by

$$\tau_{av} = (\tau_a + \tau_b)/2 = \frac{1}{2}\Big[1 + \frac{1}{2}\sqrt{1-\phi} + \sqrt{1-\phi} \cdot \frac{\sqrt{\Big(\frac{1}{\sqrt{1-\phi}} - 1\Big)^2 + \frac{1}{4}}}{1 - \sqrt{1-\phi}}\Big]. \tag{13}$$

Equation (13) presents a simple and approximate model for tortuosity of flow path in porous media. There is no empirical constant in this model. It is evident that Eq. (13) is a function of porosity only.

In Fig. 2(a), the present model predictions are compared with those from Eqs. (2)–(4). It is shown that the present simple model presents good agreement (well within the error bars[3,4] by LG simulations) with other correlations. Note that Eqs. (2) and (3) are obtained from the LG simulations, and Eq. (4) is correlated by fitting the experimental data. To examine the behaviour of the present model near the percolation threshold, $\phi_c = 0.33$, we compare the present model predictions with Eq. (3) near the percolation threshold $\phi_c = 0.33$, as shown in Fig. 2(b). It can be seen that the present simple model gives the tortuosity lower about 5% than that predicted by Eq. (3) near the percolation threshold $\phi_c = 0.33$.

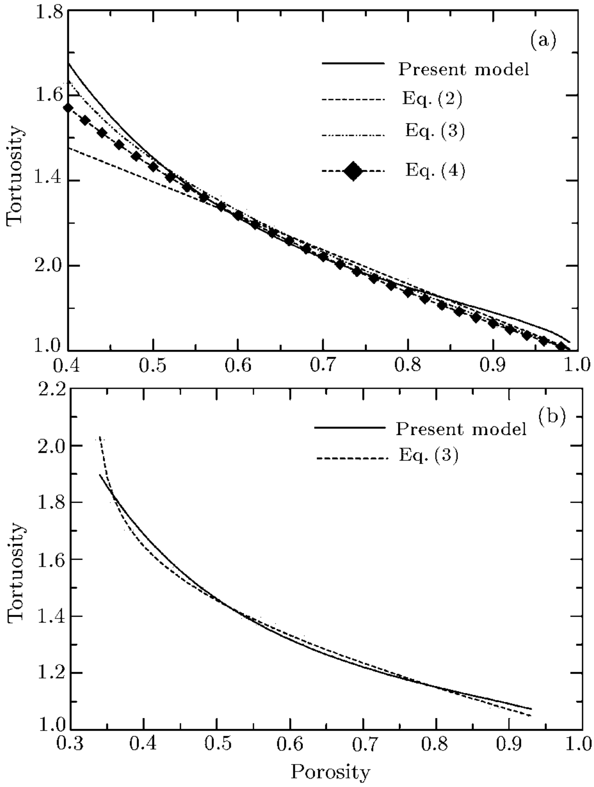

Fig. 2. (a) A comparison between the present model predictions and other models, and (b) a comparison between the present result and the model presented by Koponen *et al.*[4] near the percolation threshold $\phi_c = 0.33$.

Therefore, the present model can also give a good approximate tortuosity near the percolation threshold, and the validity of the present simple model is further verified. Since Eqs. (2) and (4) do not deal with the tortuosity near the percolation threshold, this work is comparable only with Eq. (3) near the percolation threshold.

In summary, we have shown a simple geometry model for tortuosity of flow path in porous media. This model is expressed as a function of porosity and there is no empirical constant. The model predictions are compared with those from available correlations obtained by both the LG simulations and experimental data. The present model predictions are in good agreement with those from the available correlations. In addition, the present model can also give the tortuosity with good approximation near the percolation threshold. The validity of the proposed model is thus verified.

However, it is worth pointing out that although this simple geometry model can provide the tortuosity behaviour above the percolation threshold ϕ_c, it cannot present the percolation threshold ϕ_c. If one is also interested in detailed scaling properties near the threshold, a logarithmic plot with more than one order of magnitude in variation near the threshold might be desirable. For this purpose, the Monte Carlo or LG simulations or analytical work based on percolation theory should be performed, and this might be our next work.

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