# NS 方程数值模拟多学科问题—— 离散阵风响应\*

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Abstract: Based on Navier-Stokes equations and structural and flight dynamic equations of motion, dynamic responses in vertical discrete gust flow perturbation are investigated for a supersonic transport model. A tightly coupled method was developed by subiterations between aerodynamic equations and dynamic equations of motion. First, under the assumption of rigid-body and single freedom of motion in the vertical plunging, the results of direct-coupling method are compared with the results of quasi-steady model method. Then gust responses for the one-minus-cosine gust profile are analyzed with two-freedoms of motion in plunging and pitching for the airplane configurations with and without the consideration of structural deformation.

#### **0** Introduction

Gust load is one of the important dynamic loads considered in aircraft structure design. Due to its multidisciplinary nature with aerodynamics, flight dynamics, aeroelasticity and atmospheric turbulence, up to now, only the doublet-lattice, unsteady linear aerodynamic code (DLM) coupled with the equation of motion of flexible vehicle was used for the gust response analysis [1].

Gusts in nature tend to random. The early design methods for gust loads were based on a single discrete gust having one-minus-cosine velocity profile. Recently the statistical discrete gust (SDG) method and the power spectral density (PSD) method [2] in the frequency domain are used to define the gust loads, however, which are still hard to combine with the modern Navier-Stokes numerical method.

In the recent years, for the motion of rigid vehicles, the path of stores during the separation phase has been mainly investigated with the computational fluid dynamics (CFD) algorithm coupled with a 6 degree-of-freedom (6DOF) algorithm [3]. In sofar as the authors know, the computation of gust response with the coupling method has still not been reported. In the paper, the fully implicit multiblock Navier-Stokes aeroelastic solver implemented by the authors [4], coupled with the flight and structural dynamic equations of motion, has been developed to simulate gust dynamic responses, which can model the motion of rigid or flexible vehicle. Since it is hard to find the structural data of flexible vehicle, the supersonic transport (SST) designed by National Aerospace Laboratory of Japan (NAL) [5] is taken for our calculated case. To study the effects of dynamic response due to flow perturbation and airplane motion, only the consideration of vertical plunging motion, a comparative study was first done for the airplane in the harmonic flow perturbation with the direct-coupling method and the quasi-steady model method. Then the gust responses in a one-minus-cosine gust velocity profile are analyzed with two-freedoms of motion in plunging and pitching with and without the consideration of structural deformation.

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### 1 Aerodynamic Equations and Numerical Method

Aerodynamic governing equations are taken as the unsteady, three-dimensional thin-layer Navier-Stokes equations, which is solved with LU-SGS method, employing a Newton-like subiteration. Second-order temporal accuracy is obtained by utilizing three-point backward difference in the subiteration procedure. The inviscid terms are approximated by the modified third-order upwind HLLEW scheme. For the isentropic flow, the scheme results in the standard upwind-biased flux-difference splitting scheme of Roe, and as the jump in entropy becomes large in the flow, the scheme turns into the standard HLLEW scheme. Thin-layer viscous term is discretized by second-order central difference. For multiblock-grid application, the Navier-Stokes equations are solved in each block separately. To calculate the convective and viscous fluxes in the block boundary, data communication is performed through two-level halo cells.

## 2 Equations of Motion and Numerical Method

In the present study of dynamic response, the airplane is permitted freedom in vertical plunging and pitching, and the following assumptions are made,

- a. The disturbed motion is symmetrical with respect to the airplane's longitudinal plane of symmetry.
- b. The airplane is initially in horizontal flight at cruise velocity.
- c. The vertical gust perturbation is normal to the flight path, and is uniform in the spanwise direction.
- d. For the consideration of structural deformation, only the structural deformation of the wing is considered and its deformation is approximated to the elastic plate model.

#### 2.1 Direct-coupling method

With the above assumptions, the equilibriums of total force along the z-axis and total pitching moment about the y-axis are:

$$\iint \ddot{w}(x, y, t) \rho dx dy = \iint \Delta p(x, y, t) dx dy$$
 (3a),

$$\iint \ddot{w}(x, y, t) \rho x dx dy = \iint \Delta p(x, y, t) x dx dy$$
 (3b)

For the equilibrium of an element, we obtain:

$$w(x,y,t) - w(0,0,t) - x \frac{\partial w(0,0,t)}{\partial x} = \iint_{\mathbb{R}} C(x,y,\xi,\eta) [\Delta p(\xi,\eta,t) - \rho(\xi,\eta)\ddot{w}(\xi,\eta,t)] d\xi d\eta$$
(3c)

In the system of equations, the unknown quantity w(x, y, t) represents the disturbed displacement of elastic airplane from its original equilibrium state. The pressure change of  $\Delta p(x, y, t)$  based on cruise condition is calculated by the unsteady aerodynamic equations, which depends on the instantaneous values of displacement, velocity, acceleration of airplane, as well as the past history of the motion.

Introducing natural modes with the Rayleigh-Ritz method, we have  $w(x, y, t) = \sum_{i=1}^{n} \phi_i(x, y) q_i(t)$ ,

where  $\phi_i(x, y)$  is normalized natural mode shapes of the airplane including rigid modes and  $q_i(t)$  generalized displacement. Then Equations (3a-3c) can be deduced to

$$\ddot{q}_i + 2\zeta_i \omega_i \dot{q}_i + \omega_i^2 q_i = F_i / M_i, \quad (i = 1, 2, \dots, n; \omega_1 = \omega_2 = 0)$$
(4)

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with the initial conditions  $q_i(0) = \dot{q}_i(0) = 0$  and where  $M_i = \iint \phi_i^2(x, y) \rho(x, y) dx dy$ 

 $F_i = \iint \Delta p(x, y, t) \phi_i(x, y) dxdy$ . The first equation in Eq. (4) is the equation of motion in vertical plunging. In

the equation, the generalized mass  $M_1$  represents the mass of airplane, and  $q_1$  the plunging displacement. Similarly, the second equation is the equation of motion in pitching.  $M_2, q_2$  represent the pitching moment of inertia and angular displacement in pitching, respectively.  $\omega_i, \varsigma_i$  are the natural frequency of structural modes and the damping ratio in the *i*th mode, which are 0 for the first two equations of motion of rigid body.

The subiteration method can also be used for Eq. 4. The resulting numerical scheme is

$$\begin{bmatrix} 1 & -\phi \Delta t \\ \phi \Delta t \alpha_i^2 & 1 + 2\phi \alpha_i \zeta_i \Delta t \end{bmatrix} \Delta S = -\phi \left\{ (1 + \phi) S^p - (1 + 2\phi) S^n + \phi S^{n-1} + \Delta t \begin{bmatrix} 0 & -1 \\ \alpha_i^2 & 2\alpha_i \zeta_i \end{bmatrix} S^p - \Delta t \begin{bmatrix} 0 \\ F_i^p / M_i \end{bmatrix} \right\}$$
(5)

where  $S = [q, \dot{q}]$ ,  $\Delta S = S^{p+1} - S^p$ . As  $p \to \infty$ , a full implicit second-order temporal accuracy scheme for the numerical simulation of dynamic response is formed by the coupling solutions of fluid equation and the equations of motion. Numerical experiments indicate, in general, the calculated results are nearly unchangeable as  $p \ge 3$ .

If the airplane is assumed as the rigid body, then only the first two equations of Eq. 4 coupled with the aerodynamic equations need to be solved. If the pitching motion can be further neglected, the dynamic response is only considered in the motion of vertical plunging. For the simpler case, the quasi-steady model method can be introduced as follows.

#### 2.2 Quasi-steady model method

If the time lag in buildup of lift is neglected and the incremental lift is considered only due to the change of angle of attack, then the model equation of motion can be written simply as.

$$M\ddot{z} = \frac{1}{2} \rho_{\infty} V_{\infty}^2 SC_{L\alpha} \left( \frac{w(t)}{V_{\infty}} - \frac{\dot{z}}{V_{\infty}} \right)$$
 (6)

Here w(t) represents the vertical perturbation velocity profile. The normalized equation can be written as:  $\ddot{z} + C_1 C_{L\alpha} \dot{z} = C_1 C_{L\alpha} w(t)$  (7)

where  $C_1 = \frac{\rho_\infty Sc}{2M}$ ,  $C_{L\alpha}$  is the derivative of lift coefficient which can be determined by steady flow calculation or wind tunnel experiment. Through the comparison of this method with the direct-coupling method, the dynamic responses under the quasi-steady assumption can be studied.

#### 3 Results and Discussions

Dynamic responses in vertical flow perturbation are studied for the SST experimental model [5]. For the solution of structural deformation, the data of the structural oscillating natural modes and frequencies are provided by the FEM method. The first five structural modes and natural frequencies are considered in the following calculations. The aircraft is initially assumed at cruise flight, and then encounters a gust turbulence atmosphere. So the calculation of gust dynamic response needs to start from the cruise steady flowfield.

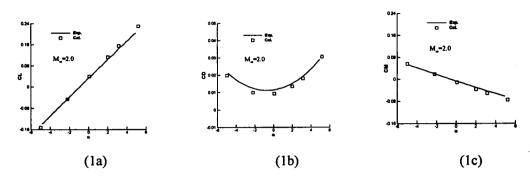


Fig. 1 The comparison of the predicted aerodynamic coefficients with experiments

The H-H type multiblock grid with 30 blocks was generated for the SST configuration. The comparison of the predicted coefficients of lift, drag and pitching moment with the experimental data of wind tunnel is depicted in Fig. 1. The agreement between experiment and calculation is fairly good. The cruise lift coefficient at cruise condition of  $M_{\infty} = 2.0$ ,  $\alpha = 2^{\circ}$  is calculated as  $C_{L0} = 0.112$ , which is in correspondence with the experimental value of 0.110. For the quasi-steady model equation of motion of Eq. 6, the derivative of lift coefficient needs to be known. Based on the curve of lift coefficient, the derivative can be approximately calculated as  $C_{L\alpha} = 2.15$ .

#### 3.1 Dynamic response for harmonic perturbation

The SST experimental model is designed for cruise at  $M_{\infty}=2.0$ ,  $\alpha=2^{\circ}$  and the flight altitude of 15,000m. A future SST is expected to cruise at a supersonic speed only over the sea and to cruise at a transonic speed over the land. Due to the strong nonlinearity of transonic flows, the transonic dynamic response may be interested. We assume the experimental airplane can cruise at  $M_{\infty}=0.9$  and the flight altitude of 9,000m. At the cruise flight, due to the equilibrium of various forces and moments, the cruise lift should be equal to the total weight of airplane, therefore the cruise angle of attack  $\alpha=2.14^{\circ}$  at  $M_{\infty}=0.9$  can be calculated based on the curve of lift versus angles of attack. The numerical results also show the pitching moment about the gravity center of the airplane still exists for this case. In fact, to guarantee the cruise of the airplane at transonic Mach number, the high-lift-system and the empennage deflection should be used to keep the equilibrium of forces and moments, however, these influences are not considered in the paper.

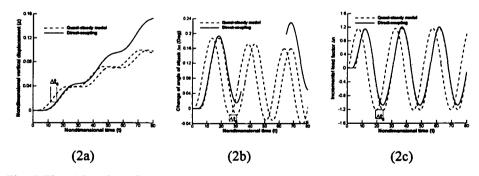


Fig. 2 Time histories of vertical displacement, incidence and incremental load factor

A vertical harmonic flow perturbation is added to the steady flow of the aircraft at  $M_{\infty} = 0.9$  and  $\alpha = 2.14^{\circ}$  with  $w(t) = w_0 \sin(\omega t)$ . The 'quasi-steady model' method is decided by a second-order ordinary equation of Eq. (7). Its initial conditions can be assumed as  $z_{t=0} = 0$ ,  $\dot{z}_{t=0} = 0$ . After the slope of lift curve and

gust profile are given, then the time histories of vertical displacement z(t), the change of angle of attack due to motion  $\Delta \alpha(t) \approx \dot{z}(t)/V_{\infty}$  and the incremental load factor  $\Delta n(t) = \ddot{z}(t)/g$  are calculated. The 'direct-coupling method' is to get time-accurate solution by coupled unsteady CFD code with the equation of plunging motion and gust profile directly. Fig. 2 shows the time histories of two methods. The dynamic responses of 'quasi-steady model' method after the translation of lag time are also depicted in the figure. Overall frequency responses of the two methods are nearly the same, whose nondimensional value is about 0.04. The incremental load factor (equivalent to the lift coefficient) has no large change except at the position of the minimum, but comparing direct-coupling method, the quasi-steady method predicts the slower growth of displacement with time, the reverse tendency of change of angle of attack. On the other hand, the vertical displacement grows near linearly with the time, which is contrary to the real fact, therefore the effect of pitching motion should be considered. We know only the direct-coupling method can treat multi-freedom motions.

#### 3.2 Dynamic response for one-minus-cosine gust

The early design methods for gust loads were based on the discrete gust having one-minus-cosine velocity

profile, namely, 
$$W_g = \frac{1}{2}W_0\left(1-\cos\frac{2\pi\alpha}{2H}\right)$$
,  $W_0$  is the design cruise gust velocity, which is specified as

50ft/s at the altitudes from sea level to 20,000ft and then decreases linearly as the functions of altitudes [6]. In the present calculation of cruise altitude of 9,000 m,  $W_0$  is assumed as 50ft/s. The gust gradient distance H is taken as the 12.5 times mean geometric chordlengths based on the experimental evidence [6]. Before and after the discrete gust pulse, there is no gust flow perturbation. In the following, we need to study how the airplane moves in plunging and pitching, how the loads change and how the structure deforms under the discrete gust profile. In the paper, total four cases named as 'Rigid+Plung', 'Flexible+Plung' 'Rigid+Plung+Pitch', and 'Flexible+Plung+Pitch' are simulated. The fourth case is the most complicated case, which need to simulate the motion of the aircraft in plunging and pitching and its structural deformation simultaneously.

The time histories of the load coefficients of lift, drag, pitching moment and bending moment are shown in Fig. 3, which indicates the loads are nearly unchangeable with or without the consideration of structural deformation due to the stronger structural rigidity of the SST model airplane. When the airplane flights through the gust pulse, forces and moments also experience a pulse, but a little large maximum load is predicted without the consideration of the motion in pitching. After the pulse response, the loads tend to recover the equilibrium state or emerge to oscillate in decay for the methods with and without the consideration of the motion in pitching. The displacement in plunging and the angular displacement in pitch are depicted in Fig. 4. Without the consideration of motion in pitching, the displacement in plunging increases near linearly with the time, which is obviously contrary to fact. Similar to the above analyses of harmonic perturbation, the method without the pitching motion cannot simulate correctly the response motion of the aircraft. Under the consideration of airplane motion in plunging and pitching, the responses of the airplane appear two-freedoms of oscillation. As the airplane plunges up, the airplane pitches down simultaneously, which can be used for the explanation of the change of loads in Fig. 3 in the corresponding phase, namely due to the decrease of angle of attack, the lift, drag and bending moments decrease and pitching moment increases. On the contrary, when the airplane plunges down and pitches up, due to the increase of angle of attack, the lift, drag and bending moment increase and the pitching moment decreases.

The maximum amplitudes of the plunging and pitching oscillation are about 0.1 times mean aerodynamic chordlength and 1.8 degree, respectively. The numerical results also show that the pitching oscillation decays much faster than that of the plunging oscillation.

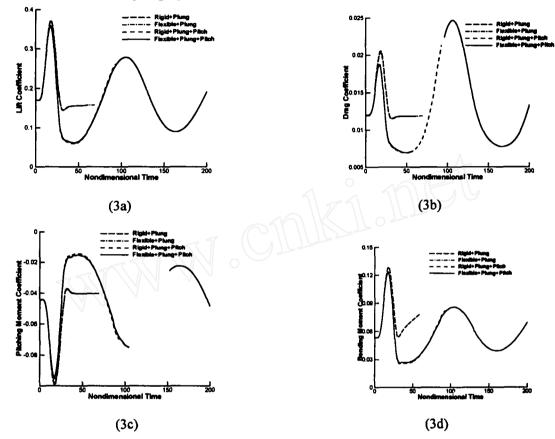
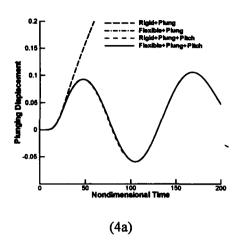


Fig. 3 Time histories of the coefficients of lift, drag, pitching moment and bending moment for rigid and flexible configurations at  $M_{\infty} = 0.9$ ,  $\alpha = 2.14^{\circ}$ 

Fig. 5 gives the time histories of structural deformation of the generalized displacements. The structural deformation occurs mainly in the first two modes. Although the deformation is smaller, the airplane experiences a larger structural deformation in the gust process, which has the same changeable tendency of the lift coefficient in Fig. 3a. After the pulse response, structural deformation oscillates to revert the original equilibrium state without the consideration of the motion in pitching, and with two-freedoms of motion, the structural deformation oscillates in much more complicated form, which couples the natural structural oscillation of high frequency and the airplane motion of long period in plunging and pitching. The nondimensional value of high frequency is 0.158 and the low frequency of the aircraft motion is 0.012, which is the same as the frequency of load responses. Finally, through the comparison of responses with and without pitching motion, we know both methods simulated the complete different response process of structural deformation after the gust pulse.



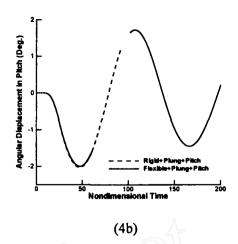
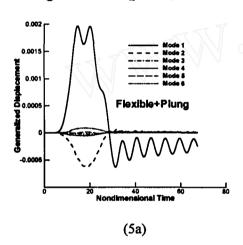


Fig. 4 Time histories of the vertical plunging displacement and angular displacement in pitch for rigid and flexible configurations at  $M_{\infty} = 0.9$ ,  $\alpha = 2.14^{\circ}$ 



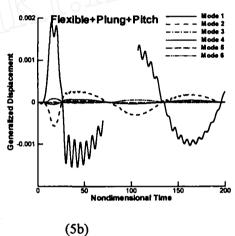


Fig. 5 Time histories of structural deformation of the first six modes for the flexible configuration with and without the pitching motion at  $M_{\infty} = 0.9$ ,  $\alpha = 2.14^{\circ}$ 

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