

含孔边裂纹球壳的断裂分析*

柳春图 韩闻生

(中国科学院力学研究所 北京 100080)

摘 要

在依据 Reissner 理论得出的球壳裂纹尖端应力应变场展开式^[1]基础上,采用局部—整体分析法和权函数方法分别计算承受内压的含孔边裂纹球壳的应力强度因子.在有限元的模式中考虑剪切变形的影响,并对奇异元模式的应力应变场展开式的项数选择、奇异元最佳尺寸的选取进行了分析.本文计算和分析了在不同几何尺寸,不同开孔大小以及不同剪切参量条件下承受内压的含孔边裂纹球壳的应力强度因子及其变化规律.

关键词:含孔边裂纹球壳;局部—整体分析法;权函数方法;应力强度因子

1 引 言

球壳上的圆形接口广泛存在于压力容器中,是产生破坏性裂纹的热点部位.因此,必须按断裂力学的方法进行球壳孔边裂纹的断裂分析,从而对压力容器结构完整性评定和剩余寿命估计提供定量的科学依据.对于含裂纹壳体的断裂分析,早期文献是采用 Kirchhoff 经典理论^[2,3],忽略剪切变形的影响,会导致一些理论上的缺陷,如剪应力中有 $r^{-3/2}$ 阶奇异性,弯曲应力幅角与拉伸应力幅角不同.近年来一些文献从考虑剪切变形的 Reissner 理论出发^[4,5],给出了裂纹尖端应力应变场的首项,讨论了无限大扁壳的情况,克服了经典理论的不足.但是对整个应力场仍不清楚,而且没有求得有限尺寸壳体的解答.柳春图等^[1]通过求解 Reissner 理论的扁壳方程,获得了球壳裂纹尖端应力应变场包括 I、II、III 型的一般解,从而较全面地揭示了裂纹尖端附近的力学性质,它类似于平面问题中 Williams 展开式的作用.文^[1]又结合有限元,计算和分析了含裂纹球壳的应力强度因子.

目前尚未见到含孔边裂纹球壳的断裂分析的文献报道.本文针对于此,在文^[1]的基础上,采用局部—整体分析法及权函数方法,对承受内压的含孔边裂纹球壳的应力强度因子进行计算和分析.

* 本文得到国家基金委的资助

来稿日期:1991-01-14

2 有限元模式的建立

裂纹的存在给有限元在断裂力学中的应用带来一定的困难. 尽管各种畸变元技术应运而生, 但裂纹附近的加密网格划分使整体有限元分析的计算量巨大, 而且这种畸变元对裂纹附近的应力状态只能给出近似的描述. 本文采用的局部—整体分析法在裂纹尖端建立一个奇异元, 以文[1]提出的应力应变场作为其应力模式和位移模式, 从而准确地模拟了裂纹尖端的真实力学性质. 这样在裂纹尖端附近, 有限数目的网格就足以保证计算结果的精度, 大大减小了计算量. 在远离裂纹的球壳其它部分采用考虑剪切变形的厚壳元作为常规元.

考虑半径 R , 厚度 h , 内压力 q 的球壳. 球壳上圆孔半径 R_0 , 孔边径向裂纹长为 a . 由于对称性, 取其 $1/8$ 进行有限元网格划分. 如图 1 建立整体坐标系 $OXYZ$, 节点坐标由球面方程给出:

$$X = R \cos \theta \sin \phi, Y = R \sin \theta \sin \phi,$$

$$Z = R(1 - \cos \theta).$$

取裂纹尖端为坐标原点, 裂纹的切向为 x 轴建立奇异元的局部坐标系 $oxyz$. 奇异元的位移和应力模式由文献[1]导出, 详见附录.

$$\text{记 } \{d\} = \{u, v, w, \varphi_x, \varphi_y\}^T$$

$$\{f\} = \{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y\}^T$$

$$\{a\} = \{u_0, v_0, w_0, \varphi_{x0}, \varphi_{y0}, \bar{K}_{\frac{1}{2}}^{(e)}, \bar{K}_1^{(e)}, \dots,$$

$$\bar{K}_{\frac{1}{2}}^{(p)}, \bar{K}_1^{(p)}, \dots, \bar{a}_1, \bar{a}_2\}^T$$

奇异元的弹性应变能为:

$$U = \frac{1}{2} \int_A \frac{1}{B} [N_x^2 + N_y^2 - 2\nu N_x N_y + 2(1 + \nu) N_{xy}^2] dA + \frac{1}{2} \int_A \frac{1}{(1 - \nu^2)D} [M_x^2 + M_y^2 - 2\nu M_x M_y + 2(1 + \nu) M_{xy}^2] dA + \frac{1}{2} \int_A \frac{1}{C} (Q_x^2 + Q_y^2) dA$$

$$= \frac{1}{2} \int_A \{f\}^T [D]^{-1} \{f\} dA$$

$$= \frac{1}{2} \{a\}^T \left\{ \int_A [F]^T [D]^{-1} [F] dA \right\} \cdot \{a\}$$

$$= \frac{1}{2} \{a\}^T [K_s] \{a\}$$

其中: 弹性柔度矩阵:

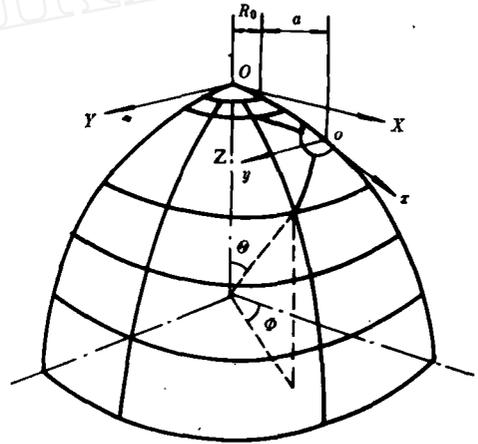


图 1 整体坐标系

$$u_R(x, a) = \frac{\sigma_0}{E \sqrt{2}} \left[4F_H(a) \sqrt{a(a-x)} + G(a) \frac{(a-x)^{3/2}}{\sqrt{a}} \right]$$

其中,

$$G(a) = [S_1(a) - 4F_H(a) \sqrt{a} S_2(a)] \sqrt{a} / S_3(a)$$

$$S_1(a) = \pi \sqrt{a} \sigma_0 \int_0^a [F_H(a)]^2 a da$$

$$S_2(a) = \int_0^a \sigma_R(x) \sqrt{a-x} dx$$

$$S_3(a) = \int_0^a \sigma_R(x) (a-x)^{3/2} dx$$

式中 $\sigma_0 = qR/2h$, σ_R 为孔边应力. 选取参考状态下的应力强度因子为

$$K_R(a) = F_H(a) \sigma_0 \sqrt{\pi a} = M_H F_B \sigma_0 \sqrt{\pi a}$$

M_H 为含裂纹球壳的校正系数表达式^[1]:

$$M_H = \sqrt{1 + 0.59\lambda^2 (1 + 1.2\chi^3 \lambda e^{-1})}$$

式中 $\lambda = \sqrt{12(1-\nu^2)} a / \sqrt{Rh}$

$$\chi = \frac{D}{Ca^2}$$

F_B 为平板孔边裂纹的形状因子^[10]

$$F_B = F_1 / (F_2 + \frac{a}{r}) + F_3 \quad (a/r \leq 10)$$

其中 $F_1 = 0.6870, F_2 = 0.2772, F_3 = 0.9446$

4 数值分析

近似取孔的半径 $R_0 = 0.01a$, 模拟无孔情况, 计算结果如下:

$k_1 / \frac{qR}{2h} \sqrt{\pi a}$ 的对比

λ	$K=1.0$			$K=2.0$		
	文[1]	有限元	权函数	文[1]	有限元	权函数
0.2	1.222	1.185	1.149	1.301	1.300	1.223
0.4	1.305	1.240	1.239	1.381	1.326	1.284
0.6	1.592	1.558	1.465	1.673	1.600	1.539
1.0	1.800	1.763	1.638	1.913	1.899	1.836

局部一整体法与文[1]相差在 5% 以内, 权函数法与文[1]相差不超过 9%.

在奇异元的模式中, 取展开式的本征值分别到 2、3、4 阶, 对球壳几何参数为 $h/R = 0.1, a/R_0 = 1, R/R_0 = 0.1$ 的情况计算:

$$K_1/\frac{qR}{2h}\sqrt{\pi a}$$
 随本征值的变化

本征值	$K_1/\frac{qR}{2h}\sqrt{\pi a}$	相差
2	2.07062	
3	2.10200	2%
4	2.10820	3%

分别用局部一整体分析法和权函数方法计算承受内压的含孔边裂纹球壳的应力强度因子。球壳各项几何尺寸的选择依据 ASME 锅炉与压力容器的规范^[1], 取 h/R 的范围为 $1/8-1/12$, R_0/R 为 $0.1-0.3$, a/R_0 为 $0.1-2.2$, 计算结果见图 2、3、4。可以看出: 应力强度因子 ($K_1/\frac{qR}{2h}\sqrt{\pi a}$) 为无量纲应力强度因子, 称鼓胀系数随 a/R_0 的增大而先平缓后呈近似线性增加; 随 R_0/R 的增大而增大, 且在 R_0/R 较大时, 应力强度因子与 R_0/R 近似成正比; 随 h/R 的增大而减小。同时还表明: 权函数法的结果与局部一整体法的结果相差最大不超过 9%。我们知道, 有限元是一种计算量很大的数值分析方法, 对于大多数工程问题, 有必要寻求一种计算量小, 精度高的方法。上面计算结果表明, 本文提出的处理含孔边裂纹球壳的权函数方法是一种具有足够精度的简便方法。

为考察剪切变形对鼓胀系数的影响, 对几何参数为 $a/R_0=0.1$, 含孔边裂纹球壳进行了计算, 得到不同剪切刚度 $\chi=\frac{D}{Ca}$ 时的鼓胀系数 (图 5)。可以看出, 当 χ 不变时, 鼓胀系数随 λ 的增大而增大; 当 λ 不变时, 鼓胀系数随 χ 的增大而增大。

我们把 $\chi=0.001$ 的情况近似认定没有考虑剪切效应影响情况。在本文数值范围内, 最大的鼓胀系数与没有考虑剪切效应的经典理论相比, 两者相差就达到 42%。

5 结 论

1. 采用局部一整体分析法首次解决了含孔边裂纹球壳问题。在本文计算中, 裂纹尖端的奇异元最佳尺寸为 $0.1a$ 。
2. 在权函数方法中, 仅考虑薄膜应力, 获得的承受内压的含孔边裂纹球壳的鼓胀系数具有足够的精度, 且简便、省机时, 便于应用。
3. 采用局部一整体分析法和权函数方法分别求得的鼓胀系数的数值和规律是吻合的, 一致的。鼓胀系数随 a/R_0 的增加而平缓变化而后呈近似线性增长; 随 R_0/R 的增加而增加, 在 R_0/R 较大时, 鼓胀系数近似与 R_0/R 成正比; 随 h/R 的增大而减小。
4. 含孔边裂纹球壳的鼓胀系数随 χ 的增大而增大, 随 λ 的增大而增大, 经典理论的鼓胀系数是偏不安全的。在本文数值范围内, 考虑剪切效应影响与不考虑剪切效应的影响, 两者的鼓胀系数最大相差达到 42%。

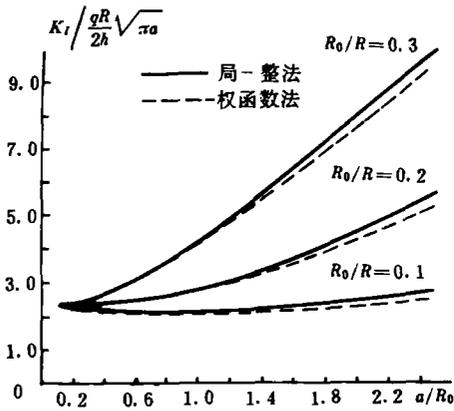
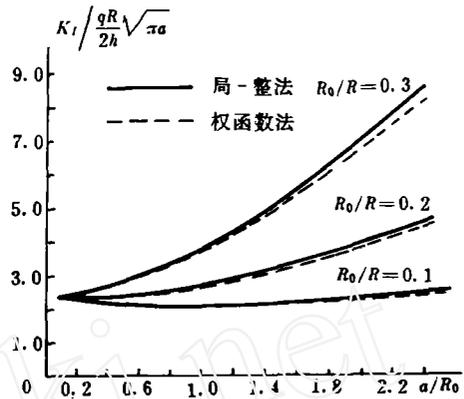
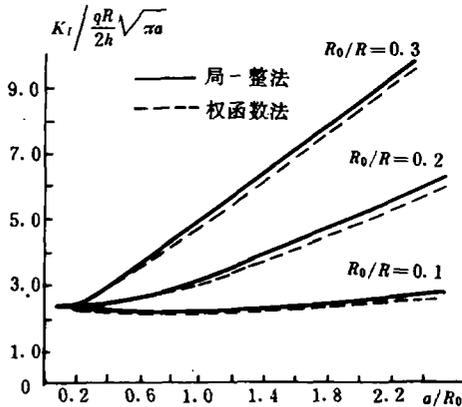
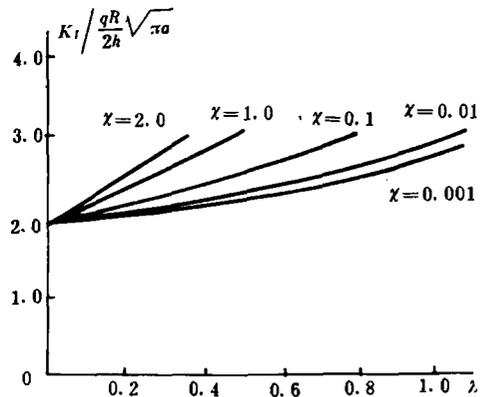
图 2 计算结果($h/R = \frac{1}{10}$)图 3 计算结果($h/R = \frac{1}{8}$)图 4 计算结果($h/R = \frac{1}{12}$)

图 5 计算结果

附 录

奇异元模式是根据文献[1]建立的. 下图中裂纹尖端处的半圆即为奇异元. 以裂纹尖端 o 点为原点, 裂纹的切向为 x 轴建立奇异元局部坐标系 oxy .

奇异元半径 r_s 取为 $0.1a$, 在具体计算中, 通过变更奇异元半径 r_s 来寻求 r_s 的最佳尺寸. 引入无量纲参数, 处理文献[1]给出的位移函数、应力函数.

令,

$$\bar{R} = R/a \quad \bar{h} = h/a \quad \bar{k} = ka = 1/\bar{R}$$

$$\bar{D} = \frac{D}{D} \equiv 1$$

$$\bar{B} = \frac{Ba^2}{D} = \frac{12(1-\nu^2)}{h^2}$$

$$\bar{C} = \frac{Ca^2}{D}$$

有,

$$4u^2 = \frac{2\bar{C}}{(1-\nu)\bar{D}}, \chi = \frac{3+\nu}{1-\nu}, \nu = \frac{1}{3}$$

定义应力系数

$$a^{\lambda-1}K_{\lambda}^{(e)} = \frac{8D}{k}(\lambda_1^{\lambda}K_{\lambda_1}^{(1)} + \lambda_2^{\lambda}K_{\lambda_2}^{(2)})$$

$$a^{\lambda-1}K_{\lambda}^{(b)} = 1(1+\nu)D(\lambda_1^{\lambda}K_{\lambda_1}^{(1)} + \lambda_2^{\lambda}K_{\lambda_2}^{(2)})$$

$$\lambda = \frac{1}{2}, 1, \frac{3}{2}, 2, \dots$$

$$a^{\lambda-1}\alpha_{\lambda-1} = \tilde{\alpha}\lambda - 1, \lambda = 2, 3, \dots$$

再定义

$$\bar{K}_{\lambda}^{(e)} = \frac{a^2}{D}K_{\lambda}^{(e)} \quad \bar{K}_{\lambda}^{(b)} = \frac{a}{D}K_{\lambda}^{(b)} \quad \bar{\alpha}_{\lambda-1} = \alpha_{\lambda-1}$$

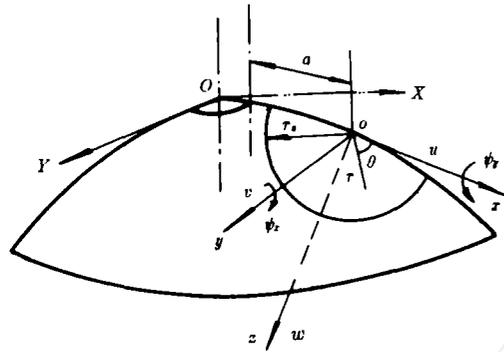


图 6 局部坐标系

式中 $K_{\lambda_1}^{(1)}, K_{\lambda_2}^{(2)}, \tilde{\alpha}_{\lambda-1}, \lambda_1, \lambda_2$ 的定义见文献[1].

将力学量进行无量纲化

$$\bar{u} = u/a, \dots \quad \bar{N}_z = N_z a^2/D, \dots \quad \bar{M}_z = M_z a/D, \dots \quad \bar{Q}_z = Q_z a^2/D, \dots$$

在不发生混淆的情况下,仍沿用原符号.根据文献[1]的位移,应力函数,求得 I 型问题的位移和应力场展开式(4 阶).

(i) 位移模式

$$\begin{aligned} u = & u_0 + \frac{(1+\nu)}{2\bar{B}}\bar{K}_{\frac{1}{2}}^{(e)}\left(\frac{r}{a}\right)^{\frac{1}{2}}\left[(2\chi-1)\cos\frac{\theta}{2}-\cos\frac{3\theta}{2}\right] \\ & + \frac{2}{\bar{B}}\bar{K}_1^{(e)}\left(\frac{r}{a}\right)\cos\theta \\ & + \frac{(1+\nu)}{6\bar{B}}\bar{K}_{\frac{3}{2}}^{(e)}\left(\frac{r}{a}\right)^{\frac{3}{2}}\left[(2\chi+1)\cos\frac{3\theta}{2}-3\cos\frac{\theta}{2}\right] \\ & + \frac{1}{2\bar{B}}\bar{K}_2^{(e)}\left(\frac{r}{a}\right)^2\left[(3+\nu)\cos 2\theta-(1+\nu)\right] \\ & + \frac{\bar{a}_1}{2\bar{R}}\left(\frac{r}{a}\right)^2\cos 2\theta \\ v = & v_0 + \frac{(1+\nu)}{2\bar{B}}\bar{K}_{\frac{1}{2}}^{(e)}\left(\frac{r}{a}\right)^{\frac{1}{2}}\left[(2\chi+1)\sin\frac{\theta}{2}-\sin\frac{3\theta}{2}\right]-\frac{2\nu}{\bar{B}}\bar{K}_1^{(e)}\left(\frac{r}{a}\right)\sin\theta \\ & + \frac{1+\nu}{6\bar{B}}\bar{K}_{\frac{3}{2}}^{(e)}\left(\frac{r}{a}\right)^{\frac{3}{2}}\left[(2\chi-1)\sin\frac{3\theta}{2}-3\sin\frac{\theta}{2}\right] \\ & -\frac{\nu}{\bar{B}}\bar{K}_2^{(e)}\left(\frac{r}{a}\right)^2\sin 2\theta+\frac{\bar{a}_1}{2\bar{R}}\left(\frac{r}{a}\right)^2\sin 2\theta \\ w = & w_0 -\frac{\bar{K}_{\frac{1}{2}}^{(e)}}{\bar{R}\bar{C}}\left(\frac{r}{a}\right)^{\frac{3}{2}}\left(\cos\frac{\theta}{2}+\frac{1}{3}\cos\frac{3\theta}{2}\right)+\frac{1}{2\bar{R}\bar{C}}\bar{K}_1^{(e)}\left(\frac{r}{a}\right)^2(1-\cos 2\theta) \\ & -\frac{1}{3\bar{R}\bar{C}}\bar{K}_{\frac{3}{2}}^{(e)}\left(\frac{r}{a}\right)^{\frac{5}{2}}\left(\cos\frac{\theta}{2}-\frac{1}{5}\cos\frac{5\theta}{2}\right)-\frac{1}{4\bar{R}\bar{C}}\bar{K}_2^{(e)}\left(\frac{r}{a}\right)^3(\cos\theta-\cos 3\theta) \end{aligned}$$

$$\begin{aligned}
& + \frac{\overline{K}_{\frac{1}{2}}^{(b)}}{3(1-\nu^2)\overline{D}} \left(\frac{r}{a}\right)^{\frac{3}{2}} [3(1-\nu)\cos\frac{\theta}{2} - (7+\nu)\cos\frac{3\theta}{2}] \\
& + \frac{\overline{K}_1^{(b)}}{2(1-\nu^2)\overline{D}} \left(\frac{r}{a}\right)^2 [(1-\nu) + (1+\nu)\cos 2\theta] \\
& + \frac{\overline{K}_{\frac{3}{2}}^{(b)}}{15(1-\nu^2)\overline{D}} \left(\frac{r}{a}\right)^{\frac{5}{2}} [5(1-\nu)\cos\frac{\theta}{2} - (9-\nu)\cos\frac{5\theta}{2}] \\
& + \frac{\overline{K}_2^{(b)}}{12(1-\nu^2)\overline{D}} \left(\frac{r}{a}\right)^3 [3(1-\nu)\cos\theta + (1+3\nu)\cos 3\theta] + \overline{a} \left(\frac{r}{a}\cos\theta + \overline{a}_2 \left(\frac{r}{a}\right)^2 \cos 2\theta\right) \\
\psi_z = \psi_{z0} & + \frac{(1+\nu)}{2(1-\nu^2)\overline{D}} \overline{K}_{\frac{1}{2}}^{(b)} \left(\frac{r}{a}\right)^{\frac{1}{2}} [(2\chi-1)\cos\frac{\theta}{2} - \cos\frac{3\theta}{2}] + \frac{2}{(1-\nu^2)\overline{D}} \overline{K}_1^{(b)} \left(\frac{r}{a}\right) \cos\theta \\
& + \frac{1+\nu}{6(1-\nu^2)\overline{D}} \overline{K}_{\frac{3}{2}}^{(b)} \left(\frac{r}{a}\right)^{\frac{3}{2}} [(2\chi+1)\cos\frac{3\theta}{2} - 3\cos\frac{\theta}{2}] \\
& + \frac{1}{2(1-\nu^2)\overline{D}} \overline{K}_2^{(b)} \left(\frac{r}{a}\right)^2 [(3+\nu)\cos 2\theta - (1+\nu)] + \overline{a}_1 \mu^2 \left(\frac{r}{a}\right)^2 (\cos 2\theta - 1) \\
\psi_r = \psi_{r0} & + \frac{(1+\nu)}{2(1-\nu^2)\overline{D}} \overline{K}_{\frac{1}{2}}^{(b)} \left(\frac{r}{a}\right)^{\frac{1}{2}} [(2\chi+1)\sin\frac{\theta}{2} - \sin\frac{3\theta}{2}] - \frac{2\nu}{(1-\nu^2)\overline{D}} \overline{K}_1^{(b)} \left(\frac{r}{a}\right) \sin\theta \\
& + \frac{1+\nu}{6(1-\nu^2)\overline{D}} \overline{K}_{\frac{3}{2}}^{(b)} \left(\frac{r}{a}\right)^{\frac{3}{2}} [(2\chi-1)\sin\frac{3\theta}{2} - 3\sin\frac{\theta}{2}] \\
& - \frac{\nu}{(1-\nu^2)\overline{D}} \overline{K}_2^{(b)} \left(\frac{r}{a}\right)^2 \sin 2\theta
\end{aligned}$$

(ii) 应力模式

$$\begin{aligned}
N_x & = \overline{K}_{\frac{1}{2}}^{(e)} \left(\frac{r}{a}\right)^{-\frac{1}{2}} \left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{5\theta}{2}\right) + 2\overline{K}_1^{(e)} \\
& \quad + \overline{K}_{\frac{3}{2}}^{(e)} \left(\frac{r}{a}\right)^{\frac{1}{2}} \left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right) + 2\overline{K}_2^{(e)} \left(\frac{r}{a}\right) \cos\theta \\
N_y & = \overline{K}_{\frac{1}{2}}^{(e)} \left(\frac{r}{a}\right)^{-\frac{1}{2}} \left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{5\theta}{2}\right) + \overline{K}_{\frac{3}{2}}^{(e)} \left(\frac{r}{a}\right)^{\frac{1}{2}} \left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right) \\
N_{xy} & = -\overline{K}_{\frac{1}{2}}^{(e)} \left(\frac{r}{a}\right)^{-\frac{1}{2}} \left(\frac{1}{4}\sin\frac{\theta}{2} - \frac{1}{4}\sin\frac{5\theta}{2}\right) \\
& \quad - \overline{K}_{\frac{3}{2}}^{(e)} \left(\frac{r}{a}\right)^{\frac{1}{2}} \left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right) - 2\overline{K}_2^{(e)} \left(\frac{r}{a}\right) \sin\theta \\
M_x & = -\left[\overline{K}_{\frac{1}{2}}^{(b)} \left(\frac{r}{a}\right)^{-\frac{1}{2}} \left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{5\theta}{2}\right)\right. \\
& \quad \left. + 2\overline{K}_1^{(b)} + \overline{K}_{\frac{3}{2}}^{(b)} \left(\frac{r}{a}\right)^{\frac{1}{2}} \left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right) + 2\overline{K}_2^{(b)} \left(\frac{r}{a}\right) \cos\theta\right] \\
M_y & = -\left[\overline{K}_{\frac{1}{2}}^{(b)} \left(\frac{r}{a}\right)^{-\frac{1}{2}} \left(\frac{3}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{5\theta}{2}\right) + \overline{K}_{\frac{3}{2}}^{(b)} \left(\frac{r}{a}\right)^{\frac{1}{2}} \left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right)\right] \\
M_{xy} & = \overline{K}_{\frac{1}{2}}^{(b)} \left(\frac{r}{a}\right)^{-\frac{1}{2}} \left(\frac{1}{4}\sin\frac{\theta}{2} - \frac{1}{4}\sin\frac{5\theta}{2}\right) + \\
& \quad + \overline{K}_{\frac{3}{2}}^{(b)} \left(\frac{r}{a}\right)^{\frac{1}{2}} \left(\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right) \\
& \quad + 2\overline{K}_2^{(b)} \left(\frac{r}{a}\right) \sin\theta + 2\mu^2(1-\nu)\overline{D}\overline{a}_1 \left(\frac{r}{a}\right) \sin\theta \\
Q_z & = -\frac{1}{2R} \overline{K}_{\frac{1}{2}}^{(e)} \left(\frac{r}{a}\right)^{\frac{1}{2}} (3\cos\frac{\theta}{2} + \cos\frac{3\theta}{2})
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{6R}\overline{K}_3^{(e)}\left(\frac{r}{a}\right)^{\frac{3}{2}}\left(3\cos\frac{\theta}{2}+\cos\frac{3\theta}{2}\right)-\frac{\overline{K}_2^{(e)}}{2R}\left(\frac{r}{a}\right)^2(1-\cos 2\theta) \\
& +\frac{\overline{C}}{(1-\nu^2)\overline{D}}\overline{K}_1^{(b)}\left(\frac{r}{a}\right)^{\frac{1}{2}}\left(\cos\frac{3\theta}{2}-5\cos\frac{\theta}{2}\right) \\
& +\frac{\overline{C}}{(1-\nu^2)\overline{D}}\overline{K}_3^{(b)}\left(\frac{r}{a}\right)^{\frac{3}{2}}\left(3\cos\frac{\theta}{2}-7\cos\frac{3\theta}{2}\right) \\
& +\frac{\overline{C}}{(1-\nu^2)\overline{D}}\overline{K}_2^{(b)}\left(\frac{r}{a}\right)^2(1-\cos 2\theta)+\overline{C}\left[1+\mu^2\left(\frac{r}{a}\right)^2(1-\cos 2\theta)\right]\overline{\alpha}_1 \\
& +2\overline{C}\overline{\alpha}_2\left(\frac{r}{a}\right)\cos\theta \\
Q_r = & -\frac{1}{2R}\overline{K}_1^{(e)}\left(\frac{r}{a}\right)^{\frac{1}{2}}\left(\sin\frac{\theta}{2}+\sin\frac{3\theta}{2}\right)-\frac{2}{R}\overline{K}_1^{(e)}\left(\frac{r}{a}\right)\sin\theta \\
& -\frac{1}{2R}\overline{K}_3^{(e)}\left(\frac{r}{a}\right)^{\frac{3}{2}}\left(\sin\frac{\theta}{2}+\sin\frac{3\theta}{2}\right)-\frac{1}{R}\overline{K}_2^{(e)}\left(\frac{r}{a}\right)\sin 2\theta \\
& +\frac{\overline{C}}{(1-\nu^2)\overline{D}}\overline{K}_1^{(b)}\left(\frac{r}{a}\right)^{\frac{1}{2}}\left(\sin\frac{\theta}{2}+\sin\frac{3\theta}{2}\right) \\
& +\frac{\overline{C}}{(1-\nu^2)\overline{D}}\overline{K}_3^{(b)}\left(\frac{r}{a}\right)^{\frac{3}{2}}\left(\sin\frac{\theta}{2}+\sin\frac{3\theta}{2}\right)-2\overline{\alpha}_2\left(\frac{r}{a}\right)\sin\theta
\end{aligned}$$

参 考 文 献

- [1] 柳春图等, 中国科学, 4(1987), 389-397
- [2] Follas, E. S., Int. J. Fract., 1(1965), 20-46
- [3] Erdogan, F. and Kibler, J., Int. J. Fract., 5(1969), 229-237
- [4] Sih, G. C. and Hagendoif, Thin Shell Structure, 1974, 365-375
- [5] Delale, F. and Erdogan, F., Int. J. Solids Struct., 15(1979), 907-926
- [6] 朱伯芳, 《有限单元法原理与应用》, 水利电力出版社, 1979
- [7] J. Rice, Int. J. Solids Struct., 8(1972), 751
- [8] W. 弗留盖着, 壳体中的应力, 中国工业出版社, 1965
- [9] J. Petroski and J. Achenbach, Eng. Fract. Mech. 10(1978), 257
- [10] Grandt, A. F., Int. J. Fract., 11(1975), 283
- [11] 荆树峰译, 国外压力容器缺陷评定标准, 劳动出版社, 1982

Study on Spherical Shells with Cracks Emanating From a Hole

Liu Chuntu Han Wensheng

(Institute of Mechanics, Chinese Academy of Sciences)

Abstract

On the basis of the expansion of stress-strain fields at crack tip in a spherical shell [1], the stress intensity factor of cracked spherical shells which have cracks emanating from a hole is obtained by using the local-global analysis method and the weight function method. Transverse shear deformations are taken into account in finite element modes. The choosing of the length of the expansion of stress-strain fields and the size of special element are analysed.

Key words: local-global analysis method, weight function method, SIF.

Sudden Unbalance Responses of Flexible Rotor Supported in Squeeze Film Dampers

Meng Guang

(Vibration Research Center, Northwestern Polytechnical University)

Abstract

In order to express the continuous process of sudden unbalance, a real function which can be differentiated for infinite times is introduced. It not only has the properties of step function, but can hold the continuity of the jumping process of sudden unbalance.

The sudden unbalance responses of flexible rotor—SFDB system are analysed in this paper. For the case of constant rotating speed, it is found that the larger the bearing parameter B , stiffness ratio K , external damping ratio and mass ratio and the smaller the quantity of sudden unbalance, the shorter the transient process of sudden unbalance response and the smaller the transient amplitude. But the quantity of sudden unbalance has no influence on the new steady state response.

Sudden unbalance may cause nonsynchronous response to the system. If the new steady state response after sudden unbalance is still nonsynchronous, the amplitude of the new steady state response is almost the same with the transient amplitude although the transient process is shorter.

For the case of accelerating through bistable region, if the sudden unbalance occurs before bistable region, as the sudden unbalance may cause large vibration amplitude, the system response will jump to the large solution of the bistable response and pass through the bistable region along the large solution with the increasing of rotating speed, then it will jump down to the small solution of the