Acta Mechanica Sinica (2006) 22:385–391
DOI 10.1007/s10409-006-0001-1

RESEARCH PAPER

Effects of physical parameter range on dimensionless variable sensitivity in water flooding reservoirs

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Received: 8 October 2005 / Revised: 7 March 2006 / Published online: 24 May 2006

Abstract The similarity criterion for water flooding reservoir flows is concerned with in the present paper. When finding out all the dimensionless variables governing this kind of flow, their physical meanings are subsequently elucidated. Then, a numerical approach of sensitivity analysis is adopted to quantify their corresponding dominance degree among the similarity parameters. In this way, we may finally identify major scaling law in different parameter range and demonstrate the respective effects of viscosity, permeability and injection rate.

Keywords Physical parameter range · Dimensionless variable · Sensitivity analysis · Water flooding reservoir · Two-phase flow in porous media

1 Introduction

People have developed a number of recovery enhancement methods such as water, chemical, steam, carbon dioxide floodings, etc. when completing the primary stage of reservoir exploitation. In most circumstances, the mixing of injected and existing fluids in reservoir may result in a complex driving system. Numerous literatures have reported scaling laws of porous media flows in this kind of complex driving system for physical modeling. In regard to water flooding, Geertsma et al. [1] derived similarity criteria for flows in cold and heated water flooding by using both inspection and dimension analysis. Kong et al. [2], Zhu et al. [3] and Shen et al. [4] made good supplements for the scaling criteria of water flooding. As for thermal exploitation, Stegemeier et al. [5] presented the scaling criteria for low pressure model, whereas Pujol and Beberg [6] did the same for high pressure model. Kimber et al. [7] obtained a set of scaling laws for steam and steam additive recovery reservoir for highly viscous oil. Similar studies were performed for steam flooding [8,9], electricity heating (Wang et al. [10,11]) and in-situ combustion [12,13] as well. Islam et al. [14] got the scaling criteria in polymer, emulsion and foam flooding experiments by the inspection. Islam et al. [15] gained the scaling criteria of surfactant-enhanced alkaline/polymer multiple flooding flows. Slavash et al. [16], Rojas et al. [17], Erdal et al. [18], Grattoni et al. [19] and Ekwere [20] dealt with the similarity law for CO_2 flooding reservoir.

Though numerous work has been reported on scaling law, few literatures have involved the dominance degree of scaling criteria quantitively, which is important in physical modeling of reservoir. Generally speaking, there are a great many dimensionless variables representing the similarity of multi-phase flow in porous media with physical and chemical processes involved. Moreover, it is often very hard or sometimes even impossible to keep all of them in the laboratory experiment identical to the field test. For example, the precise scaling of both transverse dispersion and geometry may impractically require a huge model and an extremely long time interval in tests [21]. At the same time, modeling the ratios of the capillary and the driving forces to the gravitational force can induce a considerably high permeability, which can scarcely be realized in laboratory [4].

The project supported by the Innovative Project of CAS (KJCX-SW-L08) and the National Basic Research Program of China(973).

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An efficient and practical way out is to single out the dominant variables and relax secondary ones in experiments. However, they very likely vary with physical parameter range. That is to say, the dominant variables for one reservoir may not be appropriate for another. Unfortunately, few literatures have coped with this issue thus far.

Based on the scaling criteria of water flooding reservoir, the authors have demonstrated the variation tendency of dominance degree for each dimensionless variable with physical parameter range in the following sections. To begin with, qualitative analysis of dimensionless variables is theoretically performed. The sensitivity factors are then calculated numerically in order to identify dominant and secondary ones in different physical parameter ranges. Finally, the effects of oil viscosity, permeability and injection rate are considered in more detail.

2 Similarity analysis

Let's consider the scaling criteria of a 3-D water flooding reservoir with the effects of the gravity, capillarity and compressibility of media involved. The dimensionless variables are found to be as follows [22,23]

$$
\begin{aligned}
&\pi_1 = \frac{K_{\mathrm{cwo}}}{K_{\mathrm{row}}}, \quad \pi_2 = \frac{K_{\mathrm{o}}}{K_{\mathrm{cwo}}}, \quad \pi_3 = \frac{K_{\mathrm{w}}}{K_{\mathrm{row}}},\\
&\pi_4 = \frac{y_{\mathrm{R}}}{x_{\mathrm{R}}}, \quad \pi_5 = \frac{x_{\mathrm{R}}}{z_{\mathrm{R}}}, \quad \pi_6 = \frac{x_{\mathrm{p}}}{x_{\mathrm{R}}},\\
&\pi_7 = \frac{y_{\mathrm{p}}}{y_{\mathrm{R}}}, \quad \pi_8 = \frac{r_{\mathrm{eo}}}{x_{\mathrm{R}}}, \quad \pi_9 = \frac{r_{\mathrm{o}}}{x_{\mathrm{R}}},\\
&\pi_{10} = \frac{s_{\mathrm{cw}}}{\Delta s}, \quad \pi_{11} = \frac{s_{\mathrm{ro}}}{\Delta s}, \quad \pi_{12} = \frac{s_{\mathrm{wi}} - s_{\mathrm{cw}}}{\Delta s},\\
&\pi_{13} = \frac{\sigma\sqrt{\frac{\phi_0}{K}}\cos\theta K_{\mathrm{row}} h}{q_{\mathrm{I}}\mu_{\mathrm{w}}}, \quad \pi_{14} = \frac{\mu_{\mathrm{o}}}{\mu_{\mathrm{w}}}, \quad \pi_{15} = \frac{\rho_{\mathrm{o0}}}{\rho_{\mathrm{w0}}},\\
&\pi_{16} = \frac{K_{\mathrm{row}} h}{q_{\mathrm{I}}\mu_{\mathrm{w}}}\rho_{\mathrm{w0}} g z_{\mathrm{R}}, \quad \pi_{17} = \frac{C_{\mathrm{o}} q_{\mathrm{I}}\mu_{\mathrm{w}}}{K_{\mathrm{row}} h}, \quad \pi_{18} = \frac{C_{\mathrm{w}} q_{\mathrm{I}}\mu_{\mathrm{w}}}{K_{\mathrm{row}} h},\\
&\pi_{19} = \frac{C_\phi q_{\mathrm{w}}\mu_{\mathrm{w}}}{K_{\mathrm{row}} h}, \quad \pi_{20} = \frac{p_{\mathrm{w0}} K_{\mathrm{row}} h}{q_{\mathrm{I}}\mu_{\mathrm{w}}}, \quad \pi_{21} = \frac{p_{\mathrm{o0}} K_{\mathrm{row}} h}{q_{\mathrm{I}}\mu_{\mathrm{w}}},\\
&\pi_{22} = \frac{p_{\mathrm{wf}} K_{\mathrm{row}} h}{q_{\mathrm{I}}\mu_{\mathrm{w}}}, \quad \pi_{23} = \frac{p_{\mathrm{oi}} K_{\mathrm{row}} h}{q_{\mathrm{I}}\mu_{\mathrm{w}}}, \quad \pi_{24} = J(\bar{s}_{\mathrm{w}}), \qquad (1)
\end{aligned}
$$

where p, μ,ρ, K, ϕ and s represent pressure, viscosity, density, effective permeability, porosity and saturation with subscripts w and o indicating water and oil phases, respectively. K_{cwo} refers to the effective permeability of oil phase under the condition of the irreducible water saturation, K_{row} that of water phase under the condition of the residual oil saturation, x_{R}, y_{R} and z_{R} reference length scale in three coordinate directions, respectively, x_{p} and y_{p} the location of production well, r_{o} the well radius, s_{wi} the initial water saturation, σ and θ the interfacial tension and contact angle between water and oil phases, g the gravitational acceleration, q_{I} the injection rate. The subscript 0 indicates physical quantities at a certain condition. C_{o}, C_{w} and C_ϕ are the compressibility of oil, water and rock, respectively, p_{oi} the initial oil pressure and $J(s_{\mathrm{w}})$ the capillary force function. $\Delta s = 1 - s_{\mathrm{cw}} - s_{\mathrm{ro}}$ means the mobile oil saturation, in which s_{ro} and s_{cw} denote residual oil saturation and the irreducible water saturation, respectively.

From the physical point of view, π_1, π_2 and π_3 are the permeability similarity parameters, π_4, π_5, π_6, π_7, π_8 and π_9 the similarity in geometry, π_{10} and π_{11} the ratios of the irreducible water and residual oil saturation to the mobile oil saturation, π_{12} the reduced initial water saturation, π_{14} and π_{15} the ratios of the viscosity and density of water to oil, π_{13} and π_{16} the ratios of the capillary and gravity forces to the driving force, π_{17}, π_{18} and π_{19} the relative volume variation ratios of oil, water and rock under the reservoir pressure, respectively. π_{20}, π_{21}, π_{22} and π_{23} denote the respective ratios of the reference pressure of oil and water, the bottom pressure of the production well, and the initial pressure to the reservoir pressure difference. π_{24} is the dimensionless capillary force function. More specifically, we may make analysis in more detail as follows.

π_{11} can be apparently put in the following form

$$
\frac{s_{\mathrm{ro}}\phi}{(1 - s_{\mathrm{cw}} - s_{\mathrm{ro}})\phi}, \qquad (2)
$$

where the numerator and the denominator mean the volume portions of the residual and mobile oil. Therefore, π_{11} denotes the volume ratio between the residual and mobile oil. Similarly, π_{10} is the volume ratio between the irreducible water and mobile oil.

π_{13} can be rewritten as

$$
\frac{\sigma/\sqrt{K/\phi_0}}{q_{\mathrm{I}}\mu_{\mathrm{w}}/(K_{\mathrm{row}} h)}, \qquad (3)
$$

where $\sqrt{K/\phi_0}$ is the average pore radius and hence $\sigma/\sqrt{K/\phi_0}$ turns out the capillary force in the same pore. The denominator is equivalent to the pressure difference needed to keep the injection rate q_{I} in the reservoir, i. e.

$$
\frac{q_{\mathrm{I}}\mu_{\mathrm{w}}}{K_{\mathrm{row}} h} \sim \Delta p. \qquad (4)
$$

Consequently, π_{13} implies the relative importance between capillary force and displacement pressure. In most situations, capillary force is far less than displacement pressure and so π_{13} is secondary. However, if the permeability is high or the injection rate is small enough,

the capillary force may become comparable to or even exceed driving pressure. In this case, π_{13} cannot be assumed negligible any more.

π_{14} plays an important role to maintain stability between oil and water interface in the process of displacement, thus directly affecting sweeping efficiency. When the interface becomes unstable in a certain range of π_{14}, fingering phenomenon occurs, thus leads to the diminution of sweeping area and efficiency. On the contrary, the stable displacement can enhance efficiency.

π_{16} is obviously equivalent to $\rho_{w0} g z_R/\Delta p$, which signifies the ratio of the static and dynamic pressure. In a thinner reservoir or under higher injection rate, the flow is mainly governed by the driving force. As reservoir thickness grows or the injection rate drops, the effect of the gravity becomes more evident such that π_{16} turns to dominant.

π_{17}, π_{18} and π_{19} can be rewritten in the same form

$$\frac{C_l q_I \mu_w}{K_{row} h} = C_l \Delta p, \quad l = o, w, \phi. \tag{5}$$

With the growth of the injection rate or underground pressure, the relative volume variation of water, oil and rock media magnifies and then π_{17}, π_{18}, π_{19} become significant.

The variation tendency of the dominance degree of π_{22} with physical parameter range can be clarified in the same way:

$$\frac{p_{wf} K_{row} h}{q_I \mu_w} \sim \frac{p_{wf}}{\Delta p} \sim \frac{1}{p_{inj}/p_{wf} - 1}, \tag{6}$$

which reflects the relative importance of the injection pressure p_{inj} and the well bottom pressure p_{wf}. π_{22} becomes significant with increasing injection pressure or decreasing production pressure.

Up to now, we can see that the dominance degree of each dimensionless variable is not always invariant within the physical parameter range. Therefore, numerical approach is applied to display this kind of variation quantitatively instead of qualitatively as previously stated.

3 Numerical approach to analyze the sensitivity of dimensionless variables

Based on sensitivity analysis proposed in our previous work [23], the variation tendency of the dominance degree of dimensionless variables in different physical parameter range is investigated numerically. The sensitivity factor of the dimensionless variable π_i can be defined as

$$S_i = \frac{\partial \left[f(\pi_1, \pi_2, \ldots, \pi_N)/f_p \right]}{\partial(\pi_i/\pi_{ip})}, \quad i = 1, 2, \ldots, N, \tag{7}$$

where $f(\pi_1, \pi_2, \ldots, \pi_N)$ is a target function concerned in the experiment. The subscript p represents the prototype. In a water flooding experiment, the target function can be selected as oil recovery, which looks like

$$f(\pi_1, \pi_2, \ldots, \pi_N) = \int_0^{T_D} \eta(\pi_1, \pi_2, \ldots, \pi_N, t_D) \mathrm{d}t_D, \tag{8}$$

where $\eta(\pi_1, \pi_2, \ldots, \pi_N, t_D)$ represents the oil recovery curve, T_D the dimensionless time span of development. Then, the sensitivity factor is calculated according to

$$S_i = \frac{\int_0^{T_D} |\eta_m - \eta_p| \mathrm{d}t_D \Big/ \int_0^{T_D} \eta_p \mathrm{d}t_D}{|(\pi_{im} - \pi_{ip})/\pi_{ip}|}, \tag{9}$$

where the subscript m represents the model. For a given set of physical parameters, the relative variation of the target function with respect to a specific dimensionless variable is calculated and the sensitivity factors are determined in this way. Comparing the sensitivity factors in different physical parameter range, we may yield the variation tendency of the dominance degree for each dimensionless parameter.

As usual, the dimensionless governing equations of porous media flows during water flooding can be written as

$$\begin{aligned}
&\pi_1\pi_4 \frac{\partial}{\partial x_D}\left(\rho_{oD}\frac{K_{oD}}{\mu_{oD}}\frac{\partial p_{oD}}{\partial x_D}\right) + \frac{\pi_1}{\pi_4}\frac{\partial}{\partial y_D}\left(\rho_{oD}\frac{K_{oD}}{\mu_{oD}}\frac{\partial p_{oD}}{\partial y_D}\right)\\
&\quad + \pi_1\pi_4\pi_5^2 \frac{\partial}{\partial z_D}\left(\rho_{oD}\frac{K_{oD}}{\mu_{oD}}\frac{\partial p_{oD}}{\partial z_D}\right)\\
&\quad + \pi_{16}\pi_1\pi_4\pi_5^2 \frac{\partial}{\partial z_D}\left(\rho_{oD}^2\rho_{o0D}\frac{K_{oD}}{\mu_{oD}}\right)\\
&\quad + \pi_1\rho_{oD}\frac{\pi K_{oD}(p_{wfD} - p_{oD})}{2\ln r_{eoD}/r_{oD}}\delta(x_D - \pi_6)\delta(y_D - \pi_7)\\
&\quad = \frac{\partial(\phi_D \rho_{oD} \bar{s}_o)}{\partial t_D} + \pi_{11}\frac{\partial(\rho_{oD}\phi_{oD})}{\partial t_D},
\end{aligned} \tag{10}$$

$$\begin{aligned}
&\pi_4 \frac{\partial}{\partial x_D}\left(\rho_{wD}K_{wD}\frac{\partial p_{wD}}{\partial x_D}\right) + \frac{1}{\pi_4}\frac{\partial}{\partial y_D}\left(\rho_{wD}K_{wD}\frac{\partial p_{wD}}{\partial y_D}\right)\\
&\quad + \pi_4\pi_5^2 \frac{\partial}{\partial z_D}\left(\rho_{wD}K_{wD}\frac{\partial p_{wD}}{\partial z_D}\right) + \pi_{16}\pi_4\pi_5^2\frac{\partial}{\partial z_D}(\rho_{wD}^2 K_{wD})\\
&\quad + \rho_{wD}\left[\frac{1}{4}\delta(x_D)\delta(y_D) + \frac{\pi K_{wD}(p_{wfD} - p_{wD})}{2\ln r_{eoD}/r_{oD}}\right.\\
&\quad \left.\times\delta(x_D - \pi_6)\delta(y_D - \pi_7)\right] = \frac{\partial(\phi_D\rho_{wD}\bar{s}_w)}{\partial t_D} + \pi_{10}\frac{\partial(\rho_{wD}\phi_D)}{\partial t_D}.
\end{aligned} \tag{11}$$

The dimensionless capillary force equation is put in the form

$$p_{cD} = (p_{oD} - p_{wD}) = \pi_{13}\sqrt{\phi_D} J(\bar{s}_w), \tag{12}$$

Table 1 Major physical parameters

Parameters	L	W	σ	q_{I}	k_{row}	k_{cwo}
	/m	/m	$\times 10^{-2}/(\mathrm{N\cdot m^{-1}})$	$\times 10^{-3}/(\mathrm{m^3\cdot s^{-1}})$	$\times 10^{-12}/\mathrm{m^2}$	$\times 10^{-12}/\mathrm{m^2}$
Prototype	140	140	2.50	8	0.37	0.78
Parameters	ρ_{o}	ρ_{w}	μ_{o}	C_ϕ	p_{w0}	p_{o0}
	$/(\mathrm{kg\cdot m^{-3}})$	$/(\mathrm{kg\cdot m^{-3}})$	$\times 10^{-3}/(\mathrm{Pa\cdot s})$	$\times 10^{-10}/\mathrm{Pa^{-1}}$	$\times 10^6/\mathrm{Pa}$	$\times 10^6/\mathrm{Pa}$
Prototype	800	1 000	5	6.0	12.0	12.0
Parameters	h	g	C_{o}	C_{w}	p_{wf}	p_{oi}
	/m	$/(\mathrm{m\cdot s^{-2}})$	$\times 10^{-10}/\mathrm{Pa^{-1}}$	$\times 10^{-10}/\mathrm{Pa^{-1}}$	$\times 10^6/\mathrm{Pa}$	$\times 10^6/\mathrm{Pa}$
Prototype	10	9.8	8.0	5.0	10.0	12.0

with dimensionless saturation relation

$$\bar{s}_{\mathrm{o}} + \bar{s}_{\mathrm{w}} = 1. \tag{13}$$

The dimensionless initial conditions are specified by

$$p_{\mathrm{oD}}|_{t_{\mathrm{D}}=0} = p_{\mathrm{oiD}}, \qquad \bar{s}_{\mathrm{w}}|_{t=0} = \bar{s}_{\mathrm{wi}}, \tag{14}$$

and the corresponding boundary conditions are

$$\begin{aligned} &\frac{\partial p_{l\mathrm{D}}}{\partial x_{\mathrm{D}}} = 0, \quad \frac{\partial p_{l\mathrm{D}}}{\partial y_{\mathrm{D}}} = 0, \\ &\frac{\partial p_{l\mathrm{D}}}{\partial z_{\mathrm{D}}} + \pi_{16}\rho_{l\mathrm{D}} = 0, \quad l = \mathrm{o}, \mathrm{w} \end{aligned} \tag{15}$$

where

$$\begin{aligned} \rho_{\mathrm{wD}} &= 1 + C_{\mathrm{wD}}(p_{\mathrm{wD}} - p_{\mathrm{w0D}}) + \pi_{16}C_{\mathrm{wD}}z_{\mathrm{D}}, \\ \rho_{\mathrm{oD}} &= 1 + C_{\mathrm{oD}}(p_{\mathrm{oD}} - p_{\mathrm{o0D}}) + \pi_{16}C_{\mathrm{oD}}\rho_{\mathrm{o0D}}z_{\mathrm{D}}, \\ \phi_{\mathrm{D}} &= 1 + C_{\phi\mathrm{D}}\left(\frac{p_{\mathrm{wD}} + p_{\mathrm{oD}}}{2} - \frac{p_{\mathrm{w0D}} + p_{\mathrm{o0D}}}{2}\right) \\ &\quad + \frac{\pi_{16}}{2}(\rho_{\mathrm{o0D}}C_{\phi\mathrm{D}}z_{\mathrm{D}} + C_{\phi\mathrm{D}}z_{\mathrm{D}}). \end{aligned}$$

In the above equations, the subscript D denotes dimensionless variable. The governing equations are discretized using finite difference scheme and solved by the conventional implicit pressure-explicit saturation method (IMPES) [23,24]. The main physical parameters are listed in Table 1.

4 Results and discussion

Based on previous numerical sensitivity analysis, the variation tendency of the dominance degree of dimensionless variables with physical parameter range such as oil viscosity, permeability and injection rate is shown in the following in turn.

4.1 Oil viscosity effect

Water flooding reservoirs usually can be classified as low ($\mu_{\mathrm{o}} < 3\,\mathrm{mPa\cdot s}$), moderate ($3 < \mu_{\mathrm{o}} < 30\,\mathrm{mPa\cdot s}$) and high viscosity ($\mu_{\mathrm{o}} > 30\,\mathrm{mPa\cdot s}$) [25]. Assuming μ_o equal to 2.5, 5 and 50 mPa · s, respectively, and keeping the other parameters unchanged, the sensitivity factors for water flooding reservoir with low, moderate and high viscosity can be calculated as shown in Fig. 1. A line of 0.1 demarcating the sensitivity factor level of the dominant and secondary dimensionless variables is also given in the same diagram. That is to say, the dimensionless variable is regarded as a dominant one if its sensitivity factor is greater than 0.1. Otherwise, it is deemed as a secondary one. This rule also applies in the following texts. It is obviously seen from Fig. 1 that the sensitivity factors, indicating the dominance degree of each dimensionless variable, change with oil viscosity. In particular, the dominant dimensionless variables are different for reservoirs with different oil viscosity. Namely, they are π_1, π_2, π_3, π_{12}, π_{14} and π_{15} for low and moderate oil viscosity reservoirs and π_1, π_2, π_3, π_{10}, π_{12} and π_{14} for high oil viscosity reservoir.

Obviously, π_{10} turns from a secondary dimensionless variable for low and moderate viscosity reservoirs to a dominant one for high viscosity reservoir. This is simply because high viscosity reduces oil recovery and total exploited oil volume. Hence, π_{10} representing the ratio of irreducible water and mobile oil become significant and may dramatically affect the final oil recovery. On the contrary, the high oil recovery in low viscosity reservoir has larger amount of total exploited oil volume, meaning that the irreducible water can be neglected.

Also we may notice that the dimensionless variable π_{15} turns from a dominant one in the low and moderate viscosity reservoirs to a secondary one in high viscosity reservoir. As mentioned above, π_{15} represents the density ratio of the oil to water phases, and also can be explained as the inertial force ratio. As oil viscosity increases, the driving force needed increases tremendously with almost unchanged density ratio. Therefore, the driving pressure to overcome viscosity exerts more influence on the reservoir flows than the inertial force does. The effect of density on the oil recovery is weakened as oil viscosity rises.

The sensitivity factors of π_{17}, π_{18} and π_{19} may increase with oil viscosity by almost two-order of magnitude. The reason of which is that to overcome viscous resistance,

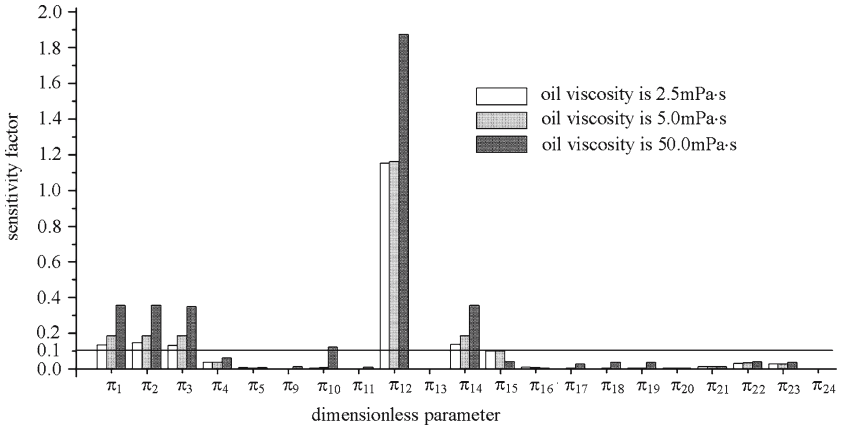

Fig. 1 The effect of the variation of oil viscosity on the dominance degree where *white, gray* and *black* indicate low, moderate and high oil viscosity, respectively

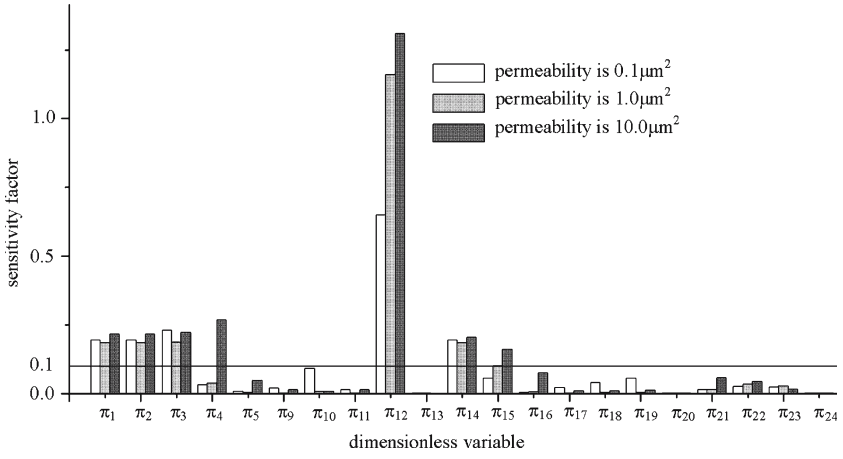

Fig. 2 The effect of the variation of permeability on the dominance degree where *white, gray* and *black* indicate low, moderate and high permeability, respectively

the augment of the driving pressure causes relatively large volume variation in water, oil and rock.

4.2 Permeability effect

We presumably choose 0.1, 1 and 10 Darcy as low, moderate and high permeability to study the effect of permeability on the dominance degree of dimensionless variables. We can see in Fig. 2 that the dominant variables are $\pi_1, \pi_2, \pi_3, \pi_{10}, \pi_{12}$ and π_{14} in low permeability reservoir, $\pi_1, \pi_2, \pi_3, \pi_{12}, \pi_{14}$ and π_{15} in moderate permeability reservoir, and $\pi_1, \pi_2, \pi_3, \pi_4, \pi_{12}, \pi_{14}$ and π_{15} in high permeability reservoir.

π_4 becomes a dominant one in high permeability case. As we know, the sweeping area increases with the permeability. Therefore, whether the geometry is similar or not exerts an evident effect on the oil recovery. The larger the permeability, the more important the geometric similarity.

π_{10} turns close to a dominant variable from a secondary one with the decreasing of permeability. For low permeability case, the resistance to flow grows and results in the decrease of oil recovery and total exploited oil volume. Therefore, the variation of the irreducible water volume exerts evident influences on the final oil recovery.

π_{15} turns from a dominant variable in the moderate and high permeability cases to a secondary one in the low permeability reservoir. This is because the augmentation of the pressure difference in the lower permeability reservoir may weaken the effect of the inertial force. The sensitivity of π_{16} increases with permeability. The higher the permeability, the smaller the driving force and the role of gravity becomes more evident. In addition, we should pay particular attention to π_{17}, π_{18} and π_{19} in low permeability reservoirs when the driving force, namely the pressure difference, is increasing. The situation definitely implies that we should consider the compressibility effect of water, oil and rock.

4.3 Injection rate effect

To fit for different rock characteristics or exploitation requirement, the injection rate may be adjusted time and again. Then, new balance among all kinds of forces is reestablished with of the driving pressure in oil reservoir. Selecting 8×10^{-4}, 8×10^{-3} and $8 \times 10^{-2}\,\mathrm{m^3/s}$ as low, moderate and high injection rates, we explore the variation tendency of the sensitivity factors of all

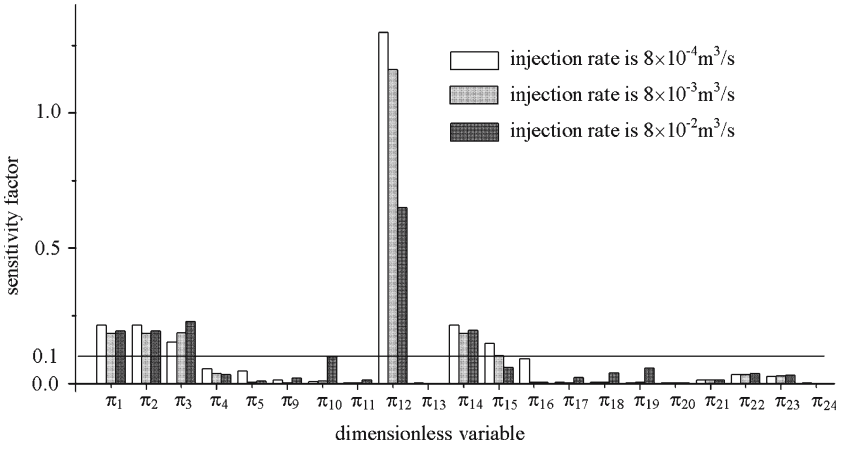

Fig. 3 The effect of the variation of injection rate on dominance degree where *white, gray* and *black* indicate low, moderate and high injection rate cases, respectively

dimensionless variables with injection rate. The results are plotted in Fig. 3, showing that $\pi_1, \pi_2, \pi_3, \pi_{12}, \pi_{14}, \pi_{15}, \pi_{16}$ are the dominant variables for low injection rates, and π_1, π_2, π_3, π_{12}, π_{14}, π_{15} and π_1, π_2, π_3, π_{10}, π_{12}, π_{14} for moderate and high injection rates, respectively.

The sensitivity of π_{16} representing the relative importance between the gravity and driving force, increases with the drop of injection rate. Apparently, the effects of the gravity cannot be neglected for low injection rate case with smaller driving force.

π_{10} is dominant at high injection rate. To some extent, oil recovery increases with the injection rate. However, this is not always true. Very high injection rate may cause early happening of water break-through and flooding out, which reduces oil recovery and total exploited oil volume. The fact once again accounts for the dominance of π_{10} at high injection rate as stated in Sect. 4.1 and 4.2.

With the increasing of the injection rate, π_{15}, the density ratio may become secondary from a dominant one. The reason is that at high injection rate, namely high driving force, the inertial force exerts less effect on reservoir flow than the driving force.

Once again, we find out that the sensitivity factors of π_{17}, π_{18} and π_{19} increase with the injection rate. Since the pressure difference is proportional to injection rate, water, oil and rock can no longer be regarded as incompressible in high pressure environment. The compressibility or the volume variation of water, oil and rock should be taken into consideration.

5 Concluding remarks

As we know, efficient technical measures taken for the enhancement of oil recovery are heavily dependent on the understanding of oil, water and polymer seepage flows in the reservoir. Physical modeling has become an additional powerful tool as new apparatus of visualization such as PIV, CT, NMR etc. appear. As a result, the study of similarity law of multiphase flows in porous media is put on agenda.

The present study in previous sections shows that all the similarity variables actually possess a definite physical meaning. Namely, they indeed imply some kinds of ratios of geometric scales, or of medium properties, or of driving forces, which obviously vary with the range of physical parameters. In the present article, we have examined the effects of most significant factors in practical petroleum engineering such as viscosity, permeability and injection rate. For highly viscous or low permeability oil reservoir or under high injection rate condition, displacement pressure may be larger than other external forces such as capillary and gravitational forces, thus leading to some relevant similarity variables negligible. In contrast, rock deformation and liquid compressibility can not be neglected any more now and a model accounting for fluid-solid interaction should be given priority to. Furthermore, high viscosity, low permeability and high injection rate always mean or are equivalent to low oil recovery. That is to say, the amount of residual oil and irreducible water may occupy a significant portion of total exploited oil volume. Then the dimensionless variables indicating rock wetability of oil and water should be taken into consideration. The contrary argument can be drawn for low viscous and high permeability oil reservoir and under low injection rate conditions. The above conclusions may serve as a guideline in the physical modeling for water flooding oil reservoir, thus finding out dominant dimensionless variables. Generally speaking, the present numerical sensitivity methods may also be used in the study of other complicated physical problems in order to identify major similarity parameters.

Please notice that the present analysis needs revision if the fluid becomes non-Newtonian for high viscous reservoir or startup pressure is comparable to other

driving forces under low permeability condition. Probably, additional similarity parameters may play important roles in these circumstances.

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